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MONTE CARLO SIMULATIONS FOR EVALUATION OF UNCERTAINTY EFFECTS IN STRUCTURAL DESIGNS

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ABSTRACT

This paper presents a practical approach for evaluating the effect of uncertainty in design variables on the behaviour of structure. The proposed approach shows how Monte Carlo simulations can be effectively used to evaluate the uncertainty effects. A case study of a structure under static loading has been used to demonstrate the approach. The results have shown that using the proposed approach can serve as a practical tool for virtual testing of structure designs. This will help the designer in estimating and understanding the complicated and unforeseen uncertainty effects on structure design performance.

KEY WORDS

Monte Carlo simulation; Structure analysis; Structure design; Design uncertainty; Approximate methods; Response surface methods; Design of experiments

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INTRODUCTION

Uncertainties in design variables can have a profound effect on structure performance. This effect can even be catastrophic as in the two space shuttle disasters; Challenger in 1986 and Columbia in 2003 where unforeseen variations in system conditions lead to two major disasters [1].

The existence of uncertainties in structure design is inevitable. Sources of uncertainties include but not limited to: loads, geometry and material data. A formal design approach for accounting for uncertainties is the safety factor approach which is usually based on experience. The safety factor is easy to implement and is largely used. However the designer is usually unaware of the effects of uncertainties within design and/or system variables. Understanding these effects will help the designer in developing better and cost effective designs [2].

The analysis of structure design under uncertainty is essentially based of probability theory. The main idea is to predict the likelihood of the occurrence of structure performance criterion (e.g., reaching yield stress and/or resonance frequency, etc.) under uncertainties. This study is usually termed structural reliability and the reader is directed to textbooks [3, 4] for more details on the topic.

The study of structure behavior under uncertainties can be organized in three steps. Firstly, design variables are identified where each variable is assigned a distribution range. For example, a variable may be assigned a Gaussian distribution with mean and standard deviation values. Secondly, design variables are implemented in the evaluation of structure response. This step can be executed analytically but only for simple structures with simple analytical solutions. However, for complex structures where finite element analysis is employed to evaluate structure response, this step is impractical due to the large computational cost. In this paper a method for overcoming this difficulty application is presented in the following section. Thirdly, statistical analysis is used to analysis the resulted structure response under uncertainties in the design variables. This step can help the designer in answering some questions, such as: Which design variable has the largest and/or smallest influence on structure response? Will the structure fail under given values of uncertainties? What is the probability of this structure failure? The designer can use this information in developing better and cost effective designs. In this paper these questions are examined in a case study.

The proposed method replaces the computationally expensive finite element model with a data driven model which is an approximate (meta-model) model, i.e., model of a model. Approximate models are easy to calculate models instead of the difficult to calculate initial models. The process of developing approximate models starts with producing or gathering system response data at different points in the design space. The quality of the approximate models largely depends on the location of these points; hence the science of design of experiments (DOE) [5, 6] is employed to maximize the information gain with minimum number of points.

There are several techniques for building approximate models, the most known are response surface method [7, 8], Kriging [9] and Neural networks [10, 11]. For a comprehensive review of the topic of approximate models and their applications, the reader is referred to references [12, 13] for more information. The response surface method (RSM) has been selected here as the technique for building the approximate model. RSM is an approximation technique used to construct response functions in the form of polynomial (usually second order) functions. The model can be written in the following form:

$$\hat{y} = X\beta \tag{1}$$

where X is the $n \times m$ design matrix, in which n is the number of undertaken experiments and m is the number of unknown model coefficients β and $\hat{y}$ is a vector containing the approximate response values. Regression analysis [14] is used to find the values of β those minimizing the value of error between actual response values (y) and approximate values $\hat{y}$. The values of β can be calculated using the following equation:

$$\beta = (X^T X)^{-1} X^T y \tag{2}$$

METHOD

The proposed method is applied to a case study of designing a hook under static loading and is illustrated in Fig. 1. At first the hook is modeled using finite element analysis and ANSYS® software is used for simulations [15]. Design of experiments is used to strategically locate the design points in the design space which will enable the building of the approximate model with minimum number of simulations. Regression analysis is then used to find the values of the unknown coefficients (β) previously mentioned in Equation (1). The response surface model can then be used to simulate the structure at as many different design points as may be required by Monte Carlo simulations since the computational cost is negligible compared to the initial FE model. Finally, the simulation results are analyzed using descriptive statistics and useful information are extracted and serve as useful tools for component development.

MODEL DESCRIPTION

The hook as shown in Fig. 2 is required to hold a load of 6000 N and four geometric dimensions are selected as design variables. The initial values of the baseline design variables are: Angle (135^0), thickness (20 mm), depth (20 mm) and lower radius (50 mm). The hook is made from steel with Young's modulus of elasticity (200 GPa), Yield strength (250 MPa). The CAD model has been created using ANSYS design modeler software where the aforementioned design variables have been expressed as variables and not as only numerical values, which facilitated running ANSYS at different design points in batch mode. The hook is modeled using ANSYS® software where 14424 solid elements with 23985 nodes are used to mesh the hook model. The inner cylinder is constrained in both radial and axial directions and the load is applied as a concentrated load at the inner area of the hook as shown in Fig. 3. ANSYS® runs finite element simulations to calculate the Von-Mises stress in the hook body. The maximum equivalent stress value is used to calculate the safety factor based on maximum equivalent stress failure theory for ductile materials.

APPROXIMATE MODEL CREATION

In order to create the approximate models, ANSYS® must be run at different design points. To facilitate this process, the four design variables and the safety factor are all coded as parameters in the ANSYS® code.

A central composite design is an efficient DOE method for developing response surface models [16]. There are several types of approximate models; linear, interaction, quadratic...etc. [17]. An interaction model is selected to capture both main and interaction effects of the design variables. A typical interaction model containing four design variables is represented as follows:

$$\hat{y} = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \\ a_{12}x_1x_2 + a_{13}x_1x_3 + a_{14}x_1x_4 + a_{23}x_2x_3 + a_{24}x_2x_4 + a_{34}x_3x_4 \quad (3)$$

Finite element simulations are conducted at each *DOE* design points. Upper and lower limits on each design variable are provided in Table 2, which also shows output response (*mass* and safety factor (*SF*)) values.

Two approximate models are created; one for mass and the other is for safety factor (SF). Regression analysis is used to calculate the unknown coefficients in Equation (3) for each model. The quality of the models (goodness of fit) is calculated using the adjusted R-square ($adjR^2$) value. This value is calculated using the following equation:

$$adjR^2 = 1 - \frac{\dfrac{SSE}{(n-p)}}{\dfrac{SST}{(n-1)}} \quad (4)$$

where

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \quad (5)$$

and

$$SST = \sum_{i=1}^{n} (y_i - \bar{y}_i)^2 \quad (6)$$

where y_i is response value computed from ANSYS® simulation, $\hat{y}$ is the approximate response value evaluated using the approximate model, $\bar{y}_i$ is the mean value of the response, n is the number of design points used to create the approximate model and p is the number of unknown approximate model coefficients.

MONTE CARLO SIMULATIONS

Monte Carlo simulations (MCS) can be defined as using randomly generated variables to estimate mathematical quantities. The term *Monte Carlo* first appeared in a paper published by Metropolis and Ulam in 1949 [18]. For more details on the method, the reader is referred to Ref. [19] written by Sobol, I.M. in a concise yet informative form.

In the present work, Monte Carlo simulations are applied through the following steps. First, each design variable is given a statistical distribution and all possible combinations are obtained as the design space for MCS. Then, using the approximate models developed in the former section, mass and safety factor can be evaluated at a small computational cost compared with running the software analysis, which is very important since MCS require very large number of evaluations. Finally, the results are analyzed using tools such as descriptive statistics and probability analysis. In the present work, all design variables are assigned normal distribution where mean values are the baseline design variables values with six standard deviations encompassing the range of each variable as shown in MATLAB® is used for running the numerical calculations and statistical analysis.

RESULTS AND DISCUSSIONS

The results shown in Table 2 are used to create the approximate model described in Equation (3). The $adjR^2$ values for mass and SF are 0.999 and 0.989 respectively. These values very close to one imply an excellent model representation. In other words, the approximate models can explain almost 100% of the model variation with design variable value changes.

Also the approximate model is used in estimating the relative importance of each design variable. This is accomplished by comparing the coefficients (a_1, a_2 through a_3a_4) in Equation (3). The values are given in Table and illustrated by a bar graph in Fig. 4. It is clear that the values of coefficients 1, 2 and 4 are the highest among all values, which indicate that the design variables associated with these coefficients (Thickness, Depth and Radius), are the most significant. It can also be deduced that interaction effects between design variables can be neglected since the values of the coefficients of interacting variables are relatively small and hence they can be safely ignored in the MCS. This will effectively minimize the required computations since the number of total function evaluations in MCS is N^D, where N is the number of levels within the range of D design variables. In the present study, assuming there are 10 levels with 4 design variables which gives 10^4 function evaluations, dropping the (Angle) variable will reduce this number to 10^3. New response surface models are created based on the three important variables (Thickness, Depth and Radius). The adjusted coefficient of determination $adjR^2$ is used to quantify the new models', the values in

Table 4 shows that the new models for both mass and SF still have an excellent quality and can be used in MCS.

In conducting MCS, each variable range is divided into ten levels spanning the six standard deviations around the values of the initial design variables. MATLAB® is used to run the numerical calculations of the response surface models of mass and SF in the form of coded polynomial functions of the three design variables (Thickness, Depth and Radius). It should be noted that using approximate models has largely reduced the computational time from 12 seconds when using ANSYS® to run the finite element model to 40 seconds for running all 1000 response surface models simulations using MATLAB®. This enabled the practical evaluation of the design at numerous design points. Moreover, by calculating the relative importance of the design variables based on polynomial coefficients enabled screening out unimportant variables, thus reducing the number of simulations required by MATLAB® by 10 folds from 10^4 to 10^3. The MCS results are analyzed and descriptive statistics, shown in

Table 5, are used to assess the effect of the uncertainties within the design variables on both mass and SF. It can be deduced that uncertainties within the design variables have affected the results to a great extent. The mean value of the safety factor (SF) has dropped to 0.52 which signals an alarm that the structure will fail in many cases as can be seen in Fig. . To calculate the probability of failure probabilistic analysis is conducted and the accumulative probability is plotted Fig. 6. It can be seen that almost half of the designs will fail (SF < 1). There is 55.4% probability that the design will fail or in other words, there is only a 45.6% probability that the structure will survive.

CONCLUSIONS

The effect of uncertainties within design variables on structural designs performance cannot be neglected. In the present work, a practical methodology has been introduced to assess this effect. The methodology succeeded in helping the designer to estimate the relative importance of the different design variables. It also enabled the computation of the expected probability of failure/survivor of the design. Present work will serve as a basis for further study on the counter measures which the designer can take to decrease the probability of failure, or to design a structure to reach a certain survivor probability.

REFERENCES

[1] Choi, S.-K., R.A. Canfield, and R.V. Grandhi, Reliability-based Structural Design. 2007, London: Springer-Verlag London Limited.
[2] Bergman, B., Robust design methodology for reliability : exploring the effects of variation and uncertainty. 2009, Chichester: Wiley.
[3] Lemaire, M., Structural reliability. 2009, London; Hoboken, NJ: ISTE ; Wiley.
[4] Thoft-Christensen, P. and M.J. Baker, Structural reliability theory and its applications. 1982, Berlin; New York: Springer-Verlag.
[5] Antony, J., Design of Experiments for Engineers and Scientists. 2003: {Butterworth-Heinemann}.
[6] Fisher, R.A., The Design of Experiments. 1971: Macmillan Pub Co.

[7] Myers, R.H. and D.C. Montgomery, Response surface methodology: process and product optimization using designed experiments. 1995: Wiley.
[8] Myers, R.H., et al., Response surface methodology: A retrospective and literature survey. Journal of quality technology, 2004. **36**(1): p. 53-77.
[9] Bakker, T., Design Optimization with Kriging Models. 2000, Delft University.
[10] Gurney, K., An Introduction to Neural Networks. 1997: {CRC}.
[11] El-Gindy, M. and L. Palkovics, Possible Application of Artificial Neural Networks to Vehicle Dynamics and Control: A Literature Review. International Journal of Vehicle Design, 1993. **14**(5/6): p. 592-614.
[12] Simpson, T.W., et al., Approximation methods in multidisciplinary analysis and optimization: a panel discussion. Structural and Multidisciplinary Optimization, 2004. **27**(5): p. 302-313.
[13] Simpson, T.W., et al., Metamodels for computer-based engineering design: survey and recommendations. Engineering with Computers, 2001. **17**(2): p. 129-150.
[14] Montgomery, D.C., E.A. Peck, and G.G. Vining, Introduction to linear regression analysis. 2006: J. Wiley & Sons.
[15] ANSYS , http://www.ansys.com. 2012.
[16] Montgomery, D.C., Design and Analysis of Experiments. 2008: Wiley.
[17] LinkMontgomery, D.C. and E.A.L.G.G. LinkPeck, LinkIntroduction to linear regression analysis. 2006: Wiley-Interscience. 612.
[18] Meteopolis, N. and S. Ulam, The monte carlo method. Journal of the American Statistical Association, 1949. **44**(247): p. 335-341.
[19] Sobol, I.M., A primer for the Monte Carlo method. 1994: CRC Press.

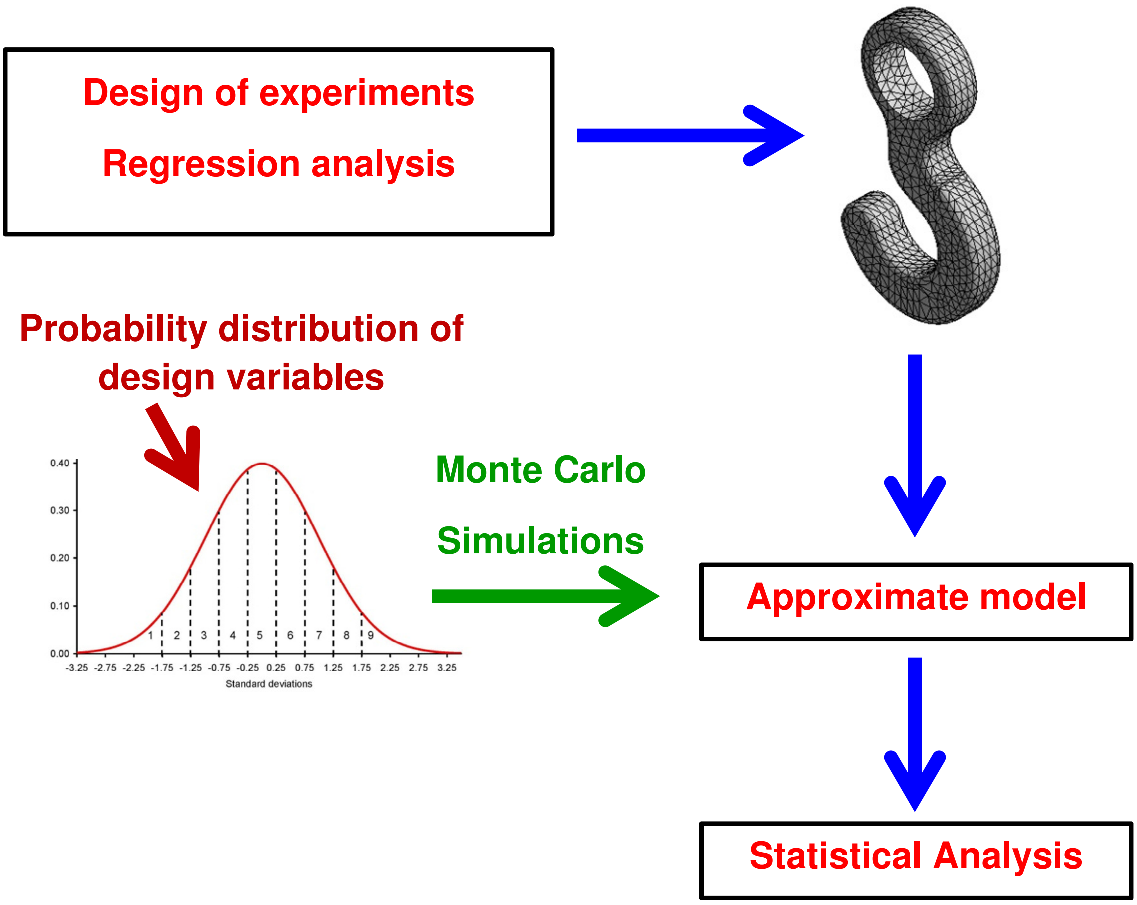

Fig. 1. A schematic drawing of the proposed method**.**

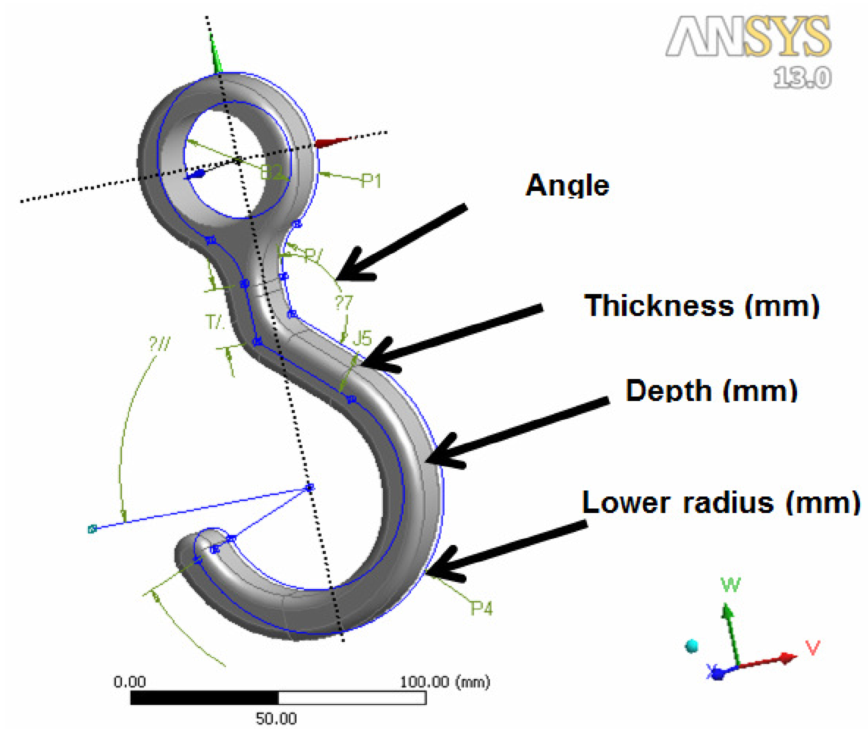

Fig. 2. Design variables of the model.

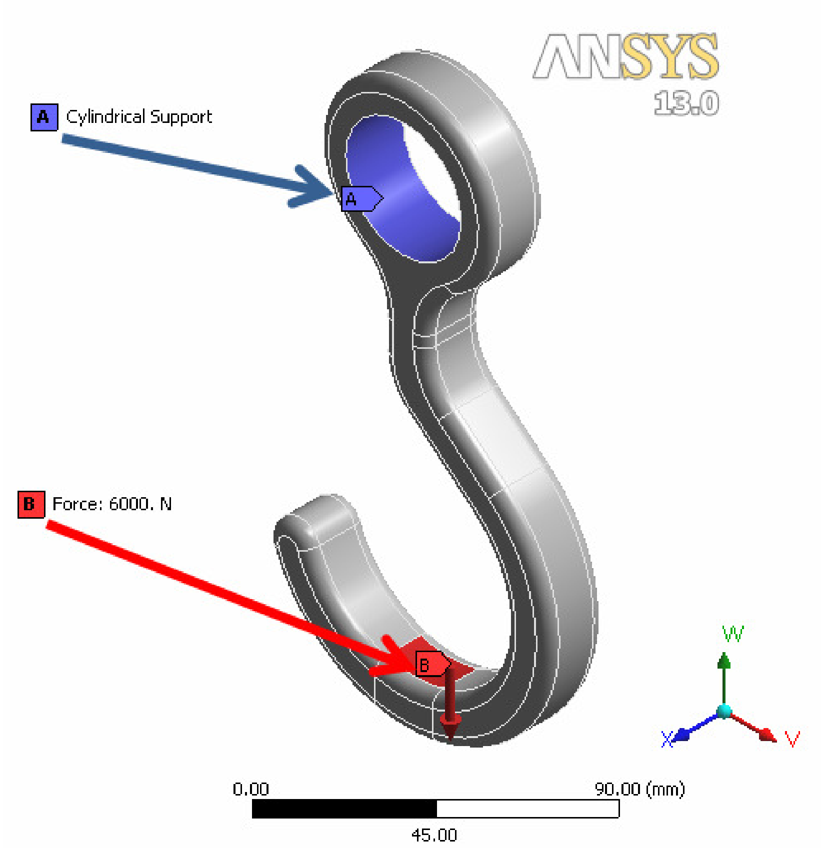

Fig. 3. Model under loads and constraints.

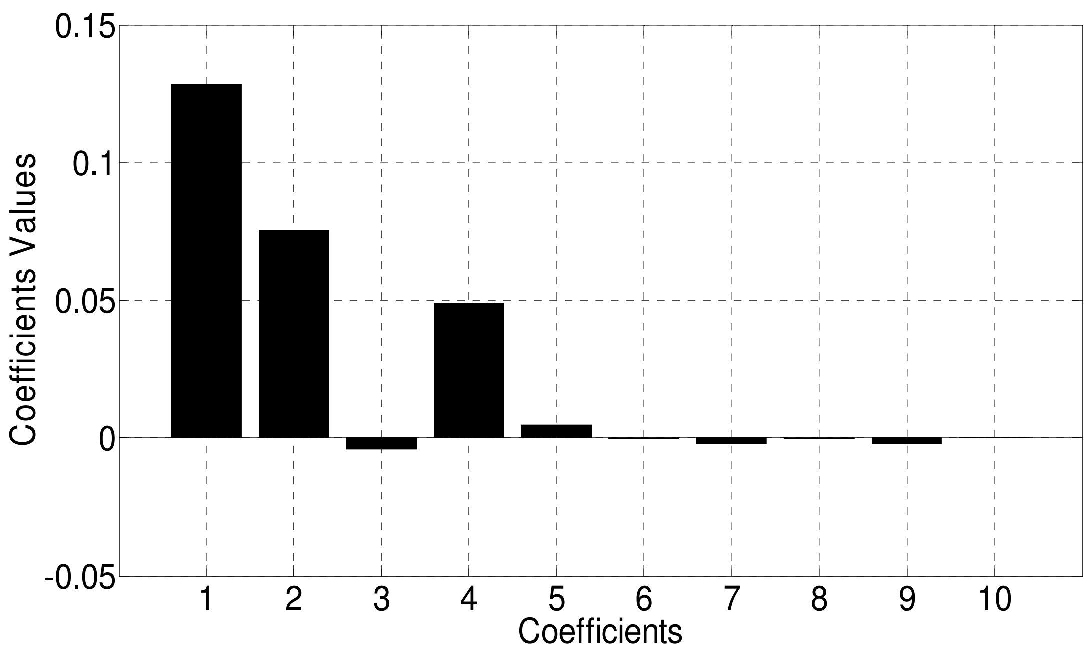

Fig. 4. Bar plot of the coefficient values of the approximate model.

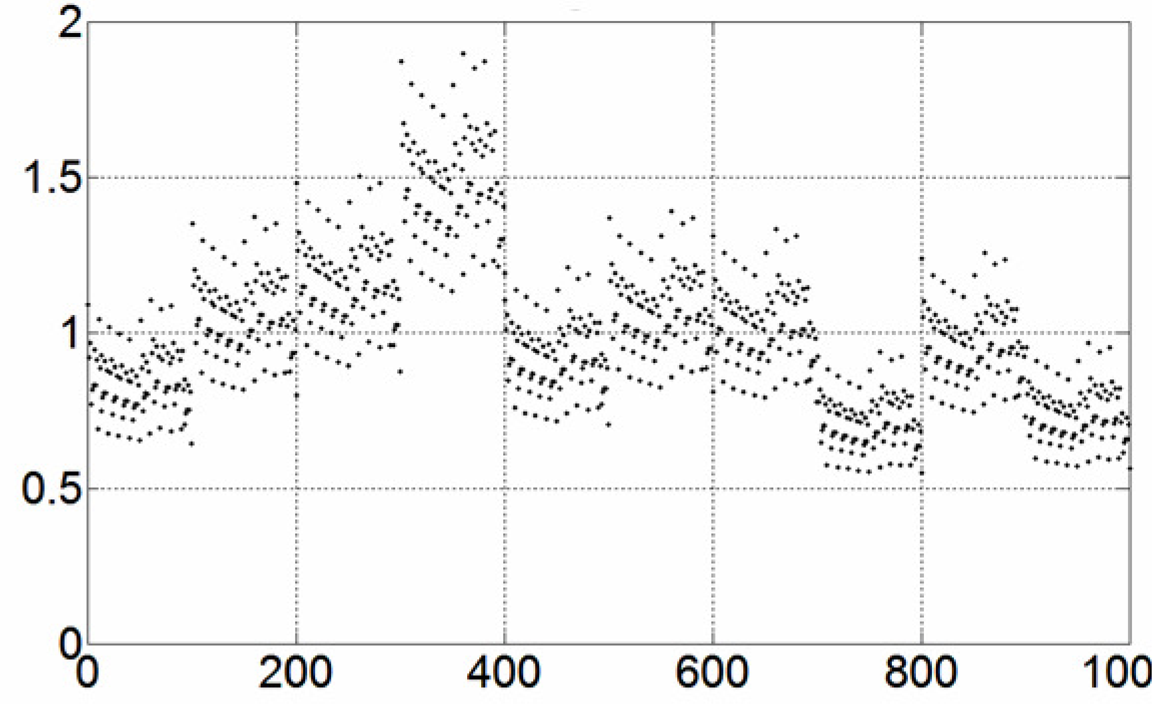

Fig. 5. A Scatter plot of safety factor values from MCS results.

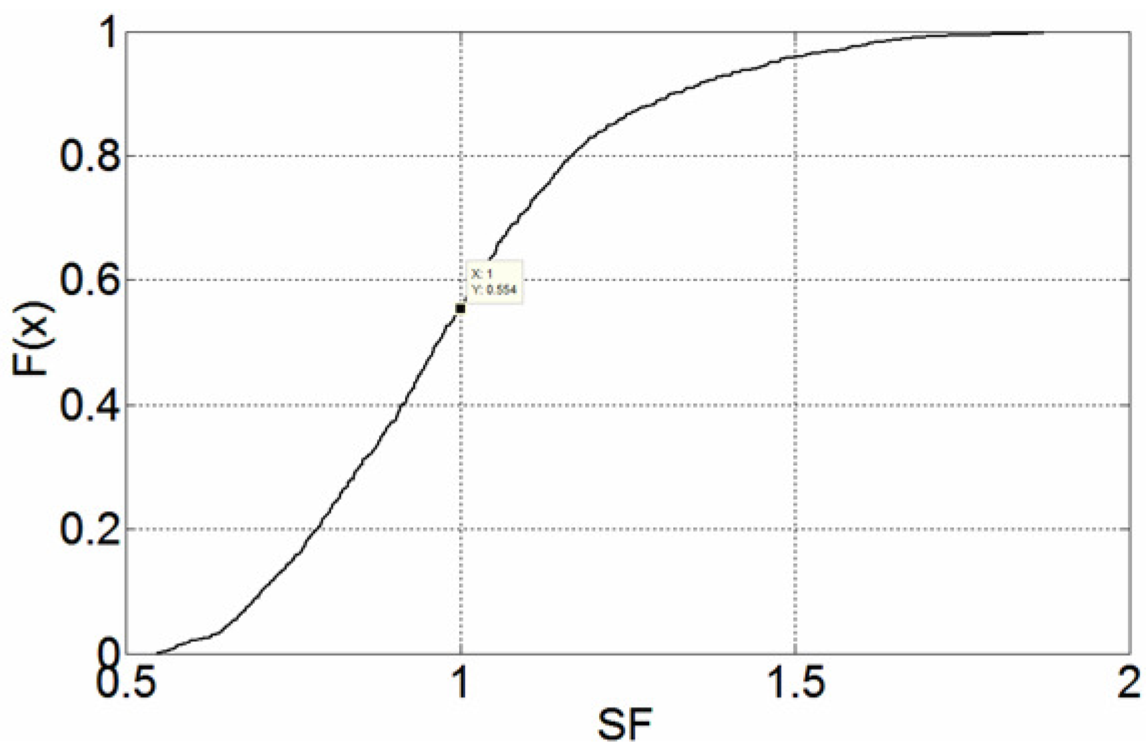

Fig. 6. A cumulative probability plot for safety factor (SF).

Table 1. Upper and lower bounds on design variables.

	Thickness (mm)	Depth (mm)	LowerRadius (mm)	Angle
Lower bound	15	15	45	120
Upper bound	25	25	55	150

Table 2. Design variables and response values at the DOE points.

ID	**Thickness (mm)**	**Depth (mm)**	**LowerRadius (mm)**	**Angle**	***mass* (gm)**	***SF***
1	20.0	20.0	50.0	135.0	0.9	0.99
2	15.0	20.0	50.0	135.0	0.8	0.56
3	25.0	20.0	50.0	135.0	1.0	1.53
4	20.0	15.0	50.0	135.0	0.7	0.72
5	20.0	25.0	50.0	135.0	1.1	1.20
6	20.0	20.0	45.0	135.0	0.8	1.08
7	20.0	20.0	55.0	135.0	1.0	0.94
8	20.0	20.0	50.0	120.0	0.9	1.02
9	20.0	20.0	50.0	150.0	1.0	0.97
10	16.5	16.5	46.5	124.4	0.6	0.56
11	23.5	16.5	46.5	124.4	0.7	1.19
12	16.5	23.5	46.5	124.4	0.9	0.87
13	23.5	23.5	46.5	124.4	1.1	1.79
14	16.5	16.5	53.5	124.4	0.7	0.50
15	23.5	16.5	53.5	124.4	0.8	1.06
16	16.5	23.5	53.5	124.4	1.0	0.75
17	23.5	23.5	53.5	124.4	1.2	1.46
18	16.5	16.5	46.5	145.6	0.6	0.60
19	23.5	16.5	46.5	145.6	0.8	1.17
20	16.5	23.5	46.5	145.6	0.9	0.85
21	23.5	23.5	46.5	145.6	1.1	1.75
22	16.5	16.5	53.5	145.6	0.7	0.50
23	23.5	16.5	53.5	145.6	0.9	1.07
24	16.5	23.5	53.5	145.6	1.0	0.76
25	23.5	23.5	53.5	145.6	1.3	1.50

Table 3. Coefficients Values of the approximate model.

Coefficients	Values
a_1	0.12835
a_2	0.07550
a_3	-0.00410
a_4	0.04864
a_1a_2	0.00474
a_1a_3	-0.00009
a_1a_4	-0.00225
a_2a_3	-0.00008
a_2a_4	-0.00213
a_3a_4	0.00014

Table 4. Values of adjusted coefficient of determination for approximate models with four and three variables.

$adjR^2$	4 variables	3 variables
mass	0.999	0.97
SF	0.989	0.991

Table 5. Descriptive statistics of MCS results**.**

	Mean	Minimum	Maximum	Standard Deviation
SF	0.60	0	1.18	0.27
mass	0.52	0.07	0.88	0.22