Military Technical College Kobry El-Kobbah, Cairo, Egypt



14th International Conference on Applied Mechanics and Mechanical Engineering.

Modeling and Simulation of a Control System for a Remotely Operated Tracked Vehicle

By

A. Eliewa*

M. Atef*

M. Elhaddad**

R. Khodary*

Abstract:

This paper deals with a proposal for building up a control module to convert the APC M113 into a remotely controlled vehicle. The driving of the vehicle has been simplified and performed through a simple control lever similar to the computer joystick which is able to control the vehicle driving systems; steering and braking system, accelerator pedal, and gear shifting lever. The necessary sensors and actuators have been added to the conventional M113.

A microcontroller is used as a control unit; the required interfacing circuits were designed. LABVIEW Software® is used as the interface between the microcontroller and the remote control station. Finally a mathematical model is prepared to simulate the proposed control module and measure its response and stability

Keywords:

Unmanned systems, Electronic control, Microcontroller.

- * Egyptian Armed Forces
- * Higher Institute of Technology-10th of Ramadan.

1. Introduction:

In recent years, there has been a great interest worldwide in the development of Autonomous Ground Vehicle System (AGVS) technologies due to their potential in civil and military applications. [1]

The objective of the control system is for the tracked vehicle to safely follow the commanded speed and heading rate (steering angular velocity). A tracked vehicle is a highly complicated non-linear and uncertain system that involves many mechanical systems, such as a diesel engine, gear and transmission, steering differential unit, track/soil interaction and brake systems. [2]

There are several attempts have been made to address various issues for the control of tracked vehicles. [3] - [8].

This paper describes a control system to convert the Armor Personnel Carrier (APC) to be remotely controlled. The control system design includes; design and selection of the hydraulic actuators, selection of the necessary sensors that give the required feedback. To check the control system response, a mathematical model is introduced.

All details of the proposed control system including design of the actuators, sensors selection, control program flowcharts, LabVIEW interfacing between the vehicle and the remote control station and finally designing the interfacing circuits are presented in [9]

2. The tracked vehicle:

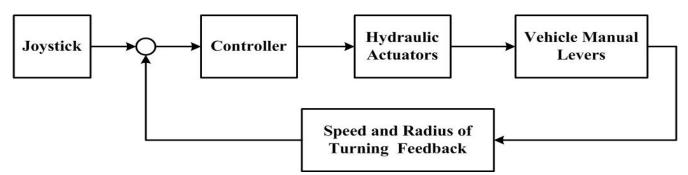
The M113 is an American armored personnel carrier; it is powered by a diesel engine of 210 hp, an automatic transmission with three forward speeds and one reverse, and a differential mechanism for steering and braking the vehicle. Steering of the vehicle is achieved by decreasing the speed of one track and increasing the speed of the other (using brakes inside the steer differential unit). The brakes are actuated mechanically and manually by the driver via two hand – command levers. The ranges of gears are selected manually but the gear ratio within the manually selected range varies automatically. The vehicle is equipped with a rear ramp for crew entrance. [10][15]

For manual driving, the accelerator pedal and the left and right steer levers are operated. Pressing / releasing the accelerator pedal increases / decreases the engine power and vehicle speed. Pulling the left / right steer levers steers the vehicle to the left / right. Pulling both the left and right steer levers reduces the vehicle speed.

3. The control system:

The control system is proposed to take the decisions to operate different actuators that drive the mechanical control linkages; so the driving is automated and simplified. The system is controlled through a control lever which is able to control the vehicle operation directly or through a wireless link from a control station in case of remote control. Figure (1) shows a simple block diagram of the proposed control system.

The proposed control system consists of the control systems of steering and braking levers,



gear shifting lever, accelerator pedal, actuators, sensors and controller.

Figure (1): Vehicle control system block diagram

3.2 Accelerator pedal control system:

The proposed accelerator control system consists of an accelerator pedal, a hydraulic subsystem which actuate the pedal, and a control subsystem which consists of a controller, sensors, and interfacing circuits. Figure (2) shows a block diagram of the accelerator pedal control system. The controller is designed to deliver the prescribed actuating signal to the hydraulic subsystem according to the input signal. The hydraulic subsystem actuates the accelerator pedal according to the input signal (the desired speed). The necessary sensors are added to detect the output signal, the measured feedback signal is the engine crankshaft speed that given by a DC tachometer, and the pedal displacement using a linear variable differential transformer (LVDT).

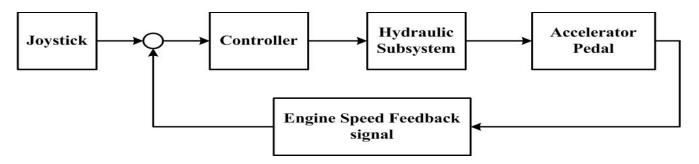


Figure (2): Accelerator pedal control system block diagram

3.1 Steering and braking control system:

The proposed steering and braking control system shown in Figure (3). The controller is designed to deliver the prescribed actuating signal to the hydraulic subsystem according to the input signal from joystick. The hydraulic subsystem actuates the levers according to the input signal, it actuate one lever (either right or left) in case of steering, and actuate both levers in case of braking. The necessary sensors are added to detect the output signal, the measured feedback signals is the speed of the right and left tracks of vehicle that measured using digital optical encoders which measure the angular velocity of the right and left output shafts and then with mathematical relations the radius of turning of vehicle is measured. The

other feedback signal is the indication of brakes if are completely applied or completely released using mechanical limit switches

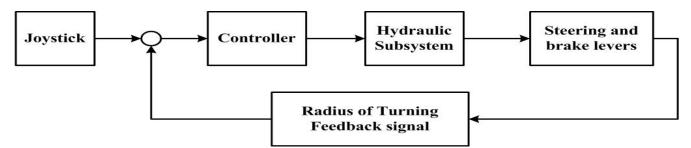


Figure (3): Steering and braking control system block diagram

4. Modeling and simulation:

The most important requirement in modeling a system is the complete understanding of the performance specifications, physical and operational characteristics of each component in the system.

4.1 Accelerator pedal control system Modeling:

The considered accelerator pedal control system can be divided into hydraulic subsystem, mechanical subsystem and controller. The hydraulic subsystem consists of a pump that supply oil continuously to the system, a proportional directional control valve that control the oil direction to the hydraulic cylinder that is the actuator connected to the accelerator pedal. The mechanical subsystem consisting of the accelerator pedal with its mechanical linkages.

The mechanical subsystem consisting of the accelerator pedal with its mechanical linkages and the engine.

As shown here, there are a lot of parameters included in the accelerator pedal control system model. So MATLAB[®] SIMULINK was used to model the equations governing the operation of the system. The overall simulink model is shown in Figure (4), where most of the accelerator pedal control system components were considered.



Figure (4): Accelerator pedal control system simulink model

4.1.1 Hydraulic subsystem mathematical modeling:

The input to the system is the flow rate of fluid supplied by the pump, and the output is the cylinder displacement as shown in Figure (5).

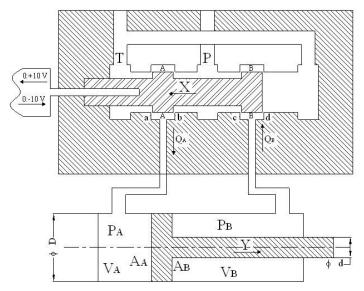


Figure (5): Schematic diagram of the hydraulic system for numerical simulation

To simulate the valve, it has been divided into two main parts, the solenoid and the directional valve block. [16], [17]

Displacement of the valve spool:

$$F_s = mx + f_s x + k_s x \tag{1}$$

Flow equations through valve restriction areas:

$$Q_a = C_d A_a(x) \sqrt{\frac{2}{\rho}} (P_A - P_t)$$
⁽²⁾

$$Q_{b} = C_{d}A_{b}(x)\sqrt{\frac{2}{\rho}(P_{s} - P_{A})}$$
(3)

$$Q_{c} = C_{d}A_{c}(x)\sqrt{\frac{2}{\rho}(P_{s} - P_{B})}$$
(4)

$$Q_{d} = C_{d}A_{d}(x)\sqrt{\frac{2}{\rho}(P_{B} - P_{t})}$$
(5)

For $x \ge 0$

$$A_a = A_c = \omega \sqrt{\left(x^2 + c^2\right)} \tag{6}$$

$$A_b = A_d = A_r = \omega c$$

For $x \le 0$

$$A_a = A_c = A_r = \omega c \tag{7}$$

$$A_b = A_d = \omega \sqrt{(x^2 + c^2)}$$

Continuity equation applied to the cylinder chambers:

$$Q_A - A_A \frac{dy}{dt} - Q_i - Q_{eA} = \frac{V_A}{B} \left(\frac{dP_A}{dt}\right)$$
(8)

$$A_{B}\frac{dy}{dt} + Q_{i} - Q_{B} - Q_{eB} = \frac{V_{B}}{B}\left(\frac{dP_{B}}{dt}\right)$$
(9)

The flow rates of Q_A and Q_B are given by:

$$Q_A = Q_b - Q_a \tag{10}$$

$$Q_B = Q_d - Q_c \tag{11}$$

The volumes V_A and V_B are given by:

$$V_A = V_{A_o} + A_A y \tag{12}$$

$$V_{B} = V_{B_{o}} - A_{B} y \tag{13}$$

Assuming that the leakage flow rate is linearly proportional to the pressure difference, the leakage flow rates could be given by the following relations:

$$Q_i = (P_A - P_B) / R_i \tag{14}$$

$$Q_{eA} = P_A / R_e \tag{15}$$

$$Q_{eB} = P_B / R_e \tag{16}$$

Equation of motion of piston:

$$P_A A_A - P_B A_B + F_L = m_P \frac{d^2 y}{dt^2} + f_P \frac{dy}{dt} + F_{Aseat}$$
(17)

$$F_{Aseat} = \begin{cases} K_{Aseat}(y - y_{max}) + f_{Aseat} \frac{dy}{dt} & \text{for } y \ge y_{max} \\ 0 & \text{for } y_{min} \le y \ge y_{max} \\ K_{Aseat}|y - y_{min}| + f_{Aseat} \frac{dy}{dt} & \text{for } y \le y_{min} \end{cases}$$
(18)

All the above equations were used in the hydraulic subsystem simulink model as shown in Figure (6).

6

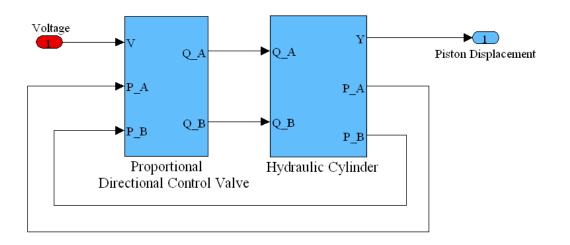


Figure (6): Hydraulic subsystem simulink model

4.1.2 Mechanical subsystem mathematical modeling:

The input of system is the accelerator pedal displacement (that equals to the piston displacement), and the output is the engine rpm.

A mathematical model describes the dynamic behavior of engine with regard to all parameters affecting the operation of engine are as follows [18]. The block diagram of engine is shown in Figure (7).

The steady state operation of engine is characterized by the combination of engine power (Ne), engine torque (T), engine r.p.m (n_e), boost pressure (P_b), specific fuel consumption (g_e), air fuel ratio (α) and engine efficiency (η_e).

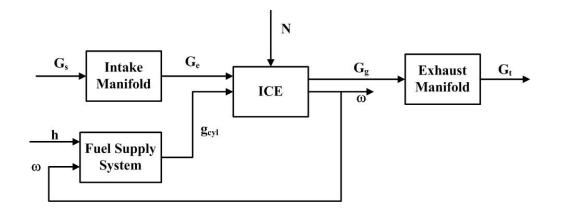


Figure (7): Engine block diagram

The investigation of the engine operation should be studied under the unsteady – state conditions. The only indication of the unsteady – state conditions is the variation of engine power with one, several or all engine parameters and time, i.e.

$$N_{e} = f\left(T, \omega, P_{b}, g_{e}, \alpha, \eta_{e}, t\right)$$
(19)

The dynamic properties of engine prime mover are characterized by the following differential equation:

$$J\frac{d\omega}{dt} = T - T_{l}$$
⁽²⁰⁾

The current values of the parameters of the above equation

$$\omega = \omega_o + \Delta \omega_T T = T_o + \Delta T_T T_l = T_{lo} + \Delta T$$

Substituting the variables current values into equation, we get:

$$J\frac{d\Delta\omega}{dt} = \Delta T - \Delta T_{l}$$
⁽²¹⁾

The reduced mass moment of inertia is calculated by simplifying the engine to a simple slider crank mechanism and a complete kinematic and kinetic analysis should be investigated [19]. The load torque depends on the angular velocity ω and the load setting N (the road slope, type of terrain, etc....), i.e.

$$T_{l} = f\left(\omega, N\right) \tag{22}$$

This equation is non – linear, but at low $\Delta \omega$, this equation may be linearly approximated using Taylor series expansion, this gives:

$$\Delta T_{l} = \frac{\partial T_{l}}{\partial \omega} \Delta \omega + \frac{\partial T_{l}}{\partial N} \Delta N$$
(23)

In this model, the applied load settings are the coefficient of rolling resistance and the road slope angle, so equation (23) will be

$$\Delta T_{l} = \frac{\partial T_{l}}{\partial \omega} \Delta \omega + \frac{\partial T_{l}}{\partial f} \Delta f + \frac{\partial T_{l}}{\partial \alpha} \Delta \alpha$$
(24)

The relationship between the load torque of engine and the angular velocity is given from following equation:

$$T_{l} = \frac{f_{c} G r_{d.s}}{i_{T} \eta_{T}}$$
(25)

$$f_c = f \cos \alpha + \sin \alpha \tag{26}$$

The relationship between the load torgue and the load settings can be calculated from the following equation:

$$T_{l} = \frac{f_{c} G r_{d.s}}{i_{T} \eta_{T}}$$
(27)

Back to equation (21), the engine torque depends on cycle fuel supply (g_{cyl}) and completeness of fuel combustion. The latter depends on the amount of air supplied into the combustion chamber. Since the cycle fuel supply is determined by the position (h) of the throttle as well as by the angular velocity (ω), and the boost pressure (P_b). But the M113 engine is a normally aspirated engine, i.e. without a turbocharger, so $P_b = 0$, so we have:

$$T = f(h, \omega)$$
(28)

The expansion of this equation into a Taylor series gives:

$$\Delta T = \frac{\partial T}{\partial h} \Delta h + \frac{\partial T}{\partial \omega} \Delta \omega$$
⁽²⁹⁾

The relationship between engine torque and angular velocity is known from the engine performance characteristics, [13]

The relationship between engine power and accelerator position is calculated from the following equation, [4]

$$p = 0.2y$$
 $y \le 50$
 $p = 10 + 1.8(y - 50)$ $y > 50$ (30)

Since the relationship between engine power and engine torque is known, so using the previous equation, the relationship between engine torque and accelerator displacement can be determined and can be calculated

Substituting by equations (24) and (29) into equation (21), this gives:

$$T_{e}\left(\frac{d\varphi}{dt}\right) + k_{e}\varphi = \chi - \theta_{e1}\alpha_{e1} - \theta_{e2}\alpha_{e2}$$
(31)

Where:

$$T_{e} = J\left(\frac{\partial h}{\partial T}\right)\left(\frac{\omega_{o}}{h_{o}}\right)$$
(32)

$$k_{e} = F_{e} \left(\frac{\partial h}{\partial T}\right) \left(\frac{\omega_{o}}{h_{o}}\right)$$
(33)

$$F_{e} = \left(\frac{\partial T_{l}}{\partial \omega} - \frac{\partial T}{\partial \omega}\right) \Delta \omega$$
(34)

$$\boldsymbol{\theta}_{el} = \left(\frac{\partial T_l}{\partial N}\right) \left(\frac{\partial h}{\partial T}\right) \left(\frac{f_o}{h_o}\right)$$
(35)

$$\theta_{e2} = \left(\frac{\partial T_l}{\partial N}\right) \left(\frac{\partial h}{\partial T}\right) \left(\frac{\alpha_o}{h_o}\right)$$
(36)

$$\varphi = \frac{\Delta \omega}{\omega_o}, \quad \chi = \frac{\Delta h}{h_o}, \quad \alpha_{e_1} = \frac{\Delta f}{f_o}, \quad \alpha_{e_2} = \frac{\Delta a}{\alpha_o}$$

All the above equations were used in the engine subsystem simulink model as shown in Figure (8)

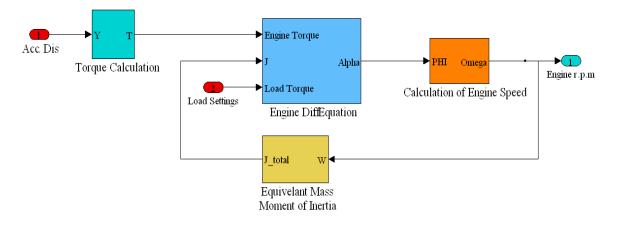


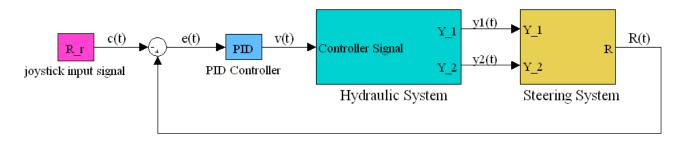
Figure (8): Engine simulink model

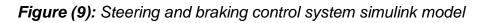
4.2 Steering and braking control system modeling:

The considered steering and braking control system can be divided into hydraulic subsystem, mechanical subsystem and controller.

As previous, there are a lot of parameters included in model. So MATLAB[®] SIMULINK was used to model the equations governing the operation of the system. The overall simulink model is shown in Figure (9) where most of the accelerator pedal control system components were considered.

The hydraulic subsystem is the same as of the accelerator pedal that previously discussed.





4.2.1 Steering differential unit mathematical modeling:

The input of system is the brake levers displacement (that equal to the pistons displacement), and the output is the radius of turning of the vehicle.

The steering mechanism (including the steer differential, steering and brake levers, and control linkages) is highly non – linear and uncertain. To achieve the required control system performance, it is important to identify and compensate the non – linearities in the system. However, in practice, precise identifications of these non – linearities are almost impossible [4], so the following empirical equation can represent the relationship between the brake levers' position and the effective braking effort

$$f = 0 \qquad \qquad \mathbf{x} \le 30$$

$$f = 100e^{-3}(x-30)/20 \qquad 30 < x \le 50$$

$$f = 100e^{-6(100-x)/100} \qquad x > 50$$
(37)

The above equation gives a relationship between the lever position and the effective braking effort, but the required relationship is between the levers displacement and the radius of turning. So from the steering characteristics [5], a relationship between lever displacement and radius of turning can be developed:

$$f_T = f_2(1+i_o) - f_1(1-i_o)\eta_R$$
(38)

$$f_2 = 0.25 \,\mu \frac{L}{B} + 0.5f \tag{39}$$

$$f_1 = 0.25 \ \mu \frac{L}{B} - 0.5f \tag{40}$$

$$\mu = \frac{\mu_{max}}{0.85 + 0.15 \frac{R}{B}}$$
(41)

$$\eta_R = \eta_{fd}^2 \times \eta_{cd}^2$$
(42)

All the above equations were used in the engine subsystem simulink model as shown in Figure (10).

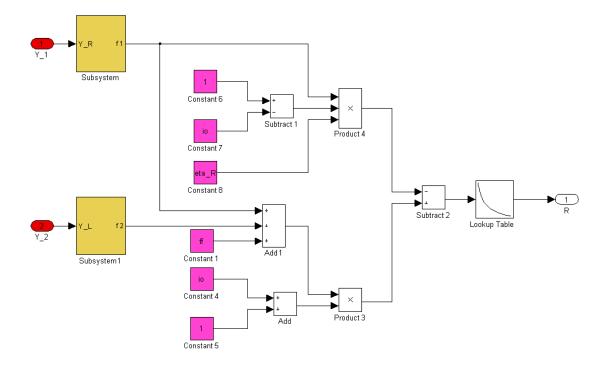


Figure (10): Steering differential unit simulink model

5. Simulation results:

5.1 Accelerator pedal control system simulation results:

These results include the step response of engine speed and of accelerator displacement. PID controller is added to the control system to eliminate the overshoot, increase rise time and decrease steady – state error as shown in the following figures.

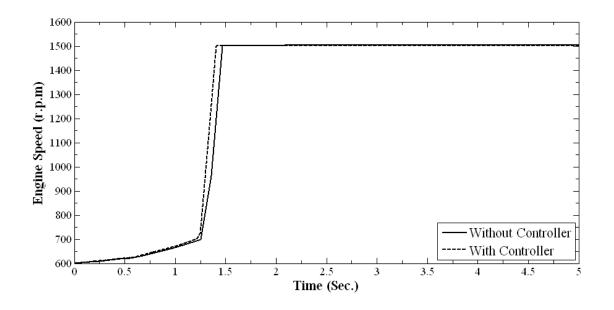


Figure (11): Step response of engine speed at 1500 r.p.m showing the effect of PID controller

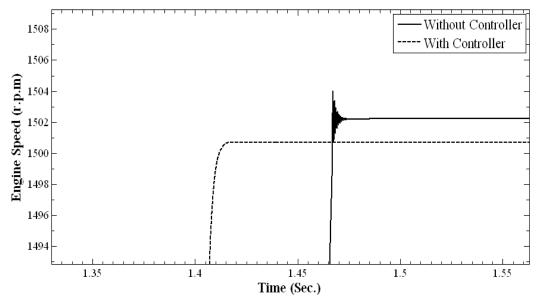


Figure (12): Magnification of step response of engine speed at 1500 r.p.m showing the effect of PID controller

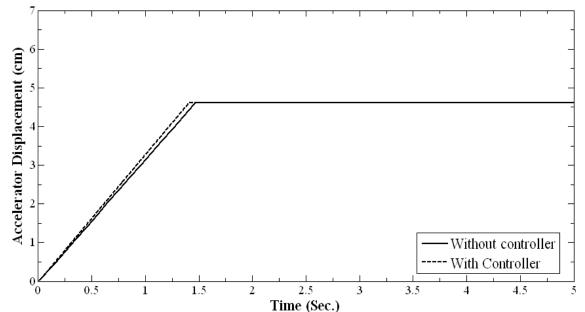


Figure (13): Step response of accelerator displacement at 1500 r.p.m showing the effect of PID controller

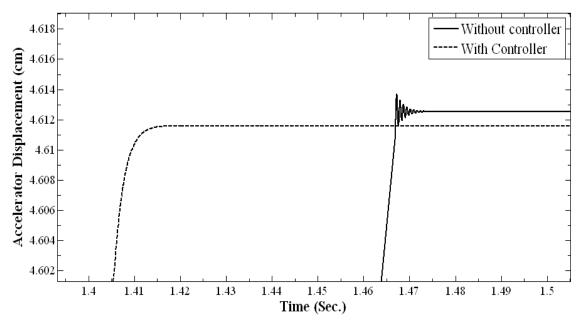


Figure (14): Magnification of step response of accelerator displacement at 1500 r.p.m showing the effect of PID controller

5.2. Steering and braking control system simulation results:

A step input signal is applied to the system, to the required radius of turning which is selected in this case to be 40m as shown in Figure (15).

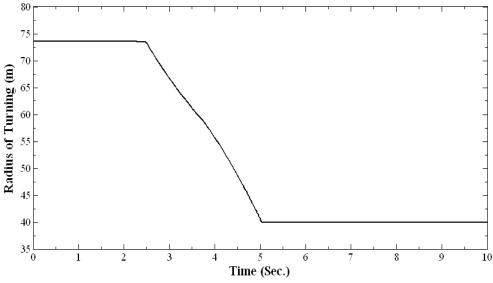


Figure (15): Step response of a steering model

As shown in this figure, and from equation (37), it was a constant output radius for about 2.35 sec. this is because of the braking force is zero when the steering lever displacement is less than 30% of its maximum, so the radius of turning is constant. Increasing the lever displacement will brake the inner track and the radius of turning is reduced until it reaches the required value. There was a small overshoot before reaching a steady – state radius (40 m). so a PID controller is used as shown in

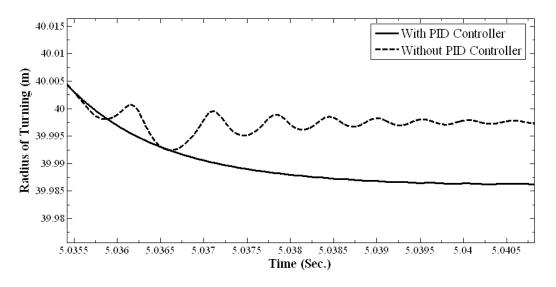


Figure (16): Effect of PID controller on Step response of a steering model

6. Conclusions:

The research outlined in this paper has demonstrated the feasibility of modifying a conventional APC M113 to be remotely controlled. This proposal is the first step for converting the APC M113 to be autonomous.

The dynamic models of accelerator pedal and steering and braking control systems are divided into two subsystems; hydraulic and mechanical to facilitate system simulation. The developed engine model can be easily adapted to fit in any tracked vehicle equipped with ICE but with some variations in some parameters.

The response of the model is good and can be compared with practical results as the system will implemented.

References:

- [1]. H. Durrant Whyte, A Critical Review of the State of the Art in Autonomous Land Vehicle Systems and Technology, SANDIA REPORT 2001, Sandia International Laboratories, USA, 2001
- [2]. J. Y. Wang, *Theory of Ground Vehicles (2nd Edition)*. John Wiley, London, 1993
- [3]. N. K. Boon, T. H. Choo, and C. C. Wah, Autonomous Unmanned Ground Vehicle and Indirect Driving, Data Horizons 2005
- [4]. Z. Gong, J. I. Guzman, S. J. Scheding, D. C. Rye, G. Dissanayake, H. D. Whyte, A Heuristic Rule – Based Switching and Adaptive PID Controller for a Large Autonomous Tracked Vehicle: Form Development to Implementation, Proceedings of the 2004 IEEE, International Conference on Control Applications, PP 1277 – 1277, Taiwan. April 2004.
- [5]. J. G. Morillon, and O. Lecointe, SYRANO: A Ground Robotic System for Target Acquisition and Neutralization, Proceedings of SPIE, Vol. 5083, Aero Sense 2003, Unmanned Ground Vehicle Technology V, April 2003.
- [6]. I. Schwartz, PRIMUS Autonomous Driving Robot for Military Applications, Proceedings of SPIE, Vol. 4024, PP 313 – 325, Aero Sense 2000, Unmanned Ground Vehicle Technology II, April 2000
- [7]. G. G. Wang, S. H. Wang and C. W. Chen, *Design of a Turning Control for a Tracked Vehicle*, IEEE Control System mMagazine, Vol. 10, No. 3 P.P 122 125, April 1990.
- [8]. J. J. Nitao and A. M. Paradi, A Teal Time Reflexive Pilot for an Autonomous Land Vehicle", IEEE Control System Magazine, Vol. 6, No. 1, PP 14 – 23, Feb. 1986.
- [9]. A. Eliewa, M. Atef, M. Elhaddad and R. Khodary, A Proposed Control System for Autonomous Unmanned Ground Vehicle, Proceedings of ASAT 2009
- [10]. www.FAS.org.
- [11]. Unit Maintenance Technical Manual of APC M113 A2, (TM 9 2350 261 20 1) Washington, Headquarters of US Army, July 1997.
- [12]. Operator's Manual of APC M113-A2, (TM 9 2350 261 10), Washington, Headquarters of US Army, July 1997.
- [13]. Technical Manual for Direct Support and General Support Maintenance engine of APC M113 – A2, (TM 9 – 2815 – 205 – 34), Washington, Headquarters of US Army, Sep. 1991
- [14]. Technical Manual for Direct Support and general Support Maintenance Automatic Transmission of APC M113-A2, (TM 9 – 2520 – 254 – 34), Washington, Headquarters of US Army, Sep. 1991
- [15]. Technical Manual for Direct Support and general Support Maintenance Steering Differential of APC M113-A2, (TM 9 – 2520 – 238 – 34), Washington, Headquarters of US Army, Sep. 1991
- [16]. M. G. Rabie, Fluid Power Engineering, McGraw Hill, New York, 2009

- [17]. T. Elmenshawy, Investigation of Dynamic Behavior of an Electro Proportional System, MSc. Thesis, Military Technical College, Cairo 2006
- [18]. V. I. Krutov, Automatic Control of Internal Combustion Engines, 1st Edition, Mir Publishers Moscow. 1987.
- [19]. R. L. Norton, Design of Machinery, an Introduction to Synthesis and Analysis of Mechanisms and Machines, 3rd Edition, WCB / McGraw – Hill Inc., U.S.A, 2004

Nomenclatures:

- A... Restriction areas, m²
- $A_A \dots$ Piston area of cab side, m²
- A_B... Piston area of rod side, m²
- A_r... Radial clearance area, m²
- B.... Track gauge, m
- B Bulk modulus
- C_d... Discharge coefficient
- c ... Spool radial clearance, m
- F_{Aseat} ... Actuator seat reaction force, N.
- F_L... Load force, N
- F_S... Solenoid force, (N)
- f_{Aseat} ... Actuator seat friction coefficient, Ns/m.
- f_c... Coefficient of total resistance
- fp ... Piston friction coefficient, Ns/m
- fs ... Friction coefficient, Ns/m
- $f_1 \dots$ Needed specific force on inner track
- f₂... Needed specific force on outer track
- G ... Weight of vehicle, ton
- Ge ... Mass flow rate of intake air to the prime mover (engine cylinders), m³/s
- G_{g} ... Mass flow rate of exhaust gases to the exhaust manifold, m^{3}/s
- $G_s \dots$ Mass flow rate of intake air to the intake manifold, m³/s
- G_t... Mass flow rate of the exhaust gases, m³/s
- G ... Gravitational acceleration, m/s²
- $g_{cyl} \dots$ Mass flow rate of the fuel supply to the prime mover
- h ... Throttle displacement, m
- $h_c \ldots$ Height from the terrain to the bottom of the vehicle, m
- i ... Total transmission ratio from gear box to the driving sprocket
- *i*_{*f.d*} ... gear ratio of final drive
- *i*_{s.m} ... gear ratio of steering mechanism
- io... Internal gear ratio of double differential steering mechanism

- J ... Reduced mass moment of inertia of engine, kg.m²
- k ... Spool spring stiffness, N/m
- K_{Aseat} Actuator seat stiffness, N/m.
- ke ... Dimensionless coefficient of self regulation
- m ... Spool mass, kg
- N ... Load settings
- P_S... Supply pressure, Pa
- Pt ... Return pressure, Pa
- p... Percentage of engine power output to the maximum
- Q ... Flow rate, m³/s
- Q_P... Pump flow rate
- $Q_a \dots$ The actual flow rate of the pump, m³/s
- Q_c... Cap flow rate, m³/s
- Qe ... External leakage flow rate, m³/s
- Q_i... Internal leakage flow rate, m³/s
- Qr ... Rod flow rate, m³/s
- Q_{th} ... The theoretical flow rate of the pump = Q_P
- q_I The internal leakage of the pump
- R ... Radius of turning, m
- R_e... Resistance to external leakage, Pa s/m³
- R_i... Resistance to internal leakage, Pa s/m³
- $T_e \dots$ Time of prime mover, sec
- T_I... Load torque, N.m
- To ... Steady state value of engine torque, N.m
- T_{lo} ... Steady state value of engine torque, N.m
- v_P ... Piston velocity, m/s
- V_A... Volume of oil filling the cylinder at cab side, m³
- $V_B \dots$ volume of oil filling the cylinder at rod side, m³
- V_{Ao} ... Initial volume of the cab side, m³
- V_{B_0} ... Initial volume of the rod side, m³
- $V_{q} \dots$ Pump nominal capacity, m³/s
- x ... Displacement, m
- y ... Percentage of the accelerator position to the maximum
- $\eta_{c.d}$... Efficiency of caterpillar drive
- $\eta_{f.d}$... Efficiency of final drive
- μ... Coefficient of lateral resistance to turning
- $\omega \dots$ Width of the port, m

- $\omega_2 \dots$ Angular velocity of crankshaft
- $\omega_{o} \dots$ steady state value of angular velocity of crank shaft, rad/sec
- $\theta_e \dots$ Dimensionless coefficient of load setting gain