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CONTROL OF UNDERACTUATED ACROBOT SYSTEM USING PARTIAL FEEDBACK LINEARIZATION

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ABSTRACT

Underactuated mechanical systems are those have fewer actuators than degrees of freedom. They arise in applications, such as space robots, mobile robots, flexible robots, walking and gymnastic robots, such as the Acrobot. The swing-up control problem is studied for the Acrobot system; as one of the most important benchmark problem of the underactuated mechanical systems. The dynamic model based on Lagrange formulation is present. Then, the design of the swing up control provided. The design methodology is based on applying partial feedback linearization in a first stage to linearized the unactuate degree of freedom pushing the Acrobot as near as possible to its equilibrium point. Then, switch to a balancing controller linear quadratic regulator (*LQR*), which forces the Acrobot to reach its equilibrium upward position.

KEY WORDS

Underactuated mechanical systems, Acrobat, Swing up control, Partial feedback linearization.

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NOMENCLATURE

F	The vector of the actuated and unactuated joints.
$h(q,\dot{q})$	Coriolis and centrifugal terms.
I_1	Moment of inertia of the first link. [kg m^2]
I_2	Moment of inertia of the second link. [kg m^2]
k_d , k_p	Positive gains.
LQR	Linear quadratic regulator.
l_1	Length of first link. [m]
l_2	Length of second link. [m]
l_{c1}	Length of the center of mass of first link. [m]
l_{c2}	Length of the center of mass of second link. [m]
$M(q)$	The inertia matrix.
m_1	Mass of the first link. [kg]
m_2	Mass of the second link. [kg]
p_e	The vertical equilibrium point.
q	The vector of generalized coordinate.
q_1	The rotation angle of the first link. [rad]
q_2	The rotation angle of the second link. [rad]
$q_{1d}(t)$	The reference trajectory for q_1.
x	The system variables.
x_L	The linearized system of the Acrobot about the vertical equilibrium point.
u_1	The outer loop control term.
$\Phi(q)$	The terms derived from the potential energy.
τ	The input generalized force.

INTRODUCTION

The underacrtuated mechanical systems are mechanical systems with fewer controls (actuators) than the degree of freedom of the system. One of the benchmark of the underactuated mechanical systems is the Acrobot (for acrobatic-robot) shown in Figure 1. The Acrobot is a highly simplified model of a human gymnast performing on a single parallel bar. By swinging her legs (a rotation at the hip) the gymnast is able to bring herself into a completely inverted position with her straightened legs pointing upwards and her center of mass above the bar. The Acrobot consists of a simple two link manipulator operating in a vertical plane. The first joint (corresponding to the gymnast's hand sliding freely on the bar) is free to rotate. A motor is mounted at the second joint (between the links) to provide a torque input to the system (corresponding to the gymnast's ability to generate torques at the hip).

The Acrobat dynamics are complex enough to yield a rich source of nonlinear control problems, in the same times it is simple enough to permit a complete mathematical analysis. Numerous researchers have proposed control strategies for the swing up control based on the energy of the system. The Acrobot was first studied by Hauser and Murray, comparing between linearization about an equilibrium point and the technique of feedback linearization, see Ref. [1]. Later, Spong put an algorithm for the swing up control of the Acrobot [2- 3- 4- 5- 6]. After that, Xin and Kaneda (2007) provide an analysis of the energy- based swing up control of the Acrobot [7].

ACROBOT DYNAMICS

For fully actuated mechanical systems a broad range of powerful techniques were developed in the last decade for the design of optimal, robust, adaptive, and learning controllers [5]. These techniques are possible because fully actuated systems possess a number of strong properties that facilitate control design, such as feedback linearization, passivity, matching conditions, and linear parameterized. For underactuated systems one or more of the above structural properties are usually lost. Moreover, undesirable properties such as higher relative degree and non-minimum phase behavior are manifested. For these reasons, control design becomes much more difficult and there are correspondingly fewer results available.

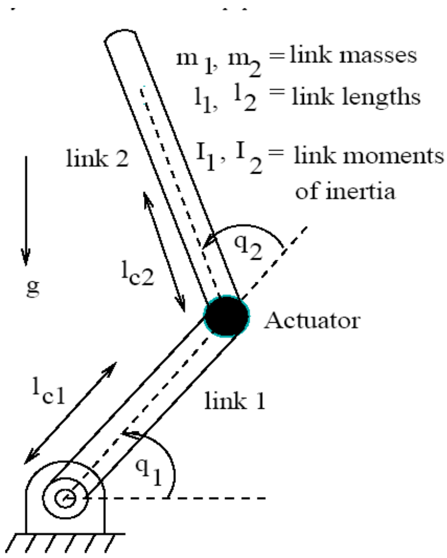

Fig. 1. The Acrobot.

Consider the Lagrange formulation of the dynamics of an n-degree-of-freedom mechanical system

$$D(q)\ddot{q}+c(q,\dot{q})\dot{q}+g(q)=B(q)\tau \quad (1)$$

where, $q \in R^n$ is the vector of generalized coordinate, $\tau \in R^m$ is the input generalized force (m<n), and $B(q) \in R^{n \times m}$ has full rank for all *q*.

For a suitable partition of the vector *q* of generalized coordinates as $q=(q_1,q_2)^T$, where $q_1 \in R^{n-m}$ and $q_2 \in R^m$, the system (1) may be written as the following:

$$M(q)\ddot{q}+h(q,\dot{q})+\Phi(q)=F\tau \quad (2)$$

where $h(q,\dot{q})$ include Coriolis and centrifugal terms, and $\Phi(q)$ contains the terms derived from the potential energy, such as gravitational and elastic generalized forces. The *M (q)*$_{(m\times m)}$ inertia matrix in generalized coordinates of the system, is symmetric, positive definite inertia matrix. The *F* vector refers to the actuated and unactuated joints.

The Acrobot model is a two link planar robot arm with an actuator at the elbow (joint 2) and no actuator at the shoulder (joint 1). The equations of the motion of the system are [2, 7, 8];

$$m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 + h_1 + \phi_1 = 0 \tag{3}$$

$$m_{21}\ddot{q}_1 + m_{22}\ddot{q}_2 + h_2 + \phi_2 = \tau \tag{4}$$

where:

$$m_{11} = I_1 + I_2 + m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos(q_2))$$
$$m_{12} = m_{21} = I_2 + m_2(l_{c2}^2 + l_1 l_{c2}\cos(q_2))$$
$$m_{22} = I_2 + m_2 l_{c2}^2$$
$$h_1 = -m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2^2 - 2m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2\dot{q}_1$$
$$h_2 = m_2 l_1 l_{c2}\sin(q_2)\dot{q}_1^2$$
$$\phi_1 = (m_1 l_{c1} + m_2 l_1)g\cos(q_1) + m_2 l_{c2} g\cos(q_1 + q_2)$$
$$\phi_2 = m_2 l_{c2} g\cos(q_1 + q_2)$$

CONTROLLER DESIGN

The control objective is to swing up the Acrobot from the vertically downward position to the vertically upward equilibrium position and then balance it at that position. This is termed as the swing-up phase and the capture phase respectively. The system is first mapping to partial feedback linearization, which is a consequence of positive definiteness of the inertia matrix [2]. Then, the balancing of the system can be achieved using a linear feedback control.

A number of previous studies of underactuated mechanical systems have been shown that the Acrobot dynamics are not feedback linearization with static state feedback and nonlinear coordinate transformation [1- 3- 9]. However, a linear response from one of the degree of freedom by suitable nonlinear feedback could be achieved.

The Non-Collocated Acrobot System

The non-collocated feedback linearization refers to linearized the unactuated degree of freedom q_1 which could be achieved under a special assumption on the inertia matrix of the Acrobot dynamics.

Consider equation (3),

$$m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 + h_1 + \phi_1 = 0 \quad (3)$$

where, m_{12} is nonzero for all values of q_2. This condition is defined by "Spong" as "*strong inertial coupling*". This condition satisfies under some restriction on the inertia parameters of the Acrobot; which is $I_2 > m_2 l_{c2}(l_1 - l_{c2})$.

From equation (3),

$$\ddot{q}_2 = \frac{-1}{m_{12}}\left(m_{11}\ddot{q}_1 + h_1 + \phi_1\right) \quad (5)$$

Substitute the expression (5) into (3) to obtain,

$$\overline{m_1}\ddot{q}_1 + \overline{h_1} + \overline{\phi_1} = \tau \quad (6)$$

where,

$$\overline{m_1} = m_{21} - \frac{m_{22}m_{11}}{m_{12}}$$

$$\overline{h_1} = h_2 - \frac{m_{22}h_1}{m_{12}}$$

$$\overline{\phi_1} = \phi_2 - \frac{m_{22}\phi_1}{m_{12}}$$

Note that, the term $\overline{m_1}$ can be shown to be strictly positive as a consequence of the positive definitions of the Acrobot inertia matrix and "*strong inertia coupling*". A feedback linearization controller can therefore defined as,

$$\overline{m_1}u_1 + \overline{h_1} + \overline{\phi_1} = \tau$$

where, u_1 is the outer loop control term that will be used to complete the generation of the swing up control low. The mapping system is given by:

$$m_{12}\ddot{q}_2 + h_1 + \phi_1 = -m_{11}u_1 \quad (7)$$

$$\ddot{q}_1 = u_1 \quad (8)$$

$q_{1d}(t)$ is defined as the reference trajectory for q_1, the tracking error for the system could be defined as,

$$y = q_1 - q_{1q} \quad (9)$$

Then, the input term can be chosen as:

$$u_1 = \ddot{q}_{1d} + k_d(\dot{q}_{1d} - \dot{q}_1) + k_p(q_{1d} - q_1) \quad (10)$$

where, k_d and k_p are positive gains. Therefore, the state variables of the new system are given as:

$$z_1 = q_1 - q_{1q}$$
$$z_2 = \dot{q}_1 - \dot{q}_{1d}$$
$$w_1 = q_2$$
$$w_2 = \dot{q}_2$$

The closed loop system may be written as:

$$\dot{z}_1 = z_2 \quad (11)$$

$$\dot{z}_2 = -k_p z_1 - k_d z_2 \quad (12)$$

$$\dot{w}_1 = w_2 \quad (13)$$

$$\dot{w}_2 = -\frac{1}{m_{12}}(h_1 + \phi_1) - \frac{m_{11}}{m_{12}}(\ddot{q}_{1d} + k_d \dot{z}_1 + k_p z_1) \quad (14)$$

Note that, a linear response from the first degree of freedom could be achieved even though it is not directly actuated but driven by the coupling forces arising from the motion of the second link. The motion of the second link may be complex and precisely defines the zero dynamics of the system. Therefore, the analysis of the zero dynamic is important to understand the behavior of the system.

Analysis of the Zero Dynamics

The zero dynamics analysis discuss in more detail the internal dynamics of the systems controlled via input-output linearization. The name "zero dynamics" is due to its relation to output zeroing and its relation to transmission zeroes [10].

The zero dynamics, with respect to the output $y = z_1$ are computed by specifying that the q_1 identically track the reference trajectory q_{1d}, so that;

$$y = z_1 = q_1 - q_{1q} = 0 \quad (15)$$

The dynamics are given by:

$$\dot{w} = f(0, w) \quad (16)$$

and are referred to the zero dynamics with respect to the output *y*. In case of swing up control, the value of reference trajectory is set to be; $q_{1d} = \frac{\pi}{2}$ (vertical upward position of the first link). The expression for the zero dynamics of the system:

$$(I_2 + m_2(l_{c2}^2 + l_1 l_{c2}\cos(q_2)))\ddot{q}_2 - m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2^{\,2} - m_2 l_2 g\sin(q_2) = 0 \tag{17}$$

The system has two equilibrium points at P_1 *(0,0)*, which is a saddle point and P_2 *(π,0)*, which is a center point as shown in Fig. 2. The phase portrait of the zero dynamics show that the typical steady state behavior for the first link to converge exponentially to (π/2) while the second link oscillate, either about the center point P_1 or outside the hemicyclic orbit of the saddle point P_2.

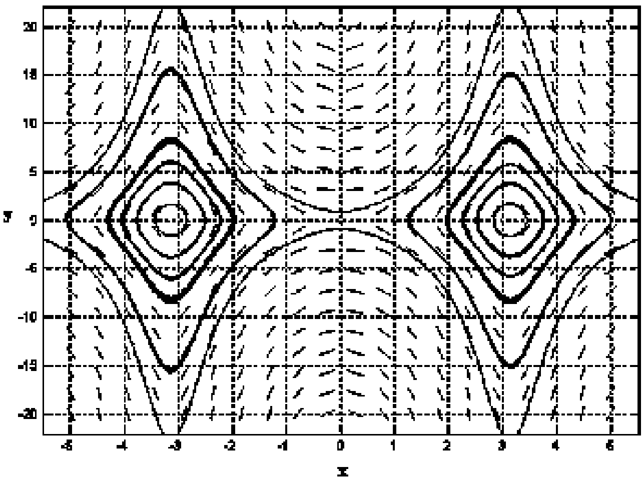

Fig. 2. Phase portrait of the zero dynamics.

The equation of the zero dynamics is independent of the gains k_d and k_p used in the outer loop control u_1. However, those gains together with the initial conditions, completely determine the particular trajectory of the zero dynamics to which the response of the complete system converges.

The steps for the swing up control design are; first, determine an appropriate set of gains k_d and k_p for the outer loop control to move the Acrobot as close as possible to its saddle point equilibrium. Then, switch from the partial feedback linearization controller to a linear quadratic regulator (*LQR*) designed to balance the Acrobot about this equilibrium point.

The Balancing Controller

The Acrobot system could be written as a standard nonlinear system, affine in the control (u=τ) [1].

$$\dot{x} = F(x) + g(x)u \tag{18}$$

where, *x* is the system variables defined as, $x = (q_1 - q_{1e}, q_2, \dot{q}_1, \dot{q}_2)^T$.

Linearized the Acrobot system (18) a bout the vertical equilibrium point $p_e = (q_{1e}, q_{2e}) = (\pi/2, 0)$; using the given parameters of the Acrobot; give the following controllable linear system:

$$\dot{x}_L = Ax_L + Bu \tag{19}$$

where, x_L is the linearized system of the Acrobot about the vertical equilibrium point.

A linear quadratic regulator (*LQR*) controller is used to balance the system at the vertical upward equilibrium position.

SIMULATION RESULTS

The simulations were performed using Matlab. The Acrobot parameters are shown in Table 1.

Table 1. Acrobot Parameters.

Link No.	m [kg]	l [m]	l_c [m]	I [kg m^2]
Link1	1	1	0.5	0.08
Link2	2	2	1	0.33

First, a study of the effect of the partial feedback linearization controller with several values of the gains k_d and k_p, shown their effect on the response of the rotation of the second link about its equilibrium value. Figure 3 shows the response of the partial feedback linearization controller for gains $k_d = 20$ and $k_p = 8$. The second link rotates 2π in the steady state.

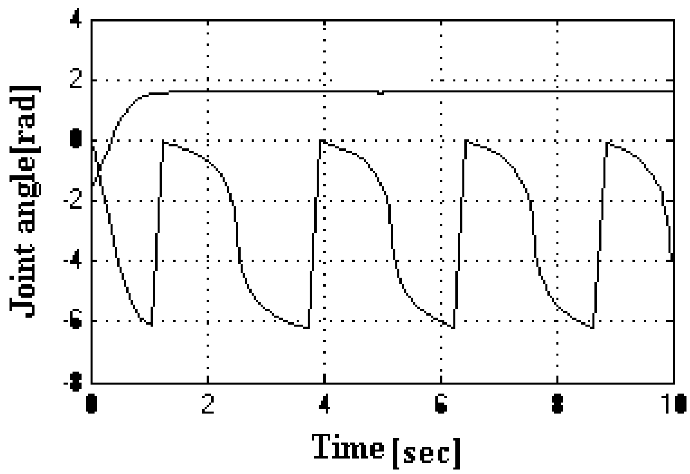

Fig. 3. Partial feedback linearization response.

The linear model about the equilibrium point (19) is given by:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 11.76 & -17.64 & 0 & 0 \\ -13.23 & 38.22 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ -2.1 \\ 4.05 \end{bmatrix}$$

The weighting and gain matrices for the LQR controller are given by:

$Q = I_{4\times4}, \quad R = 1$, yielding the state feedback controller $u = -kx$,

where

$$k = [-541.9 \quad -240.9 \quad -243.3 \quad -121.6].$$

Then, The linear control low is switched on when the Acrobot became near to its vertical position, $q_2 \approx \pm 0.174[rad] = \pm 10[\deg]$. Figure 4 shows that the partial feedback linearization followed by the balancing controller (*LQR*) succeeds to force the Acrobot to reach its vertical upward position.

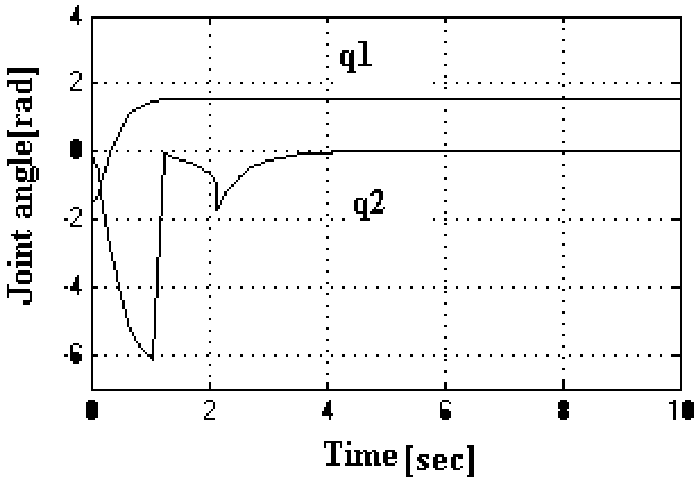

Fig. 4. Swing up and balance of the Acrobot.

CONCLUSION

A swing up strategy for the Acrobot is presented, which uses the partial feedback linearization to move the Acrobot near to its equilibrium upward point as possible as it could, then switching to a balance controller (*LQR*) to force the Acrobot to reach its upward position in steady state response. As, most researches are focus on using a linear quadratic regulator (*LQR*) as a balancing controller for the second stage of the swing up control technique, a further study is investigating the effect of several control methods; as a balancing control; in the Acrobot performance. Also, a further study is to apply one of the intelligent controls (fuzzy control) to the Acrobot system and compare the performance in those different cases.

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