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# GAS PARAMETERS PREDICTION IN THERMAL PULSE FLOW METER

V. Jinga[1], C. Samoilă[2], D. Ursuțiu[3] and A. P. Cotfas[3]

## ABSTRACT

This paper will present to the audience a processing and simulation method regarding the theoretical approach of a mathematical model that describes the phenomena taking place inside a new gas flow sensor. Because the working principle of the flow sensor is a simple one – generating with a specific frequency thermal pulses that transfer heat to the annular volume of gas and then, by measuring the gas temperature evolved in time, determining the flow rate, a mathematical model is needed to establish all the parameters that vary during this process. This involved lot of mathematics for solving some parts of the established model. The authors of this paper have proposed the specific software especially developed for simulating the gas flow sensor. The software performs mathematical prediction regarding the gas parameters after the generation of the thermal pulse and the occurrence of heat exchange, more precisely, the evolution of gas speed, temperature, density and pressure. These things are extremely important for the development of this gas flow sensor, by comparing the experimental data with the theoretical results.

## KEY WORDS

Gas flow measurement, thermal impulse flowmeter, heat transfer modeling.

[1] Ph.D. student, Faculty of Material Science and Engineering, Transilvania University, Braşov, România
[2] Professor, Faculty of Material Science and Engineering, Transilvania University, Braşov, România
[3] Professor, Faculty of Computing Science and Engineering, Transilvania University, Braşov, România

## INTRODUCTION

A new type of gas flow meter has been proposed that works on a simple basic principle of heat transfer from a heat source to the volume of gas and afterwards the evolution of the gas temperature in time (for a short period of a few seconds) has been measured to determine the correlation between temperature decreasing along the sensor and real flow. The relation between the temperature and the fluid flow has been performed through a mathematical model. This method was applied in the 80`s, but it had no success when really implementing it because of the technical problems related to the lack of accurate gas flow measurements.

Nowadays, because of the technical development concerning hardware components and real time processing methods of acquired data, this method of measure can be properly implemented, resulting a new type of flow meter, good for both static and transient regime.

Because the working principle is a simple one having a smart implementation regarding low flows of gas – a repeating cycle of thermal pulse generation, heat exchange, some data acquisitions concerning the gas temperature evolution and afterwards by data processing the gas flow rate has been calculated – this new sensor will have a series of advantages.

## PROCESS AND MODELLING

This article presents the theoretical approach regarding the evolution of gas parameters after generation of the thermal pulse and the heat transfer takes place. For better understanding, first take a look at the schematics of the process used after for the mathematical modeling, presented in Fig. 1. All the gas parameters are described in Table 1.

**Table 1.** Gas parameters description.

| **Initial gas state (before the pulse)** | **Second state (after the pulse)** | **Parameter description** | **Measuring Units [S.I.]** |
|---|---|---|---|
| $\omega_1$ | $\omega_2$ | The speed of the gas | m/s |
| $T_1$ | $T_2$ | The temperature of the gas | Kelvin |
| $p_1$ | $p_2$ | The pressure of the gas | Pa |
| $\rho_1$ | $\rho_2$ | The density of the gas | $kg/m^3$ |
| Q | - | The heat quantity | J |
| $C_p$ | - | The specific heat of the gas at constant pressure | J/(kg.K) |

As a simplifier hypothesis for this mathematical modeling, the temperature influence on other parameters, except those mentioned below, will be neglected because the temperatures are not so high.

The mathematical model for the heat transfer has been written starting from the hypothesis that as a consequnece of the heating process, the gas density will decrease ($\rho_2 > \rho_1$) and the traveling speed of the gas along the pipe will increase ($\omega_2 > \omega_1$). This fact is verified at the end of the mathematical model by the relation:

$$\rho_1\omega_1 = \rho_2\omega_2 \tag{1}$$

<=> $\rho_2 = \rho_1\omega_1/\omega_2$ when $\rho_2 > \rho_1$ then $\omega_2 > \omega_1$ from the continuity equation, which is actually the first equation of the system.

The second equation of the system is in fact the equation regarding the pulse applied to the gas mass along the flow direction between sections 1 – 2 (by neglecting the friction):

$$p_1 - p_2 = \rho_1\omega_1(\omega_2 - \omega_1) \tag{2}$$

The third equation of the system is a relation resulted from the perfect gases equation:

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2} \tag{3}$$

The last equation of the system is a more complex one and is obtained by mathematically describing the thermal flux received by the gas mass unit during the time unit, q [J/kg], in two ways:

$$q = h_{02} - h_{01} \tag{4}$$

where $h_{01}$ and $h_{02}$ represent the gas braked enthalpies in section 1 – 2.
Supposing an average of the specific gas heat, $C_p = C_{p1} = C_{p2}$, results:

$$q = C_p \times (T_2 - T_1) + \frac{1}{2} \times (\omega_2^2 - \omega_1^2) \tag{5}$$

and:

$$q = \frac{Q}{D} \tag{6}$$

where Q [J/s] represents the heat quantity regarding a period of time and D [kg/s] is the flow.

$$D = \rho_1\omega_1 A = \frac{\rho_1\omega_1 \times (2\frac{\omega_2}{\omega_1} - 1)}{2\frac{T_2}{T_1} - 1} \times A \tag{7}$$

where A [$m^2$] is the flow section (constant) and will be neglected (A = 1, only its measuring units will be taken into consideration, for the equation consistency) because the overall process takes place on the limit layer.

By introducing relation no. 7 into relation no. 6 and equalizing the result with relation no. 5, the last equation of the system is obtained:

$$C_p \times (T_2 - T_1) + \frac{1}{2} \times (\omega_2^2 - \omega_1^2) = \frac{Q}{\frac{\rho_1 \omega_1 \times (2\frac{\omega_2}{\omega_1} - 1)}{2\frac{T_2}{T_1} - 1}} \tag{8}$$

At the end of the mathematical modeling concerning the phenomena taking place inside the new gas flow sensor, all the gas parameters regarding the second state – after the thermal pulse generation and heat transfer takes place – four parameters ($\omega_2$, $T_2$, $p_2$, $\rho_2$) can be determined from the four equations system:

$$\begin{cases} \rho_1 \omega_1 = \rho_2 \omega_2 \\ p_1 - p_2 = \rho_1 \omega_1 (\omega_2 - \omega_1) \\ \frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} \\ C_p \times (T_2 - T_1) + \frac{1}{2} \times (\omega_2^2 - \omega_1^2) = \frac{Q}{\frac{\rho_1 \omega_1 \times (2\frac{\omega_2}{\omega_1} - 1)}{2\frac{T_2}{T_1} - 1}} \end{cases} \tag{9}$$

The next step was to start solve the above mathematical system in the classical way, by doing all the necessary replacements from one equation to another. From the last relation of the equations system, relation no. 8, results:

$$T_2 = \frac{T_1}{2} * [1 + \frac{(2\rho_1 \omega_1 \omega_2 - \rho_1 \omega_1^2) * (C_p T_1 + \omega_1^2 - \omega_2^2)}{C_p T_1 * (2\rho_1 \omega_1 \omega_2 - \rho_1 \omega_1^2) - 2Q\omega_1}] \tag{10}$$

From the first relation of the equations system: $\Rightarrow \rho_2 = \frac{\rho_1 \omega_1}{\omega_2}$ (11)

From the second relation of the equations system:

$$\Rightarrow p_2 = p_1 - \rho_1 \omega_1 (\omega_2 - \omega_1) \tag{12}$$

Introducing relation no. 11 and 12 in the third relation of the equations system, after doing all the necessary simplifications results:

$$T_2 = [p_1 - \rho_1 \omega_1 (\omega_2 - \omega_1)] * \frac{\omega_2 T_1}{p_1 \omega_1} \tag{13}$$

By equalizing relation no. 10 with relation no. 13 (both representing the $T_2$ variable), a new equation is obtained and after doing all the calculations, the „simplified" and final form is:

$$2\rho_1 \omega_1 * (2C_p \rho_1 T_1 - p_1) * \omega_2^3 + \rho_1 * (\omega_1^2 p_1 - 6C_p \rho_1 T_1 \omega_1^2 - 4C_p T_1 p_1 - 4Q\omega_1) * \omega_2^2 + 2 * (\rho_1 p_1 \omega_1^3 + C_p \rho_1^2 T_1 \omega_1^3 + 2Q\rho_1 \omega_1^2 + 3C_p \rho_1 T_1 p_1 \omega_1 + 2Qp_1) * \omega_2 - \omega_1 * (\rho_1 p_1 \omega_1^3 + 2C_p \rho_1 T_1 p_1 \omega_1 + 2Qp_1) = 0 \tag{14}$$

Relation no. 14 is actually a third degree equation with the unknown variable of $\omega_2$:

$a * \omega_2^3 + b * \omega_2^2 + c * \omega_2 + d = 0$ , where a, b, c, d  R and $a \neq 0$.

The equations coeficients are:

$$\begin{cases} a = 2\rho_1\omega_1 * (2C_p\rho_1T_1 - p_1) \\ b = \rho_1 * (\omega_1^2p_1 - 6C_p\rho_1T_1\omega_1^2 - 4C_pT_1p_1 - 4Q\omega_1) \\ c = (\rho_1p_1\omega_1^3 + C_p\rho_1^2T_1\omega_1^3 + 2Q\rho_1\omega_1^2 + 3C_p\rho_1T_1p_1\omega_1 + 2Qp_1) \\ d = -\omega_1 * (\rho_1p_1\omega_1^3 + 2C_p\rho_1T_1p_1\omega_1 + 2Qp_1) \end{cases} \tag{15}$$

Solving the above third degree equation and finding out the right solutions for the $\omega_2$ parameter – the gas speed after the generation of the thermal impulse, is done by using the Cardano method and formulas.

## SOFTWARE PROPOSED FOR MATHEMATICAL MODELLING:

The first approach in solving the last equation (relation no. 14) was the classical one, with the pen and paper, but there was no succes because of the complexity of the coeficients, human not being able to solve the calculations in the traditional way. So the Cardano method was implemented in a software way.

From the theoretical point of view, the solving method of the third degree equation is like this: having an equation like $a * x^3 + b * x^2 + c * x + d = 0$ , where a, b, c, d  R and $a \neq 0$, the first step is to change from the variable „x" to another variable – by replacing „x" with „$y - \frac{b}{3a}$" in the initial equation (a and b are the third degree equation coeficients), thus, after doing all the possible calculations and simplifications, resulting another third degree equivalent equation $y^3 + my + n = 0$ , where $m = \frac{c}{a} - \frac{b^2}{3a^2}$ and $n = \frac{2b^3}{27a^3} - \frac{bc}{3a^2} + \frac{d}{a}$.

After this, with the help of the Cardano method, M and N can be calculated using the formulas:

$$M = \sqrt[3]{-\frac{n}{2} + \sqrt{(\frac{m}{3})^3 + (\frac{n}{2})^2}} \text{ and } N = \sqrt[3]{-\frac{n}{2} - \sqrt{(\frac{m}{3})^3 + (\frac{n}{2})^2}} \tag{16}$$

Then, the three distinct solutions for the eqaution where „y" is the unknown variable are:

$$\begin{cases} y_1 = M + N \\ y_2 = -\frac{M+N}{2} + i * \frac{M-N}{2}\sqrt{3} \\ y_3 = -\frac{M+N}{2} - i * \frac{M-N}{2}\sqrt{3} \end{cases} \quad (17)$$

If the equation discriminant is calculated, $\Delta = \frac{n^2}{4} + \frac{m^3}{27}$ , there are three cases depending on the sign of Δ:

- If $\Delta > o$, then the third degree equation has one real solution and 2 complex and concerted ones:

$$\begin{cases} y_1 = \delta + z \\ y_2 = \varepsilon\delta + \varepsilon^2 z \\ y_3 = \varepsilon^2\delta + \varepsilon z \end{cases} \quad (18)$$

where δ is the real value of M, z is the real value of N and ε, $\varepsilon^2$ are the cube roots of 1.

- If $\Delta = 0$, then the third degree equation has all three solutions real, two of them being equal:

$$\begin{cases} y_1 = 2\sqrt[3]{-\frac{n}{2}} \\ y_2 = y_3 = -\sqrt[3]{-\frac{n}{2}} \end{cases} \quad (19)$$

- If $\Delta < 0$ then the third degree equation has three real and distinct solutions:

$$\begin{cases} y_1 = 2\alpha \\ y_2 = -\alpha - \beta\sqrt{3} \\ y_3 = \alpha - \beta\sqrt{3} \end{cases} \quad (20)$$

where α = Re{M} and β = Im{N}, meaning that $\alpha + i * \beta = \sqrt[3]{-\frac{n}{2} - \sqrt{(\frac{m}{3})^3 + (\frac{n}{2})^2}}$.

The dedicated software for solving the third degree equation was developed in the graphical programming software Lab VIEW, a product of the National Instruments company from U.S.A.

More than that, the already developed software, after determining the values of the solutions for the third degree equation – the second equation having the „y" as an unknown variable – respects the described mathematical algoritm backwards and makes the necessary calculus for determining the values for the parameters concerning the second state of the gas – after the thermal pulse generation. First the speed of the gas is determined, $\omega_2$, and then $p_2$, $\rho_2$ and $T_2$ are calculated.

The front panel of the developed software is presented in Fig. 2; the end user has the possibility of adjusting the values for the inputs of the system, meaning that the known parameters concerning the initial state of the gas (before the thermal pulse) – $\omega_1$, $p_1$, $\rho_1$, $T_1$, Q and $C_p$ can be edited acording to the reality, if the experimental conditions change.

Because the devoped software part concerns a lot of mathematical formulas and algorithms, the basic structure of the programme consists of a few Formula Nodes, where the developer can write easy mathematics and the soft will recognize the language. As an example, such a structure is presented in Fig. 3.

Testing the developed software was done for three types of gases (air, oxigen and nitrogen), where the some of the initial conditions (inputs for the four equations system) were constant and others were variable.

The input parameters that remain constant are:

- The initial temperature: $T_1 \approx 20\ ^0\ C = 257.15\ K$
- The gas density: $\begin{cases} \rho = 1.2047\ kg/m^3 \text{ for air} \\ \rho = 1.429\ kg/m^3 \text{ for oxigen} \\ \rho = 1.2506\ kg/m^3 \text{ for nitrogen} \end{cases}$
- The specific gas heat: $\begin{cases} C_p = 1011\ J/(kg.K) \text{ for air} \\ C_p = 920\ J/(kg.K) \text{ for oxigen} \\ C_p = 1040\ J/(kg.K) \text{ for nitrogen} \end{cases}$
- The heat quantity: Q = 350 J in the unit of time

The variable input parameters for the four equations system are:

- The gas speed: $\omega_1$ ranges between 0.008 m/s and 0.024 m/s (in direct relation with the gas flow that ranges between 5 l/h and 15 l/h)
- The pressure: $p_1$ ranges between 100 kPa and 300 kPa (equivalent with 1 bar – 3 bar)

The results of the tests are presented in the next three tables, first for air, second for oxigen and third for nitrogen:

**Table 2.** The results from running the developed software for air.

| **Initial Conditions:** Air | **Flow [l/h]** | **5 l/h** | **10 l/h** | **15 l/h** |
|---|---|---|---|---|
| $T_1 = 20\ ^0\ C = 257.15\ K$ | **$\omega_1$ [m/s]** | **0.008 m/s** | **0.016 m/s** | **0.024 m/s** |
| $\rho_1 = 1.2047\ kg/m^3$ | **$T_2$ [k]** | 293.071 | 275.11 | 269.124 |
| $p_1 = 300000$ Pa | **$\omega_2$ [m/s]** | $9.118*10^{-3}$ | 0.017 | 0.025 |
| $C_p = 1011\ J/(kg.K)$ | **$p_2$ [Pa]** | $3*10^5$ | $3*10^5$ | $3*10^5$ |
| Q = 350 J | **$\rho_2$ [kg/m$^3$]** | 1.057 | 1.126 | 1.151 |

**Table 3.** The results from running the developed software for oxigen.

| **Initial Conditions:** Oxigen | **Flow [l/h]** | **5 l/h** | **10 l/h** | **15 l/h** |
|---|---|---|---|---|
| $T_1$ = 20 $^0$ C = 257.15 K | **$\omega_1$ [m/s]** | **0.008 m/s** | **0.016 m/s** | **0.024 m/s** |
| $\rho_1$ = 1.429 kg/m$^3$ | **$T_2$ [k]** | 290.428 | 273.789 | 268.243 |
| $p_1$ = 300000 Pa | **$\omega_2$ [m/s]** | $9.035 \cdot 10^{-3}$ | 0.017 | 0.025 |
| $C_p$ = 920 J/(kg.K) | **$p_2$ [Pa]** | 3*10$^5$ | 3*10$^5$ | 3*10$^5$ |
| Q = 350 J | **$\rho_2$ [kg/m$^3$]** | 1.265 | 1.342 | 1.37 |

**Table 4.** The results from running the developed software for nitrogen.

| **Initial Conditions:** Nitrogen | **Flow [l/h]** | **5 l/h** | **10 l/h** | **15 l/h** |
|---|---|---|---|---|
| $T_1$ = 20 $^0$ C = 257.15 K | **$\omega_1$ [m/s]** | **0.008 m/s** | **0.016 m/s** | **0.024 m/s** |
| $\rho_1$ = 1.2506 kg/m$^3$ | **$T_2$ [k]** | 290.788 | 273.969 | 268.363 |
| $p_1$ = 300000 Pa | **$\omega_2$ [m/s]** | 9.146*10$^3$ | 0.017 | 0.025 |
| $C_p$ = 1040 J/(kg.K) | **$p_2$ [Pa]** | 3*10$^5$ | 3*10$^5$ | 3*10$^5$ |
| Q = 350 J | **$\rho_2$ [kg/m$^3$]** | 1.106 | 1.174 | 1.198 |

## FUTURE WORK AND CONCLUSIONS

For the obtained results, the conclusions are:

- If the gas flow increases resulting the increasing of gas speed after the generation of the thermal impulse ($\omega_2$), the gas temperature after the thermal impulse generation ($T_2$) will decrease because the guantity of heat regarding the thermal impulse is constant
- The gas density after the thermal impulse generations ($\rho_2$) evolves in direct relation with the flow and exact the oposite as the gas temperature after generating the thermal impulse ($T_2$)
- The gas pression suffers some very small modifications so from the results of the developed software the gas pressure appears as being constant

All the obtained results from running the developed LabVIEW software were verefied also by solving the four equations system in MathCAD and no considerable differences were observed regarding the obtained results in both cases.

Working in this way, using this dedicated new software, it is possible to see a theoretical evolution of the gas temperature – like a prediction – and to compare it with the real results of the experimental work.

The importance of this software is represented by the fact that as imputs, the user can setup all the initial gas state parameters (conditions in which the experiment will take part), the software calculates and predicts the gas parameters for the second state (after the thermal impulse) and all the data can be compared with the experimental results, thus validating the mathematical modelation of the phenomena. Also, all the calculated data through this new prediction software will be used to outline the dependende between the gas speed and temperature, first in the moment of heating when the thermal impulse is generated and afterwards during cooling down, after the thermal impulse was generated, in the moment when the gas temperature reached its peak and starts to decrease.

Because of the simplest way of working, the new developed flow sensor will have also a series of advantages. First of all, because the flow section won`t be disturbed, the flow measurement will be very accurate – no need of the soft correction from the modern flow meters – this being an opportunity for the new sensor to be used in the medical domain (where there is no room for errors) or for the calibration of other types of gas flow meters.

Second, as a consequence of the working principle, the new flow sensor will be able to do accurate measurements in a transitory regime. Third, let`s not forget the possibility of integrating the new flow meter in a S.C.A.D.A. (Supervisory Control And Data Acquisition) system with the help of wireless communication (these days, the processing modules have already integrated inside them the communication parts) for industrial applications.

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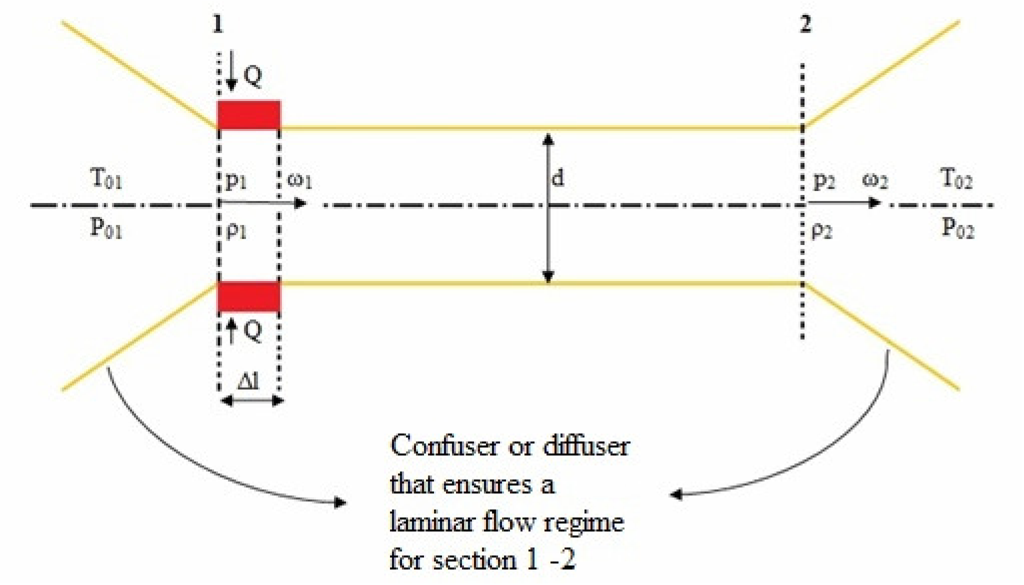


**Fig. 1.** Schematic drawing for the measured system.

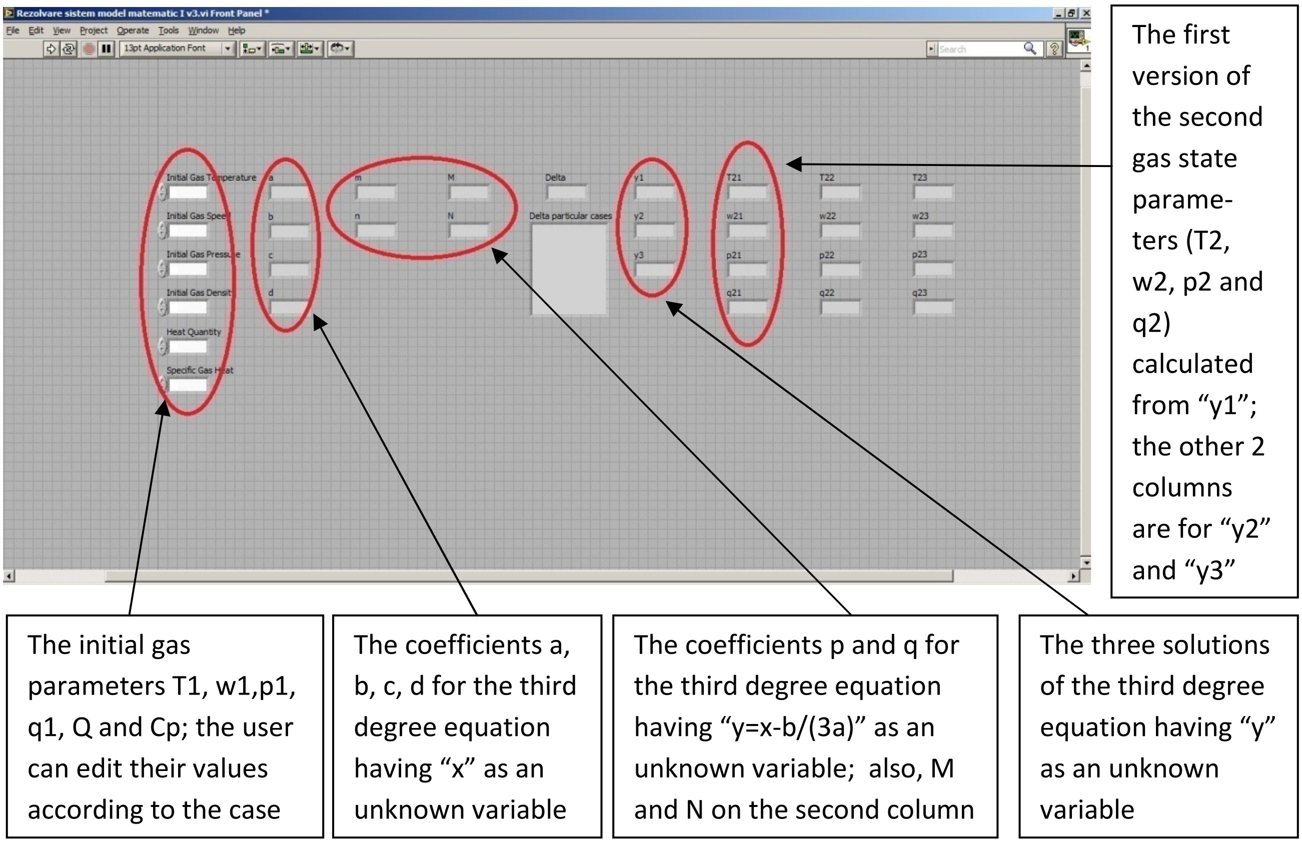


**Fig. 2.** The Front Pannel of the developed software.

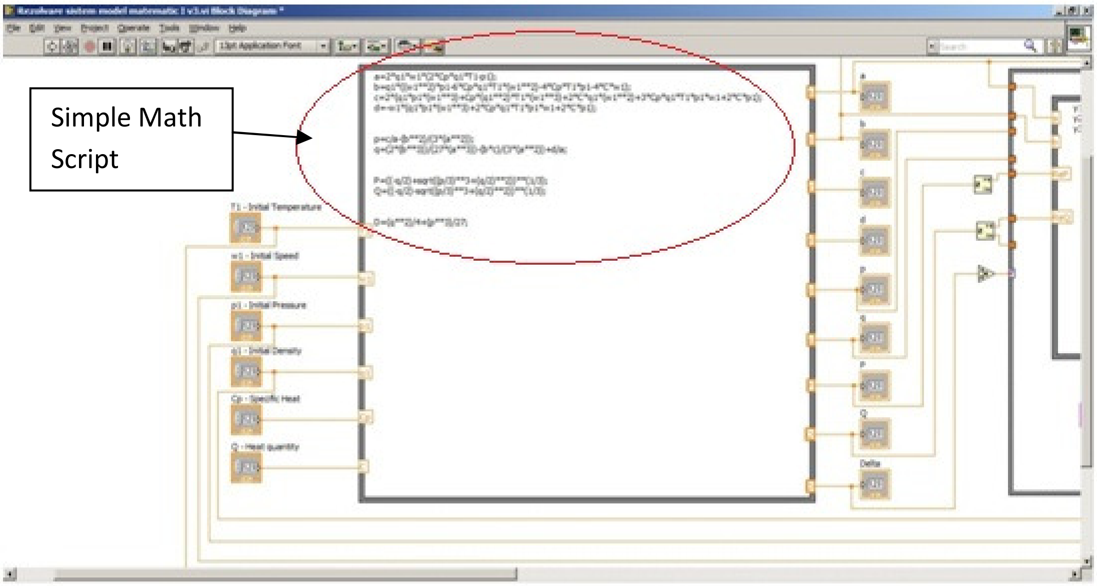


**Fig. 3**: The Back Pannel of the developed software