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ON THE EFFECTIVENESS OF POROSITY ON TRANSIENT FLOW OF A VISCOELASTIC FLUID WITH TEMPERATURE DEPENDENT VISCOSITY UNDER AN EXPONENTIAL DECAYING PRESSURE GRADIENT

H. A. Attia[*], M. E. S. Ahmed[**], K. M. Ewis[***] and I. Hamdy[***]

ABSTRACT

The transient flow and heat transfer through a porous medium of a non-Newtonian viscoelastic fluid between two infinite horizontal porous plates is studied considering a temperature dependent viscosity. The fluid is subjected to a uniform suction from above and a uniform injection from below whereas a uniform and exponential decaying pressure gradient is applied in the axial direction to drive the fluid motion. The plates are kept at two different invariant temperatures. The equations of motion and the energy equation including the dissipations are solved numerically using finite differences to yield the velocity and temperature distributions for various values of the included physical parameters.

KEY WORDS

Non-Newtonian fluids, porous medium, parallel plates, variable properties, numerical solution.

* Professor, Dept. of Engineering Mathematics and Physics, Fayoum University, El-Fayoum-63514, Egypt.

** Professor, Dept. of Engineering Mathematics and Physics, Fayoum University, El-Fayoum-63514, Egypt.

*** Lecturer, Dept. of Engineering Mathematics and Physics, Fayoum University, El-Fayoum-63514, Egypt.

NOMENCLATURE

a… constant
B_o … the magnetic field
c_p… the specific heat capacity
Ec... the Eckert number
J ... the current density
K… the Darcy permeability
k… the thermal conductivity of the fluid.
M… the porosity parameter
m… the Hall parameter
N… the viscosity exponent
Pr… the Prandtl number
Q… the viscoelastic parameter
Re… the Reynolds number
S… the suction parameter
t… Time
T… the temperature
T_1… The lower constant temperature
T_2… The upper constant temperature
u… the fluid velocity in direction x
w… the fluid velocity in direction z
ρ … the density of the fluid
σ … the fluid electrical conductivity
β … the Hall factor
μ … the viscosity of the fluid
μ_o … the coefficient of viscosity at temperature T_1

INTRODUCTION

The flow of a viscous incompressible fluid between two parallel plates has important applications in many devices such as accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and fluid droplets-sprays [1-3]. The flow between parallel plates of a Newtonian fluid with heat transfer, subjected to different physical effects, have been studied by many authors [1-9] where the results obtained are of great importance in the design of the duct wall and the cooling arrangements. The effect of uniform suction and injection through the parallel plates on unsteady Hartmann flow of a conducting Newtonian fluid was given by Attia [10,12]. The rectangular channel problem has later been extended also to fluids that undergo non-Newtonian behavior. The flow of a viscoelastic fluid has attracted the attention of many researchers [13-18] due to its important industrial applications [13].

In the present paper, we study of the transient flow and heat transfer through a porous medium of a non-Newtonian viscoelastic fluid between two infinite horizontal porous plates. A uniform suction from above and injection from below are applied perpendicular to the plates and an exponential decaying pressure gradient is applied in the axial direction. The fluid viscosity is assumed to be temperature dependent and the two plates are maintained at two constant but different temperatures. The flow through the porous medium deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [19-21]. The equations of motion and the energy equation including the Joule and viscous dissipations represent a set of coupled non-linear partial differential equations and is solved numerically using finite differences. The effects of the temperature-dependent viscosity, the viscoelasticity, the porosity of the medium, and the suction on the velocity and temperature fields are studied.

DEFINITION OF THE PROBLEM

The two infinite horizontal plates are placed at the $y=\pm h$ planes and a uniform suction from above and injection from below with a uniform velocity $V_y\mathbf{j}$ are applied. The motion is driven by an exponential decaying pressure gradient $dp/dx=-Ge^{-\alpha t}$ in the x-direction, where G and α are constants. The lower plate is maintained at a fixed temperature T_1, whereas the upper plate is maintained at another fixed temperature T_2 with $T_2>T_1$. The flow is through a porous medium where the Darcy model is assumed [19-21]. As the horizontal plates are infinite in the x and z-directions, all physical quantities, apart from the pressure, have no dependence on x or z. The non-Newtonian fluid considered is viscoelastic and a finite time is required for a strain response in the fluid when the stress imposed on the fluid boundaries changes. This amounts for an increment in the apparent viscosity especially at low shear rates. There is no velocity component in the z-direction and the velocity field may be written as

$$\vec{V}(y,t)=V_x(y,t)\vec{i}+V_y\vec{j}$$

The fluid starts its motion from rest at $t=0$, that is $V_x=0$ for $t\le 0$. The no-slip condition at the plates implies that $V_x=0$ at $y=-h$ and $V_x=0$ at $y=h$. The initial temperature of the fluid is assumed to be T_1, thus the initial and boundary conditions of temperature are $T=T_1$ at $t=0$, $T=T_1$ at $y=-h$, $t>0$ and $T=T_2$ at $y=h$, $t>0$.

MATHEMATICAL FORMULATION

The non-Newtonian fluid considered is viscoelastic and the shear stress is given by [13]

$$\tau=\mu\frac{\partial V_x}{\partial y}-\frac{\mu}{k}\frac{\partial \tau}{\partial t} \tag{1}$$

where μ is the coefficient of viscosity and k is the modulus of rigidity. In the limit as k tends to infinity or at steady state, the fluid behaves like a viscous fluid without elasticity. The coefficient of viscosity is taken to be temperature dependent according to [22]:

$$\mu=\mu_o e^{-a(T-T_1)}$$

where a is a constant and μ_o is another constant which is the coefficient of viscosity at temperature T_1. The fluid motion is governed by the momentum equation

$$\rho(\frac{\partial V_x}{\partial t}+V_y\frac{\partial V_x}{\partial y})=Ge^{-\alpha t}-f_b+\frac{\partial \tau}{\partial y} \tag{2}$$

where ρ is the fluid density, G and α are parameters describing the decaying pressure gradient, f_b is the body force due to porosity and the last term represents the viscoelastic force per unit volume which can be determined using Eq. (1) in the form,

$$\frac{\partial \tau}{\partial y}=\frac{\partial}{\partial y}(\mu\frac{\partial V_x}{\partial y})-\frac{1}{k}\frac{\partial}{\partial y}(\mu\frac{\partial}{\partial t}(\mu\frac{\partial V_x}{\partial y}))$$

where the term $(1/k^2)\partial/\partial y(\mu\partial/\partial t(\mu\partial\tau/\partial t))$ which is proportional to $(1/k^2)$ has been neglected. Knowing that

$$\frac{\partial \mu}{\partial y}=\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}=-a\mu_o e^{-a(T-T_1)}\frac{\partial T}{\partial y},\frac{\partial \mu}{\partial t}=-a\mu_o e^{-a(T-T_1)}\frac{\partial T}{\partial t}$$

The momentum equation becomes

$$\rho(\frac{\partial V_x}{\partial t}+V_y\frac{\partial V_x}{\partial y})=Ge^{-\alpha t}-\frac{\mu_o}{K}V_x+\mu_o e^{-a(T-T_1)}(\frac{\partial^2 V_x}{\partial y^2}-a\frac{\partial V_x}{\partial y}\frac{\partial T}{\partial y})+\rho\frac{\mu_o^2}{k}e^{-2a(T-T_1)}$$

$$x(2a(\frac{\partial^2 V_x}{\partial t\partial y}\frac{\partial T}{\partial y}-a\frac{\partial V_x}{\partial y}\frac{\partial T}{\partial y}\frac{\partial T}{\partial t})+a\frac{\partial^2 V_x}{\partial y^2}\frac{\partial T}{\partial t}+a\frac{\partial V_x}{\partial y}\frac{\partial^2 T}{\partial y\partial t}-\frac{\partial^3 V_x}{\partial t\partial y^2}) \tag{3}$$

where K is the Darcy permeability [19-21]. The second term in the right side of Eq. (3) represents the porosity force.
The temperature distribution is governed by the energy equation [22]

$$\rho c_p(\frac{\partial T}{\partial t}+V_y\frac{\partial T}{\partial y})=k\frac{\partial^2 T}{\partial y^2}+\mu_o e^{-a(T-T_1)}(\frac{\partial V_x}{\partial y})^2 \tag{4}$$

where c_p is the specific heat capacity of the fluid at constant volume and k is the thermal conductivity. The last term in the right side of Eq. (4) is the viscous dissipation term. Equations (3) and (4) can be made dimensionless by introducing the following dimensionless variables

$$y'=\frac{y}{h},t'=\frac{\mu_o t}{h^2\rho},V'=\frac{\rho h V_x}{\mu_o},G'=\frac{\rho G}{h^2\mu_o^2},T'=\frac{T-T_1}{T_2-T_1}$$

Introducing the following dimensionless parameters,

$S=\rho h V_y/\mu_o$, the suction parameter,

$M=\rho h^2/K$, the porosity parameter,

$\Pr=\mu_o c_p/k$, the Prandtl number,

$Ec = \mu_o^2 / (h^2 \rho^2 c_p (T_2 - T_1))$, the Eckert number,
$Q = \mu_o^2 / \rho k h^2$, the viscoelastic parameter,
$N = a(T_2 - T_1)$, the viscosity exponent.

Eqs. (3) and (4) may be written, after dropping all primes for convenience, as

$$\frac{\partial V}{\partial t} + S\frac{\partial V}{\partial y} = Ge^{-\alpha t} - MV + e^{-NT}\left(\frac{\partial^2 V}{\partial y^2} - N\frac{\partial V}{\partial y}\frac{\partial T}{\partial y}\right)$$
$$+ Qe^{-2NT}\left(2N\left(\frac{\partial^2 V}{\partial t \partial y}\frac{\partial T}{\partial y} - N\frac{\partial V}{\partial y}\frac{\partial T}{\partial y}\frac{\partial T}{\partial t}\right) + N\frac{\partial^2 V}{\partial y^2}\frac{\partial T}{\partial t} + N\frac{\partial V}{\partial y}\frac{\partial^2 T}{\partial y \partial t} - \frac{\partial^3 V}{\partial t \partial y^2}\right) \tag{5}$$

$$\frac{\partial T}{\partial t} + S\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + Ece^{-NT}\left(\frac{\partial V}{\partial y}\right)^2 \tag{6}$$

The initial and boundary conditions become

$$V(y,0) = V(-1,t) = 0, V(1,t) = 0 \tag{7a}$$

$$T(y,0) = T(-1,t) = 0, T(1,t) = 1 \tag{7b}$$

Equations (5) and (6) represent a coupled system of nonlinear partial differential equations, which are solved numerically under the initial and boundary conditions (7), using the method of finite difference. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. The Crank-Nicolson implicit method [23] is applied at two successive time levels. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step, and the iterations are continued until convergence, within a prescribed accuracy. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas algorithm [23]. The finite difference equations relating the variables are obtained by writing the equations at the midpoint of the computational cell and then replacing the different terms by their second-order central difference approximations in the *y*-direction. The diffusion terms are replaced by the average of the central differences at two successive time levels. The computational domain is divided into meshes each of dimension Δt and Δy in time and space respectively. Grid-independence studies show that the computational domain $0<t<\infty$ and $-1<y<1$ can be divided into intervals with step sizes Δt=0.001 and Δy=0.005 for time and space, respectively. Convergence of the scheme is assumed when every one of the variables *V*, *T*, and their gradients for the last two approximations differs by less than 10^{-6} for all values of *y* in $-1<y<1$ at every time step. Computations have been made for *G*=5, $\alpha = 1$, *Pr*=1 and *Ec*=0.2.

RESULTS AND DISCUSSION

Figures 1 and 2 show the time progression of the profiles of the velocity *V* and temperature *T* for the cases *N*=0 and *N*=0.5 and for *M*=0.5, *Q*=0.5, and *S*=0.5. It is found that both *V* and *T* increase with time for a short interval and then starts decreasing with time up till the steady state. It is also indicated that increasing N increases V and T and the time at which they reach their steady state values due to the decrease in viscosity.

Figures 3 to 10 present the variation of the velocity V and the temperature T at the mid-plane of the rectangular channel with time for various values of N, M, Q, and S. Both V and T do not increase monotonically with time. Figures 3 and 4 show the effect of N in both the non-porous and porous cases when Q=0.5 and S=0. It is found that both V and T increase with increasing N, that is with the decrease of viscosity. It is easy to understand that decreasing the viscosity increases the fluid velocity, but the effect of this on temperature is not straightforward. The increase of V at the mid-plane implies an increase in velocity gradients but due to the decrease in viscosity with increasing N, it is difficult to tell beforehand whether the viscous dissipation which is proportional to $e^{-NT}(\partial V/\partial y)^2$ increases or decreases. The results indicate that T increases.

Figures 5 and 6 show the effect of M on both V and T for the cases N=0 and 1 and for Q=0.5 and S=0. For the case N=0, as shown in Fig. 5a, increasing M decreases V due to the increase in the damping force while its effect on the steady state time of V is small. Also, increasing M decreases T due to the decrease in the viscous dissipation. Figures 5b and 6b present a more apparent effect of M on the velocity V and the temperature T at small time in the case of large values of N.

Figures 7 and 8 present the effect of Q on V and T at y=0 for N=0 and 1 and for M=0.5 and S=0. It is indicated that increasing N increases both V and T and their steady state times for all values of Q and that the effect of Q on V and T depends on time. For small and moderate time, increasing Q decreases both V and T. But for large time, V and T increases with increasing Q. This results in a crossover in V and T charts with time which is more apparent for V than for T. Figures 7a and 8a show that increasing Q increases the time at which they reach their steady state values. Comparing Figs. 7a with 7b and 8a with 8b shows that the influence of Q on V and T is more pronounced for small values of N.

Figures 9 and 10 present the effect of the suction parameter S on both V and T for the cases N=0 and 0.5 and for M=Q=0.5. Increasing S decreases both V and T as a result of pumping the fluid from the more slow and cold lower half region to the centre of the channel. It is also indicated from Fig. 10 that S has a negligible effect on the steady state time of T. The influence of S on V depends greatly on N, while the situation is not the same for the temperature T.

CONCLUSIONS

The transient flow and heat transfer through a porous medium of a non-Newtonian viscoelastic fluid between two parallel porous plates is studied considering the viscosity to be temperature-dependent and in the presence of an exponential decaying pressure gradient. The effect of the viscosity exponent N, the viscoelasticity parameter Q, the porosity parameter M, and the suction parameter S on the velocity and temperature fields is discussed. It was found that the variation of the viscosity exponent N has a marked effect on the steady state time of both V and

T. It was also observed that the variation of both *V* and *T* with the parameters *M*, *Q*, and *S* depends greatly on the viscosity exponent *N*. It is interesting to detect that the variation of the velocity and temperature on the viscoelasticity parameter *Q* depends upon time.

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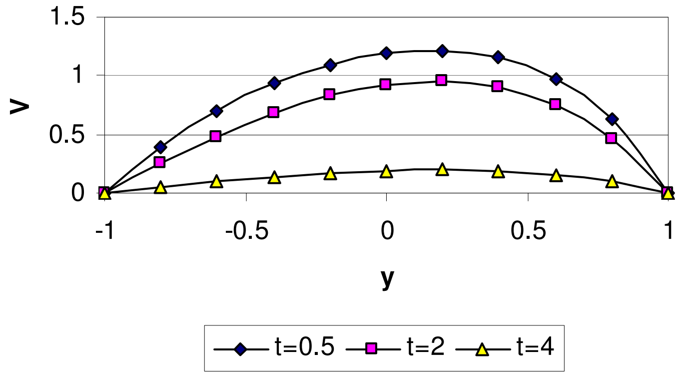
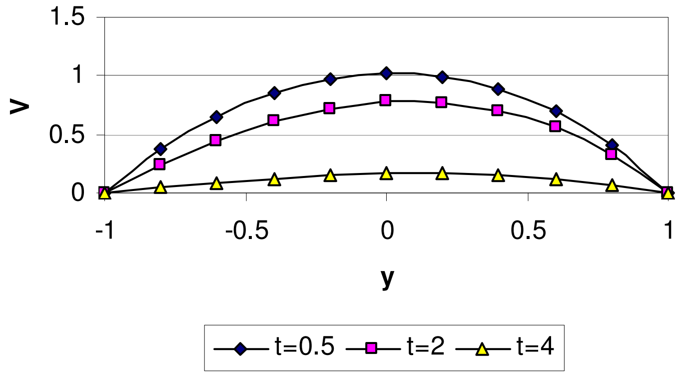
FIGURES:

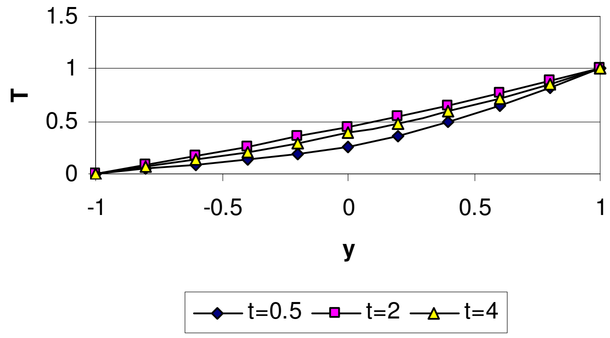


(a) *N*=0



(b) *N*=0.5

Fig. 1 Time development of the profile of *V* for various values of *N* (*M*=0.5, *Q*=0.5, and *S*=0.5).



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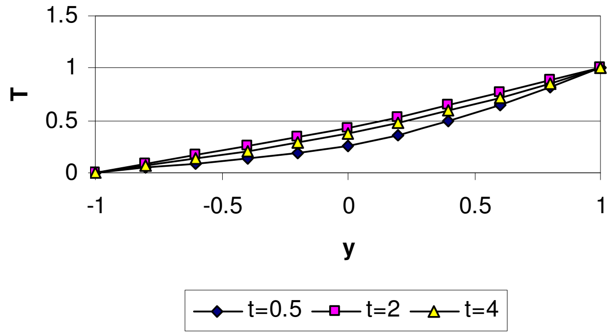
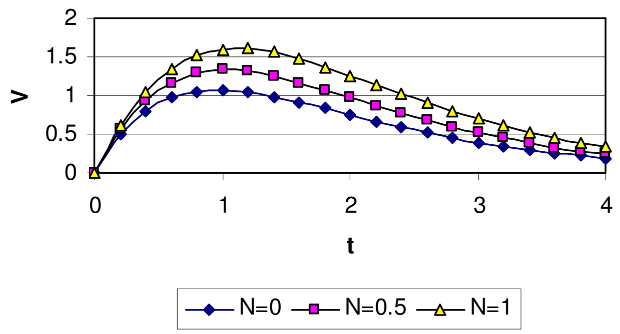
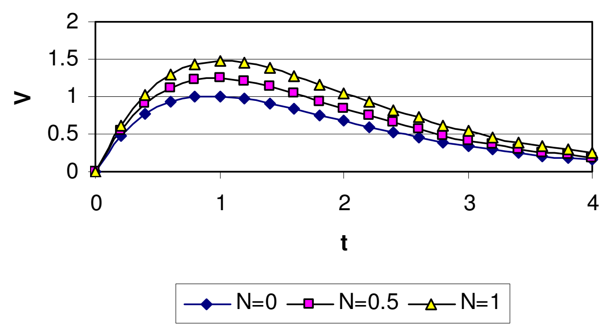


(b) *N*=0.5

Fig. 2 Time development of the profile of *T* for various values of *N* (*M*=0.5, *Q*=0.5, and *S*=0.5)

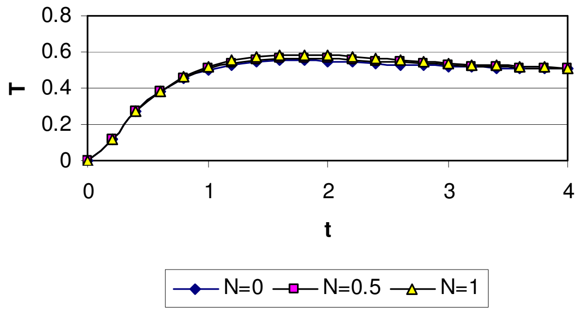


(a) *M*=0

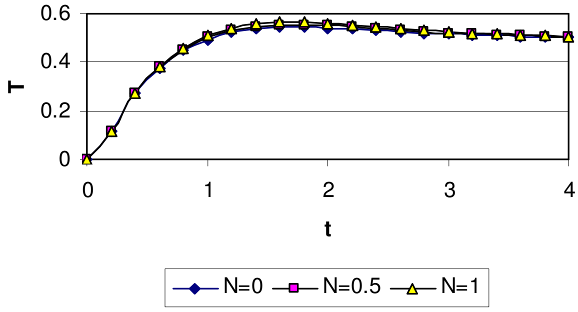


(b) *M*=0.5

Fig. 3 Time development of *V* at *y*=0 for various values of *N* and *M* (*Q*=0.5 and *S*=0)

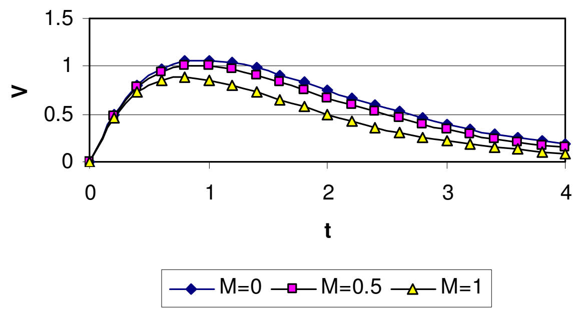


(a) *M*=0

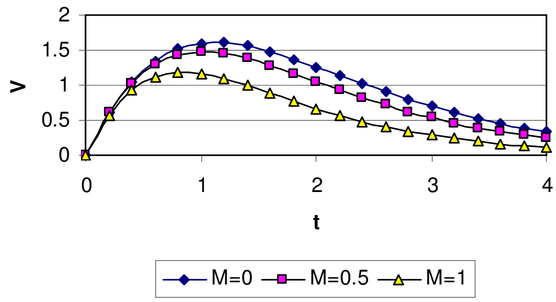


(b) *M*=0.5

Fig. 4 Time development of *T* at *y*=0 for various values of *N* and *M* (*Q*=0.5 and *S*=0)

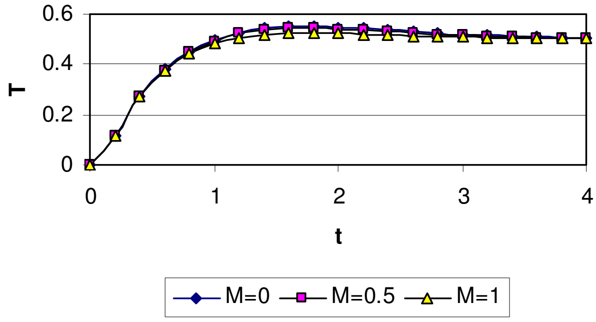


(a) *N*=0

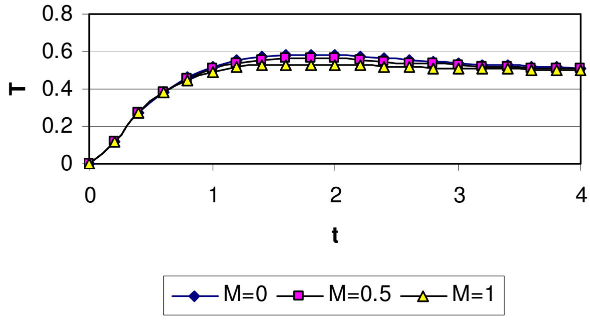


(b) *N*=1

Fig. 5 Time development of *V* at *y*=0 for various values of *N* and *M* (*Q*=0.5 and *S*=0)

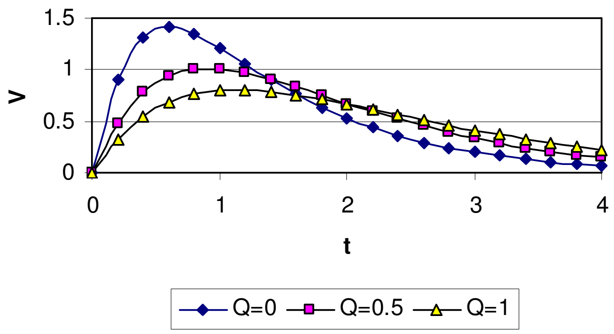


(a) *N*=0



(b) *N*=1

Fig. 6 Time development of *T* at *y*=0 for various values of *N* and *M* (*Q*=0.5 and *S*=0).



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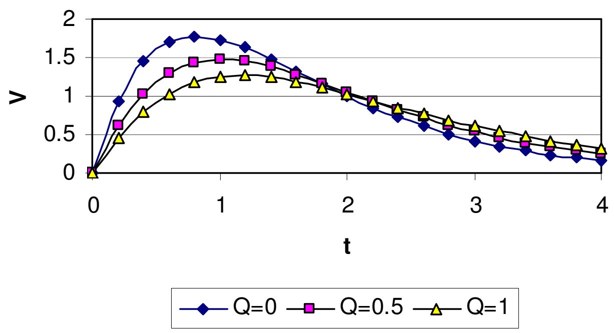
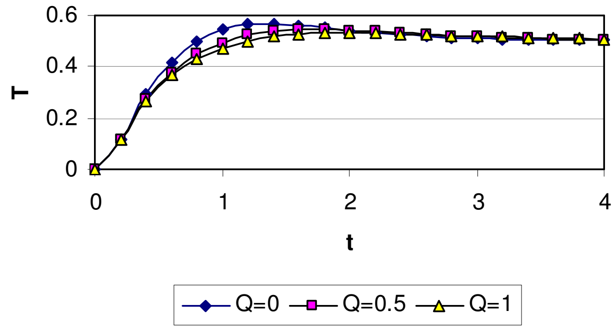
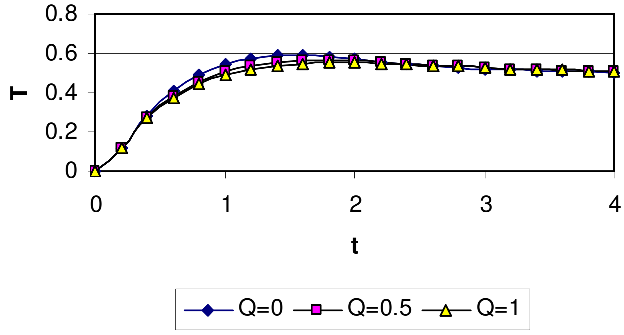


(b) *N*=1

Fig. 7 Time development of *V* at *y*=0 for various values of *N* and *Q* (*M*=0.5 and *S*=0).

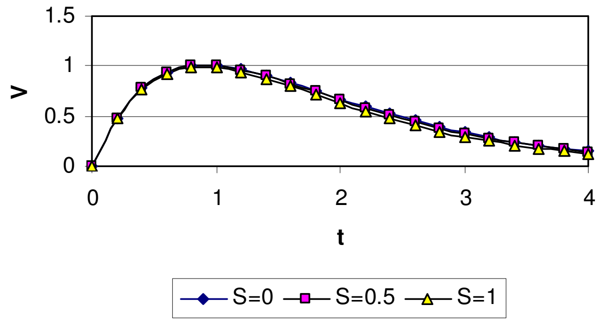


(a) *N*=0

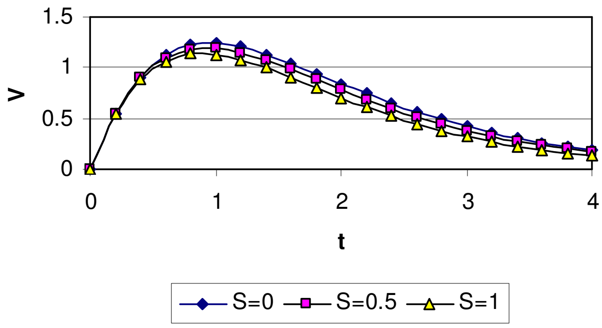


(b) *N*=1

Fig. 8 Time development of T at y=0 for various values of N and Q (M=0.5 and S=0).

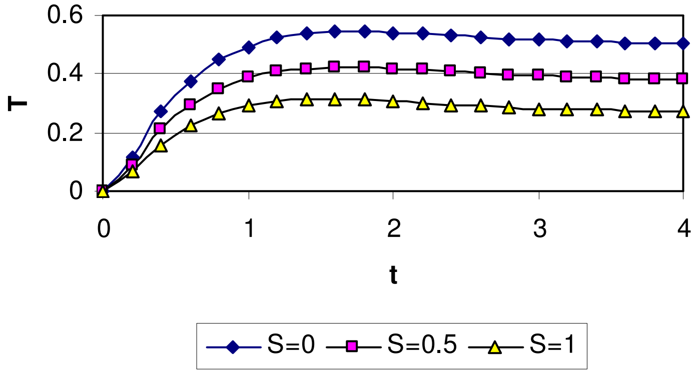


(a) *N*=0

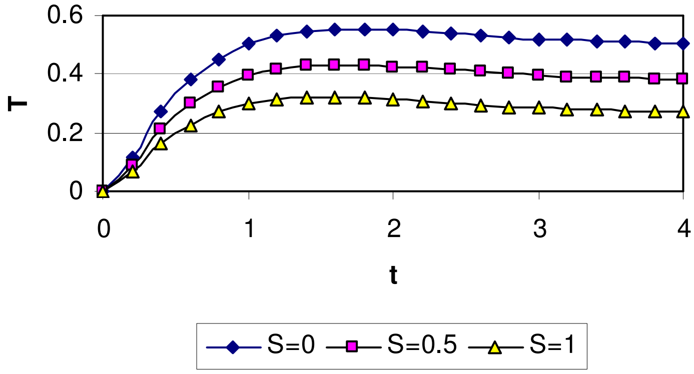


(b) *N*=0.5

Fig. 9 Time development of V at y=0 for various values of N and S (M=0.5 and Q=0.5).



(a) N=0



(b) N=0.5

Fig. 10 Time development of T at y=0 for various values of N and S (M=0.5 and Q=0.5).