EM-3

WARHEAD FRAGMENTS MASS DISTRIBUTION USING NUMERICAL COMBINATION OF MOTT'S AND HELD'S FORMULAS

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ABSTRACT

Design of the high explosive (HE) warheads is confronted with series of contradictory requirements. Modeling of fragment size, mass distribution is one of the great importance for determination of fragmenting warhead efficiency. In this work a numerical solution for Mott's formula was built using MATLAB for determination of mass distribution and average mass of fragments. A 105 mm high explosive HE projectile was tested in a pit test and the results are comparing with numerical results. The comparison show small error in average mass of fragments (0.03%). A combination of this method with Held's formula leads to calculate the warhead characteristics λ and B. This gives a good approach of calculating different fragmentation parameters for different warheads

Keywords

Average mass of fragments, mass distribution, mass of fragment, natural fragmentation, number of fragments.

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1. INTRODUCTION

Measurements of warheads performance are expensive. The results are strongly dependent on details of warhead design, necessary economies at the measurement facility, and round-to-round variations. Therefore it is essential to have a capability to make warheads performance predictions in the earliest phases of weapon preliminary design.

Natural fragmentized projectiles or warheads result in a wide range of random fragments distribution (masses and geometry). Expansion of the warhead case under detonation products pressure causes splitting of the warhead body into various sized fragments. For cylindrical steel warhead cases, initial elastic-plastic expansion of the case occurs when it is extended from the original volume to about 1.44 times. When the current case volume being risen to about 2.56 to 3.24 times of the initial warhead volume, the detonation products are released through cracks and subsequently an expanding detonation products cloud is developed beyond the fractured warhead case. [1]

The shapes of fragments from a detonated munition may vary considerably. Many will appear flattened, corresponding to the thickness of the swollen case which is stretched by the expansion that follows detonation. Some fragmentation munitions have a light casing wrapped with a notched metal helix of square cross-section to control the size to some extent and, therefore, the distribution of mass among the fragments. In general purpose munitions; the size of fragment is not controlled; consequently, fragments may vary from dust-like particles to relatively large pieces.

Ordnance warheads are generally axisymetric. This reduces performance calculations to two dimensions. Warhead fragmentation is generally one of two types: natural fragmentation or controlled fragmentation. Natural fragmentation yields a random distribution of fragment sizes (masses) that can be described analytically.

MASS OF THE WARHEAD

The total mass of the warhead (m) is determine by summation of the mass of warhead case (mb), mass of explosive charge (me) .

$$m = m_b + m_e \qquad \text{(kg)} \qquad\qquad (1)$$

3.3.1 Warhead Case Mass
Is the mass of the warhead body (case). It can be estimated by the following equation:

$$m_b = V_b \rho_b = \pi \rho_b \int \left(R^2 - r^2 \right) \, d(R - r) \qquad\qquad (2)$$

Where:
Vb – volume of the warhead case (m3)
ρb – density of the warhead case material (kg/ m3)
R – external radius of the warhead case (m)
r – internal radius of the warhead case (m)

3.3.2 Explosive Charge Mass
The mass of the explosive charge can be estimated by the following equation:

$$m_e = V_e \rho_e = \pi \rho_e \int r^2 dr \qquad\qquad (3)$$

Where:

Ve – volume of the explosive charge (m3)
ρe – density of the explosive charge (kg/ m3)

MOTT AND HELD FRAGMENTS MASS DISTRIBUTION LAW
The developed natural fragmentation model is based on the Mott's theory of break-up of cylindrical "ring-bombs", in which the average length of the resulting circumferential fragments is a function of the radius and velocity of the ring at the moment of break-up, and the mechanical properties of the metal. Following Mott and Linfoot, the "random variations" in fragment sizes are accounted through the following fragment distribution relationship. [2,3]

$$N(m) = \frac{M}{2\mu} e^{-\left(\frac{m}{\mu}\right)^{\frac{1}{2}}} \tag{4}$$

$$\mu = b^2 t^{\frac{5}{3}} D_i^{\frac{2}{3}} \left[1 + \left(\frac{t}{D_i}\right)\right]^2 \tag{5}$$

Where:
N(m) – number of fragments of mass greater than m
μ   – half average fragment mass
Gurney and Sarmousakis developed another expression for thin walled warheads (less than 0.6 in.)

$$N(m) = \frac{M}{\mu_j} e^{-\left(\frac{m}{\mu_j}\right)^{\frac{1}{2}}} \tag{6}$$

$$\mu^{\frac{1}{2}} = \frac{2At(D_o - t)^{\frac{3}{2}}}{D_i} \sqrt{1 + \frac{1}{2}\left(\frac{C}{M}\right)} \tag{7}$$

$$\mu_j = \frac{\mu}{j!} \tag{8}$$

For C/M<2, j=2, C/M>2, j=1, for thick wall (>0.015 m) j=3
Where:
C – explosive mass
M – meatal mass
Do,Di –outer and inner diameter of the warhead case (m)
t   – warhead case thickness (m)
A,b   – a function of the explosive and the case material in equation, the Gurney-Sarmousakis equation. As a rough approximation, for a mild steel case, A=338.1/PCJ, with the detonation pressure (PCJ) in kbar.
Held has developed an experimental approach to represent the fragment mass distribution as a function of a number of fragments. The method of analyzing mass distributions by means of Held's formula is explained as follows [3]

$$M(n) = M\left(1 - e^{-Bn^\lambda}\right) \tag{9}$$

Where:
M(n) – mass of fragments of number of fragments greater than n
B,λ   – constants
To determine the values of B and λ it is convenient to take a logarithm of equation (9) so that in a logarithmic representation the point of intersection n=1, or log n=0, will give constant B directly on the ordinate axis, and exponent λ can be determined from the slope of he straight line (Figure 1). That is

$$\log\left(\ln\frac{M}{M-M(n)}\right)=\log B+\lambda\cdot\log n \tag{10}$$

If the correlation coefficient of the relation Figure 1 is greater than 0.99 the result is accepted else, using the constant B and λ as originally determined, one can now calculate an optimum mass MB, that is, a total mass MB that the best fits this set of equations, as [4]

$$M_B=\frac{M(n)}{1-e^{-Bn^\lambda}} \tag{11}$$

The constants BB and λB are now determined with this new total mass MB as

$$M(n)=M_B\left(1-e^{-B_B n^{\lambda_B}}\right) \tag{12}$$

These set of Held's equations calculations are repeated until the correlation coefficient became greater than 0.99.

2. NUMERICAL SOLUTION

The equations from (1) through (8) are generated for cylindrical warhead and they are not applicable for most types of warheads configurations. Numerical solutions were made to use these equations in the warhead that contain different type of geometric shapes like line, circle, tangent etc.

The way to determine warhead geometrical parameters is by using mathematical functions to determine warhead's geometry. Procedure is straightforward that all we need is the drawing of the warhead (105 mm HE projectile is applied for this research) see Figure 2. Body path is divided in several segments. Every segment has different mathematical function. Here we have six different segments on external body path: (Figure 3)

circle
line
line
line
line
sloped line

Internal warhead body path is also divided in several segments; every segment has different mathematical function. Here also we have six different segments on internal body path: (Figure 4)

line
circle
line
sloped line
circle
line

Finally, the mathematical formulations of the functions which describe warhead external and internal body path are made the calculation is jointed together the warhead body path joint is obtained (see Figure 8). There are eight separate segments of warhead body which we will take into account.

Basis for computation method, for warhead body and explosive charge is warhead with planes perpendicular on symmetry axis. Elements between planes should be cylinder rings (for warhead body) and cylinder (for explosive charge). One must find the warhead parameter that explain in the equations (1) to (8) for every element and add them together so we can obtain parameters of the warhead.

A computer program is built up using MATLAB the language of technical computing to calculate the above warhead parameters. [5,6,7]. Held experimental technique equations (9) to (12) was used to evaluate the characteristics λ and B using accumulation number of fragments from Mott's formula.

3. RESULTS AND DISCUSSIONS

Tables 1 to 3 show the mass, and mass distribution results for 105 mm HE projectile, Table 4 it's for experimental test for 105 mm HE projectile in a pit test to get mass and size of fragments, Figure 5 show a comparison between numerical and experimental results.

The average masses for experimental and predicting technique were 5.2722 and 5.2737 respectivly with small error about 0.03%. The total number of fragments for exprimental and predecting technique were 2188 and 1955 respectivly with error about 10.7 but the different in weight about 10.6% (total weight of tested projectile fragments is 11.5355 kg and 10.31 kg for calculted one) the cause difference related to the numerical solution that just caculate the metal sandwich with explosive and neglect the nose and the base of the projectile and that is not happened in the real case.

Figure 5 show the mass distibution for exprimental, Mott's and Held's formulas, it is clear that Mott's formula distribution start form small rising gradly up to total number of fragments and this is not usually happened with real warheads that increase or decreae from group to another

4. CONCLUSIONS

A combination Held's and Mott's formulae gives a good approach of calculating different fragmentation parameters for different Fragmenting warheads. This will safe a lot of time and minimizes the cost of experiments of developing new fragmenting warheads.

References

[1] Zecevic B., et.al., "Influence of Warhead Case Material on Natural Fragmentation Performances", 15th DAAAM International Symposium, Vienna, Austria, November 2004.

[2] Baker E., Koon N., and Hirlinger J., "A method for Prediction Fragmentation Characteristics of Natural and Preformed Explosive Fragmentation Munitions", Warheads, Energetic and Combat-support Armament Center, Picatinny Arsenal, New Jersey, USA, September 2001.

[3] Richard L., "Conventional Warhead Systems Physics and Engineering Design", American Institute of Aeronautics and Astronautics, Inc, Vol. 179, USA, 1998.

[4] Manfred H., "Fragmentation Warheads", edited by Joseph C., "Tactical Missile Warheads", chapter 8, American Institute of Aeronautics and Astronautics, Inc, Vol. 155 USA, 1993

[5] Palm III J., "Introduction to MATLAB 6 for Engineers", McGraw Hill, New York, 2001. Chapra C., "Applied Numerical Method with MATLAB for Engineers and Scientists", McGraw Hill, New York, 2005.

[6] Kuncicky C., MATLAB Programming, Pearson Prentice Hall, USA, 2004

$$\log\left(-\ln\frac{M_0 - M(n)}{M_0}\right)$$

$$\ln\frac{M_0 - M(n)}{M_0} = -B \cdot n^{\lambda}$$

$$\frac{x}{y} = \lambda$$

log B

log n

Figure 1: Determination of the constant B and λ from the log-log plot

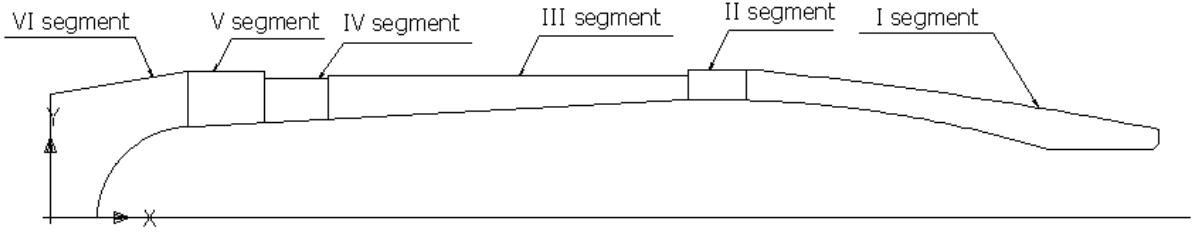Figure 2: 105 HE projectile drawing

Figure 3: External path of 105 mm projectile body
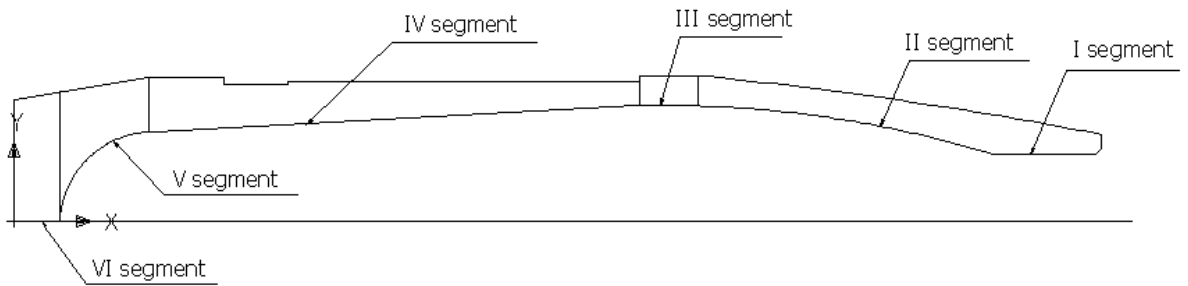

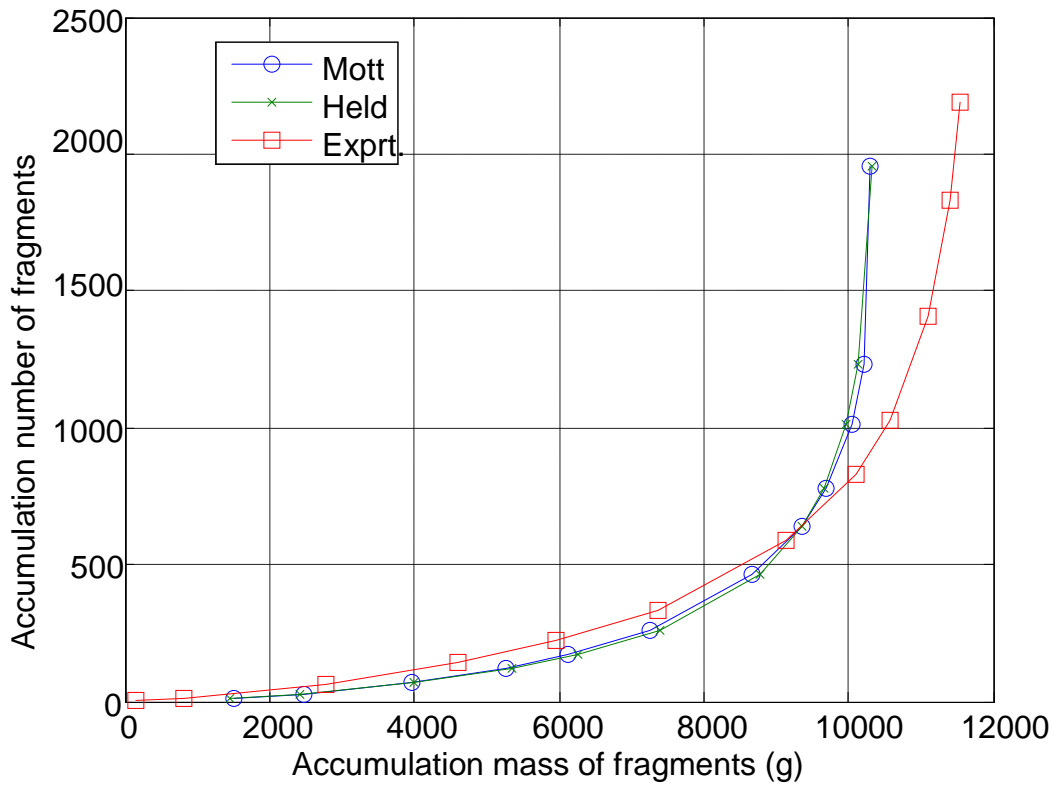Figure 4: Internal path of 105 mm projectile body

Figure 5: 105 mm HE projectile mass distribution

Table 1: 105 mm HE projectile mass

Warhead Item	Mass (Kg)
Case (steel)	11.848
Explosive (TNT)	2.1928
Total Mass	14.04
C/M	0.21272

Table 2: mass distribution for 105 mm HE projectile using Numerical technique

Mass group (g)	frag. number	mass of frag. in group	Cumulative frag. number	Cumulative mass of frag. in group (g) Mott's	Cumulative mass of frag. in group (g) Held's
Up 150	3	469.81	3	469.81	489.79
100-150	4	515.77	7	985.58	956.23
75-100	6	524.09	13	1509.7	1455.2
50-75	16	965.01	29	2474.7	2419.4
30-50	39	1503.4	68	3978.1	3989.4
20-30	53	1280.1	121	5258.2	5345.9
15-20	51	876.96	172	6135.1	6264.9
10-15	91	1112.1	263	7247.2	7397.1
5-10	201	1431.5	464	8678.7	8766.3
3-5	173	674.22	637	9352.9	9368.6
2-3	142	351.45	779	9704.4	9674.4
1-2	237	344.05	1016	10048	9979.7
0.5-1	213	154.74	1229	10203	10134
0-0.5	727	107.15	1955	10310	10335

Table3: Fragments parameters for 105 mm HE using numerical technique

Average Mass of Fragment (g)	5.2737
Total Number of Fragments	1955
λ	0.58137
B	0.083032
R2 (coreleation ceffecient)	0.99835

Table 4: Mean values for 4 tested 105 mm HE projectiles in Pit test

Mass group (g)	frag. number	mass of frag. in group	Cumulative frag. number	Cumulative mass of frag. in group (g)
75 -100	2	162.5	2	162.5
50 - 75	11	664.5	13	827
30 - 50	52	1941.3	65	2768.3
20 - 30	76	1833.3	141	4601.6
15 - 20	79	1353.8	220	5955.4
10 – 15	114	1400	334	7355.4
5 -10	254	1773.5	588	9128.9
3 – 5	245	966.5	833	10095.4
2 – 3	193	474	1026	10569.4
1 – 2	379	535.3	1405	11104.7
0 – 1	423	300	1828	11404.7
to 0.5 g	360	130.8	2188	11535.5
Average Mass of Fragment (g)			5.2722	