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The Effect of a Moving Heat Source and Relaxation Times on Viscoelastic Biological Tissues During Thermal Treatments

Ibrahim A. Abbas, Aboelnour N. Abdalla, Abdelrahman A. Abbas*

Mathematical Department, Faculty of Science, Sohag University, Sohag 82524, Egypt
**Email: abdrahmanabbas20@gmail.com*

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Abstract: This study uses mathematical modeling to attempt to provide computational information about bioheat transfer in elastic skin tissue generated by a moving heat source that depends on different relaxation times. To achieve success in implementing heat treatment procedures, it requires a precise understanding of the heat transfer mechanism and the thermo-mechanical interaction relevant to living tissues., phase delay parameters based on relaxation times were taken into account. Due to the importance of the role of the effect of different variables on thermal relaxation times, this effect on temperature distributions, thermal stress, and displacement in living tissues was studied. Because of our use of powerful mathematical methods such as Laplace transformations and others, we were able to obtain accurate evaluations and calculations of the distributions of displacement, thermal stress, and temperature. After that, the results are given in the form of a graph for displacement, temperature, and stress. Finally, to enhance our understanding of how living tissues behave in thermal environments and to customize hyperthermia treatments, these results and a good understanding of them will do this, which will lead to improved treatment results.
Keywords: Laplace transforms; thermal relaxation times; biological tissue; Thermo-mechanical interaction; bio-thermo-viscoelastic model.

List of symbols

q	The heat flux
L	Characteristic length
t	Time
λ, μ	Lamé's constants
$c_e = [(\lambda + 2\mu)/\rho]^{1/2}$	Speed of propagation of isothermal elastic waves
ρ	Density of tumor tissue
c	Specific heat of tumour tissue
ρ_b	Blood mass density
$K = \lambda + (2/3)\mu$	Bulk modulus
ω_b	Blood perfusion rate
e_{ij}	Strain tensor components
α_T	Coefficient of linear thermal expansion
θ_o	Reference temperature
θ_b	Blood temperature
c_b	Specific heat of blood
u_i	Components of displacement vector
θ	Temperature
τ_o	Relaxation times
σ_{ij}	Components of the stress tensor
k	Thermal conductivity of the tumor tissue
τ_1, τ_2	Viscoelastic relaxation times

1. Introduction

Tumors are one of the most common and challenging diseases in the modern era, posing a significant challenge to the global medical community. Scientists and doctors have faced great challenges in finding an effective and safe treatment for this complex health condition. Among the many methods used to treat tumors, the use of thermal sources is one of the promising and innovative approaches in this field. Thermal sources are considered an effective tool in tumor therapy, where they are used to heat cancerous tissues at high temperatures, leading to their destruction. A variety of thermal sources are used in this treatment, including far ultraviolet rays, infrared rays, and air heating. Numerous studies conducted by global research institutions and medical centres have demonstrated the effectiveness of thermal therapy in treating various tumors. Recent results have shown the success of this treatment in reducing tumor size, decreasing cancer cell growth, and improving the quality of life for patients. Additionally, thermal therapy is believed to help reduce the side effects of other treatments used in tumor therapy, such as chemotherapy and radiation therapy, making it a favourable option for patients.

The thermal characteristics of live tissue are only approximate due to the difficulty of monitoring in vivo. This is because necropsy can alter the thermal properties of the tissue, and there will be no perfusion effects in tissue that is analysed outside of the body. Because the thermal behaviour of biological tissues is dependent on various intricate events, such as metabolic heat generation and blood circulation, researchers have constructed some governing equations. Viscoelasticity refers to the property of materials that display both viscous and elastic qualities when deformed. Ilioushin and Pobedria's book [1] includes an explanation of the thermal viscoelasticity theory. Materials that display temperature and time dependency when subjected to a load are known as linear viscoelastic materials. All biological tissues have mechanical viscoelastic qualities, which are crucial to their distinctive activities. This is due to the presence of viscoelastic components in structural proteins, extracellular matrices, and tissue cells. Viscoelasticity has been demonstrated in even hard tissue. Pennes' [2] bio-heat transfer equation explains the

heated behaviour that depends on the original Fourier's law. Indeed, Fourier's law predicts an endlessly accelerating transfer of thermal signals, which manifestly contradicts reality. Furthermore, due to the substantial internal inhomogeneity of living biological tissues, heat nevertheless spreads through them at a finite rate. However, Several documented studies by Kaminski [3] and Luikov [4] have demonstrated that heat transfer in non-homogeneous media requires a longer relaxation period to accumulate enough thermal energy to travel to the nearest section. Pennes [2] developed a mathematical model to investigate heat response in the human forearm at rest concerning arterial blood temperature, commonly referred to as the Pennes bioheat transfer model. This is based on Fourier's law of heat conduction. Pennes' model has some shortcomings because it assumes that the spread speed of thermal disturbance is infinite. Thermal diffusion is limited in living tissues due to their highly heterogeneous internal structure. For this, other models have been offered to solve the dilemma that occurred in Pennes' concept. Vernotte [5] and Cattaneo [6] applied Fourier law to derive the C-V constitutive relationship. Thermalization The relaxation times are the times that pass between the heat flux and the temperature gradient. Furthermore, Lord and Shulman[7] developed the two well-known and thoroughly studied generalised thermocouple theories and Green and Lindsay[7]. Ezzat and El-Karamany [8] demonstrate the uniqueness of theories in the field of generalized thermo-viscoelasticity under various contexts. Ezzat et al.[9] developed a model of the equation of generalized thermo-viscoelasticity with one relaxation time and used a state-space approach to solve a one-dimensional thermal shock problem in half-space with or without heat sources Ezzat [10]. The precise equations for temperature distribution, displacement, and thermal stress components are computed using the normal mode analysis. To calculate the temperature and thermal damage in live tissue brought on by laser irradiation, Alzahrani and Abbas [11] introduced an analytical approach employing Laplace transformation, experimental temperature data, and a sequential time-based concept. Abbas [12-15] investigated thermoelastic problems caused by moving heat sources. In addition, Marin [16, 17] investigated the thermoelastic interactions in porous media. Some difficulties in this subject were handled by Abbas et al.[18, 19], Zenkour et al.[20], El-Bary et al.[21], Lotfy et al. [22-24] Youssef and El-Bary [25], Ezzat and Youssef [26], Marin et al. [27]. Several researchers have solved the various causes of linear and nonlinear thermoelasticity and their solutions[28-36].

This study's main goal is to investigate numerical temperature effects and thermal damage in biological tissue using the SPL model. By the general theory of thermos-viscoelasticity, the effects of volume relaxation are duly considered. Furthermore, the purpose of the study is to examine the transient bio-thermo-viscoelastic reactions of live, viscoelastic skin tissue under various heat-loading scenarios. One of the main elements of the approach is The Laplace transformation method is used, and the Tzuo [37] process is used to determine the reversal. In addition For various theories, the effects of changing the thermal material characteristics and the relaxation period on thermal stress, displacement and temperature are discussed. The results are presented in a comprehensive graphical format.

2. The Pennes model of biological tissue bioheat transfer

The temperature change as a function of time in the heat response produced by thermal heating or a heat source was investigated using the bioheat transmission model. Based on Fourier's law of heat conduction, Pennes created the first model of biological tissues, which is as follows:

$$k\nabla^2\theta = \left(\rho c\frac{\partial\theta}{\partial t} - Q_b - Q_m - Q_{ext}\right), \quad (1)$$

Q_m denotes the metabolic heat generated by the chemical reaction inside the tissue, Q_b Refer to the thermal sources of blood perfusion and it is constant, and ∇^2 is the Laplace operator, Moreover. The external heat source is given by Q_{ext}.

3. The Vernotte-Cattaneo model of bioheat conduction (Modified Fourier law)

Vernotte-Cattaneo (V-C) modified the traditional Fourier thermal drive law by assuming a limited speed of thermal wave propagation, this resulted in the following form of the thermal wave.

$$k\nabla^2\theta = (1+\tau_0\frac{\partial}{\partial t})(\rho c\frac{\partial\theta}{\partial t} - Q_b - Q_m - Q_{ext}), \quad (2)$$

where $\tau_0 \geq 0$ is called the relaxation time parameter and is a material property.

4. Basic equations

The thermo-viscoelasticity theory's governing equations are concerning the heat conduction model of Pennes' -1 [2] The equation for motion:

$$(\lambda+\mu)u_{j,ij} + \mu u_{i,jj} - \gamma\theta_{,i} = \rho\frac{\partial^2 u_i}{\partial t^2}. \quad (3)$$

2- Displacement-stress-temperature relations:

$$\sigma_{ij} = (\lambda u_{k,k} - \gamma\theta)\delta_{ij} + \mu(u_{i,j} + u_{j,i}). \quad (4)$$

The displacement-strain relation:

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \quad (5)$$

3- The heat conduction equation of Lord-Shulman [38]:

$$\left(1+\tau_o\frac{\partial}{\partial t}\right)q_i = -k\nabla^2\theta. \quad (6)$$

4- Pennes' equation for energy conservation [2]:

$$k\nabla^2\theta = (1+\tau_0\frac{\partial}{\partial t})(\rho c\frac{\partial\theta}{\partial t} + \gamma\theta_0\frac{\partial^2 u_{j,j}}{\partial x\partial t} - Q_b - Q_m - Q_{ext}), \quad (7)$$

where i, j = 1, 2, 3 refer to general coordinates. $\gamma = (3\lambda + 2\mu)\alpha_T$, Q_m is the heat produced by metabolic procedures, the line-moving heat source Q_{ext} that can be expressed by [13].

$$Q_{ext} = Q_o\delta(x - vt), \quad (8)$$

where v is constant velocity, δ is the delta function and Q_o is constant and Q_b refer to the thermal sources of blood perfusion that can be expressed by

$$Q_b = \omega_b\rho_b c_b(\theta_b - \theta). \quad (9)$$

The production of metabolic heat is expressed by Mitchell et al. [39]. Such viscoelastic properties of isotropic materials are The production of metabolic heat is expressed by Mitchell et

al. [39]. Such viscoelastic properties of isotropic materials are described by the Kelvin-Voigt viscoelastic model. As well as in many domains of materials science, manufacturing processes, and biological systems, viscoelasticity research is critical. When the viscoelastic effect is considered, the parameters μ, λ, and γ are as follows:

$$\mu = (1+\tau_1\frac{\partial}{\partial t})\mu_e,\ \lambda = (1+\tau_2\frac{\partial}{\partial t})\lambda_e,$$

$$\gamma = (3(1+\tau_2\frac{\partial}{\partial t})\lambda_e + 2\,(1+\tau_1\frac{\partial}{\partial t})\mu_e)\alpha_T, \quad (10)$$

where τ_1 and τ_2 denote the viscoelastic relaxation times.

5. Formulation of the problem

Given the homogeneity and regularity of the cancer layer's surface, along with its linear and thermally elastic properties, it is assumed that the tumour surface extends infinitely in the y and z directions, with the x-axis chosen to be perpendicular to the tumour surface as in **Fig. 1** [40].

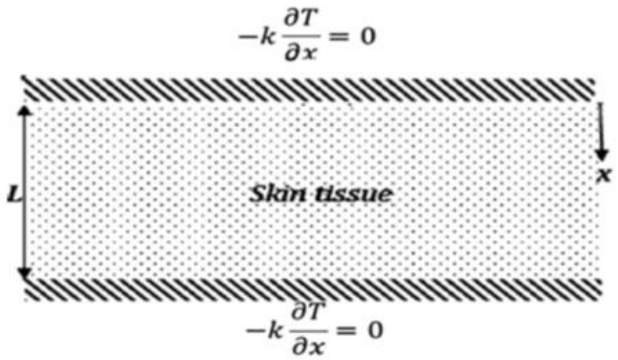

Fig. 1: Schematic diagram of biological tissue.

The problem can be treated as one-dimensional, where all functions depend solely on the position x and time t. The form of the displacement component is as follows:

$$u_x = u(x,t)\ , u_y = 0, u_z = 0. \quad (11)$$

The strain-displacement relation:

$$e = \frac{\partial u}{\partial x}. \quad (12)$$

The stress tensor's x-component is:

$$\sigma_{xx} = (\lambda+2\mu)\frac{\partial u}{\partial x} - \gamma(\theta-\theta_o). \quad (13)$$

The motion equation has the following form:

$$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial \theta}{\partial x}. \quad (14)$$

It is possible to formulate the heat equation with varying thermal conductivity as:

$$k\frac{\partial^2\theta}{\partial x^2} = (1+\tau_0\frac{\partial}{\partial t})(\rho c\frac{\partial\theta}{\partial t} + \gamma\theta_0\frac{\partial^2 L}{\partial x\partial t} - Q_b - Q_m - Q_{ext}). \quad (15)$$

6. The initial and boundary conditions

$$\sigma(x,0)=0,\ \frac{\partial\sigma(x,0)}{\partial t}=0, \theta(x,0)=\theta_b,\ \frac{\partial\theta(x,0)}{\partial t}=0.\ t\le 0. \quad (16)$$

You can express the thermal boundary condition as:

$$-k\frac{\partial\theta(0,t)}{\partial x}=0\ , -k\frac{\partial\theta(L,t)}{\partial x}=0, \sigma(0,t)=0, \sigma(L,t)=0 \quad (17)$$

The following non-dimensional variables are used for simplicity[13]:

$$(x`,u`) = ʒc(x,u), \theta` = \frac{\theta-\theta_o}{\theta_o}, (t`,\tau`_o) = ʒc^2(t,\tau_o),$$

$$\omega`_b = \frac{\omega_b}{ʒc^2}, \sigma`_{xx} = \frac{\sigma}{\lambda+2\mu},\ Q` = \frac{Q}{kT_o c_e^2 ʒ^2}, ʒ = \frac{\rho c_e}{k},$$

$$c_e = \sqrt{\frac{\lambda+2\mu}{\rho}}\ . \quad (18)$$

The governing Eqs. (13)– (15) can be stated in the following way by using the components of non-dimensional mentioned above and omitting the reference bullets for convenience:

$$\sigma_{xx} = (\lambda+2\mu)\frac{\partial u}{\partial x} - \gamma(\theta-\theta_o), \quad (19)$$

$$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial\theta}{\partial x}, \quad (20)$$

$$k\frac{\partial^2\theta}{\partial x^2} = \left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\rho c\frac{\partial\theta}{\partial t} + \gamma\theta_0\frac{\partial^2 u}{\partial x\partial t} - Q_b - Q_m - Q_{ext}\right). \quad (21)$$

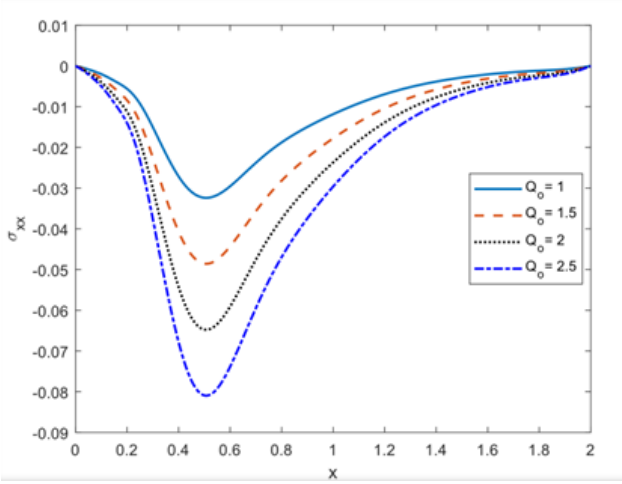

Fig. 2: Variation of stress when the heat source Q_o changes.

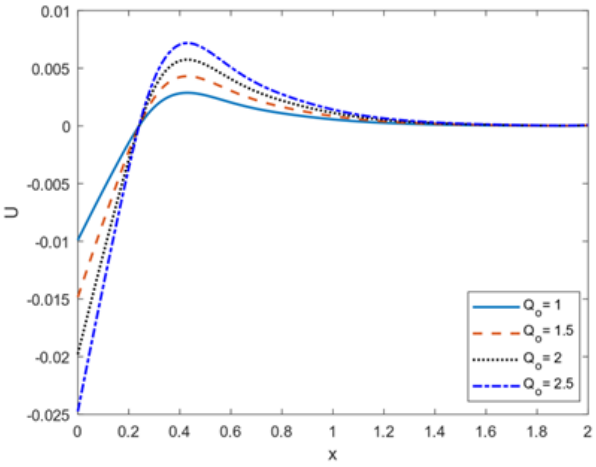

Fig. 3: Variation of displacement when the heat source Q_o changes.

7. Methods of Solution

Applying the Laplace transform with parameter S defined by the formulas:

$$\bar{F}(x,s) = L\{f(x,t)\} = \int_0^\infty f(x,t)e^{-st}\,dt,\ s>0. \quad (22)$$

Hence, we obtain the following system of differential equations.

$$\bar{\sigma}_{xx} = \zeta_1 \frac{d\bar{u}}{dx} - \zeta_2 \bar{\theta}, \quad (23)$$

$$\frac{d^2\bar{u}}{dx^2} - \zeta_3 \bar{u} = \zeta_4 \frac{d\bar{\theta}}{dx}, \quad (24)$$

$$\frac{d^2\bar{\theta}}{dx^2} - \zeta_5 \bar{\theta} = \zeta_6 \frac{d\bar{u}}{dx} - \frac{\bar{Q}_m}{s} - \frac{\bar{Q}_o}{v} e^{\frac{sx}{v}}, \quad (25)$$

with the boundary conditions

$$\bar{\sigma}(0,s) = 0, \bar{\sigma}(L,s) = 0, \frac{d\bar{\theta}(0,s)}{dx} = 0, \frac{d\bar{\theta}(L,s)}{dx} = 0. \quad (26)$$

Where $\zeta_1 = (1 + \tau_1 s)$, $\zeta_2 = \frac{\gamma \theta_o}{(\lambda_e + 2\mu_e)}$, $\zeta_3 = s^2$,

$\zeta_4 = \frac{(3\lambda + 2\mu)\alpha_T \theta_o}{(\lambda_e + 2\mu_e)}$, $\zeta_5 = (1 + \tau_o s)(s + \frac{\omega_b \rho_b c_b}{\rho c_e})$,

$$\zeta_6 = (1 + \tau_o s)(\frac{s\gamma}{\rho C_e})$$

Now, we will solve the reduced problem Eqs. (23)– (25) to get the solution of ODEs by eliminating θ from the Eqs. (24) and (25) we get

$$\frac{d^4\bar{u}}{dx^4} - \delta_1 \frac{d^2\bar{u}}{dx^2} + \delta_2 \bar{u} = \zeta_4 \delta_3 \delta_4 e^{-\delta_3 x}. \quad (27)$$

Where $\delta_1 = s^2 + \zeta_5 + \zeta_4 \zeta_6$, $\delta_2 = s^2 \zeta_5$, $\delta_3 = \frac{s}{v}$, $\delta_4 = \frac{\bar{Q}_o}{v}$,

The solutions of Eq. (27) can be written in the form:

$$\bar{u} = R_1 e^{\mu_1 x} + R_2 e^{-\mu_1 x} + R_3 e^{\mu_2 x} + R_4 e^{-\mu_2 x} + \frac{\delta_3 \delta_4 \zeta_4}{\delta_3^4 - \delta_1 \delta_3^2 + \delta_2} e^{-\delta_3 x}, \quad (28)$$

where R_1, R_2, R_3 and R_4 determined from the boundary conditions, μ_1, $-\mu_1$, μ_2 and $-\mu_2$ are the roots of the following characteristic equation:

$$\mu^4 - \delta_1 \mu^2 + \delta_2 = 0. \quad (29)$$

Where μ_1 and μ_2 are given by

$$\mu_1 = \pm\sqrt{\frac{\delta_1 + \sqrt{\delta_1^2 - 4\delta_2}}{2}}, \quad \mu_2 = \pm\sqrt{\frac{\delta_1 - \sqrt{\delta_1^2 - 4\delta_2}}{2}}$$

Using Eq. (28) in Eqs. (24) and (25), The temperature expression can be expressed as follows:

$$\bar{\theta} = k_1 R_1 e^{\mu_1 x} - k_1 R_2 e^{-\mu_1 x} + k_2 R_3 e^{\mu_2 x} - k_2 R_4 e^{-\mu_2 x} + \frac{\bar{Q}_m}{s\zeta_5} + \frac{\delta_4 (s^2 - \delta_3^2)}{\delta_3^4 - \delta_1 \delta_3^2 + \delta_2} e^{-\delta_3 x}, \quad (30)$$

Where $k_1 = \frac{\mu_1^3 - \mu_1 s^2 - \mu_1 \zeta_4 \zeta_6}{\zeta_4 \zeta_5}$ and $k_2 = \frac{\mu_2^3 - \mu_2 s^2 - \mu_2 \zeta_4 \zeta_6}{\zeta_4 \zeta_5}$

Substituting from Eqs. (28) and (30) into Eq. (23), we obtain

$$\bar{\sigma}_{xx} = T_1 R_1 e^{\mu_1 x} - T_1 R_2 e^{-\mu_1 x} + T_2 R_3 e^{\mu_2 x} - T_2 R_4 e^{-\mu_2 x} - \frac{\zeta_2 \bar{Q}_m}{s\zeta_5} + T_3 e^{-\delta_3 x}. \quad (31)$$

Where $T_1 = (1 + s\tau_1)\mu_1 - \zeta_2 k_1$, $T_2 = (1 + s\tau_1)\mu_2 - \zeta_2 k_2$ and

$$T_3 = \frac{-\zeta_1 \zeta_4 \delta_4 \delta_3^2 - \zeta_2 \delta_4 (s^2 - \delta_3^2)}{\delta_3^4 - \delta_1 \delta_3^2 + \delta_2}$$

8. Numerical Results

The research delves into examining temperature, displacement, and stress variations across skin tissue using the general theory of thermo-viscoelasticity, particularly under conditions of moving heat flow on the skin surface. Table 1 displays the values of the fundamental physical parameters employed in the current calculations. The numerical reversal process relies on the Riemann-sum approximation approach for

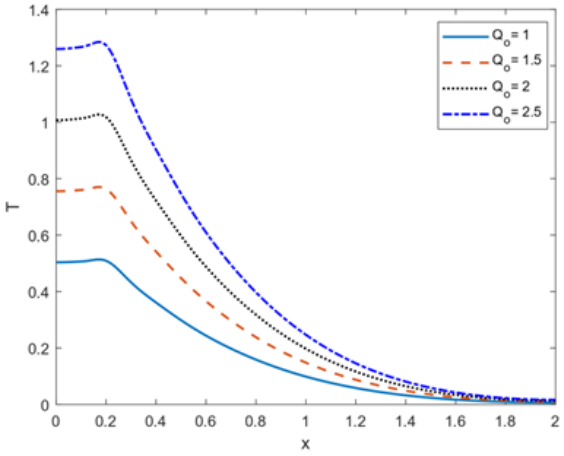

Fig. 4: Variation of temperature when the heat source Q_o changes.

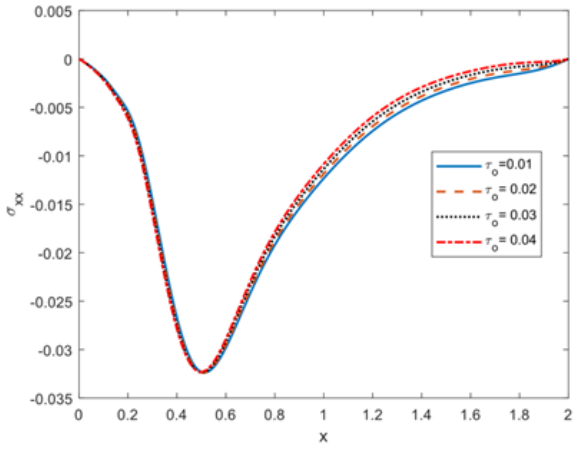

Fig. 5: Stress changes with τ_o variation

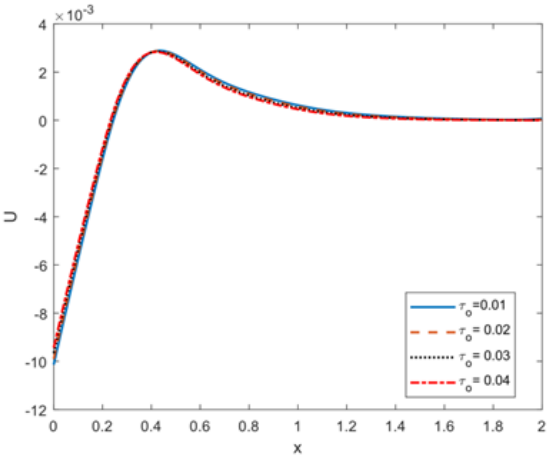

Fig. 6: Variation of displacement when τ_o changes.

studying numerical outcomes, wherein a numerical reversal procedure is employed to derive the final solution. This method facilitates the translation of any function in the Laplace domain into the time domain, as demonstrated.

$$(32) f(x,t) = \frac{e^{st}}{t}\left\{\frac{1}{2} R_e\left(\bar{F}(x,s)\right) + R_e\left[\sum_{k=0}^{N} (-1)^k \bar{F}(x, s + \frac{ik\pi}{t})\right]\right\}$$

whereas R_e is the real part. For quicker assemblage, numerical methods were decided. That $st = 3.7$ that satisfies the above equation. MATLAB software is used to carry out the

computations, and the results are shown graphically. For numerical computations, exemplary thermal property values for living tissue have been selected [32].

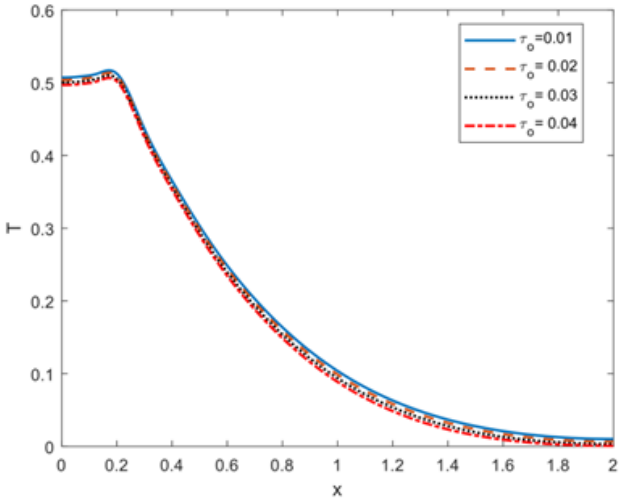

Fig. 7: Variation of temperature when τ_o change.

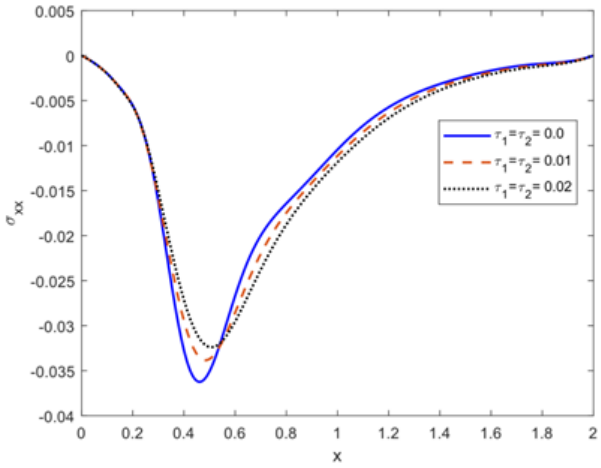

Fig. 8: Stress changes with τ_1, τ_2 variation.

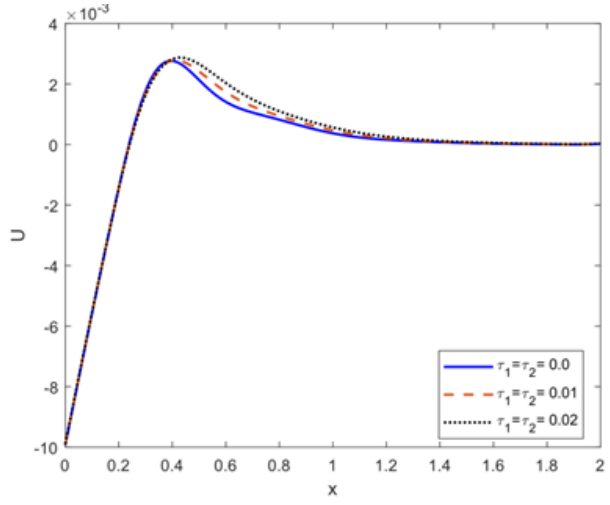

Fig. 9: Variation of displacement when τ_1, τ_2 change.

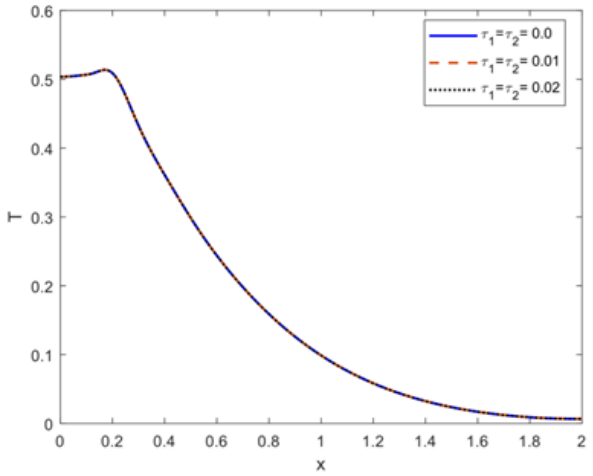

Fig. 10: Variation of temperature when τ_1, τ_2 change.

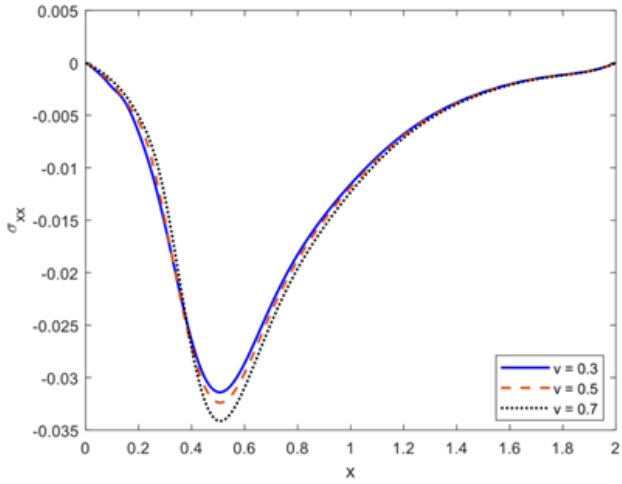

Fig. 11: Stress changes with velocity variation.

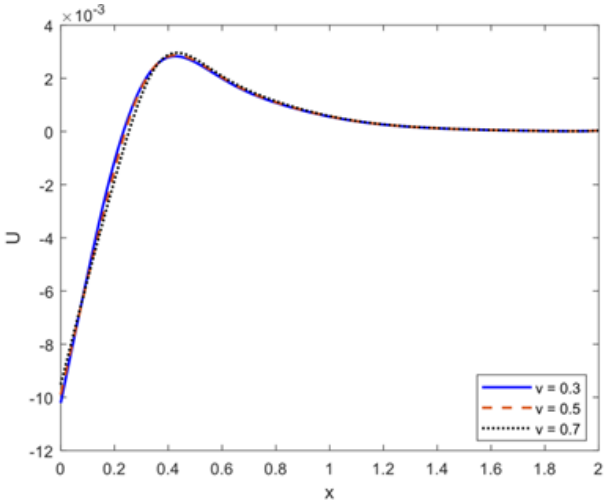

Fig. 12: Variation of displacement when velocity changes.

9. Discussion

The numerical values of the physical quantities calculated by the biothermal model will be presented, taking into account the consideration times and using the above-mentioned parameters. The onset displacement stress and temperature were determined by different values of physical information

formally examined by distance ***x***, and a numerical calculation was performed at time t = 0.4. From Figures 2 - 13, we realized that the change in physical parameters, relaxation times, τ_o, τ_1, τ_2 the change in velocity v, and the heat source Q_o, result.

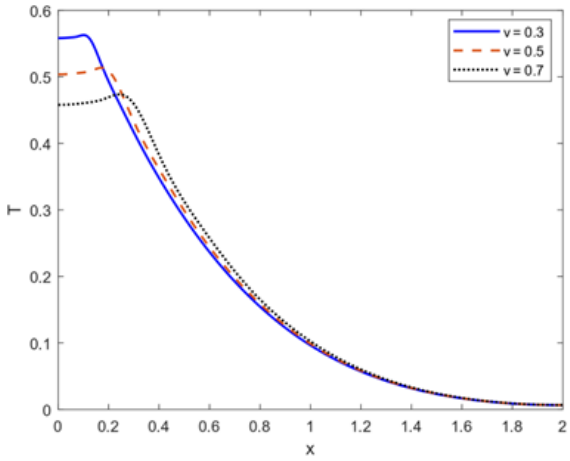

Fig. 13: Variation of temperature when velocity changes.

Table 1 The skin tissue's material characteristics.

Parameter	Value	Unit
$\boldsymbol{\rho_b}$	1060	$(kg)(m^{-3})$
$\boldsymbol{\omega_b}$	1.87×10^{-3}	(s^{-1})
$\boldsymbol{C_b}$	3860	$(J)(kg^{-1})(k^{-1})$
$\boldsymbol{C}$	3600	$(J)(kg^{-1})(k^{-1})$
$\boldsymbol{\theta_b}$	37	℃
$\boldsymbol{\tau_o}$	0.2	(s)
$\boldsymbol{\tau_1}$	0.002	(s)
$\boldsymbol{\lambda_e}$	8.27×10^{8}	$(kg)(m^{-1})(s^{-2})$
$\boldsymbol{\rho}$	1190	$(kg)(m^{-3})$
$\boldsymbol{Q_m}$	1.19×10^{3}	$(W)(m^{-3})$
k	0.235	$(W)(m^{-1})(k^{-1})$
$\boldsymbol{Q_o}$	1×10^{3}	$(W)(m^{-3})$
L	0.02	(m)
$\boldsymbol{\alpha_t}$	1×10^{-4}	(k^{-1})
$\boldsymbol{\tau_1}$	0.002	(s)
$\boldsymbol{\mu_e}$	3.446×10^{7}	$(kg)(m^{-1})(s^{-2})$

We note from the results and through the change of physical parameters that the displacement changes along the x-axis with the change of physical parameters. We can see that the displacement starts from the lowest values on the surface of the tissue (x = 0). Before returning to zero, it progressively rises to maximum values near the surface. We also note that the temperature changes with the change of physical parameters and at its beginning it is on the surface of the tissue, where it peaks (x = 0) due to the heat flow, then it decreases and continues to decrease with the increase of the distance x, and we notice a steadily decreasing temperature. Also, through the change of physical parameters, we notice the change of stress σ_{xx} along the x-axis, where it starts from its highest negative values and ends at zero to comply with the boundary conditions. Based on the provided information, we may infer that thermal relaxation periods result in a decrease in the maximum amplitude of stress, displacement, and temperature. This suggests that thermal relaxation times are a good way to lessen the impact of mechanical thermal diffusion. An increase

Figs 2 - 4, considered the first group, show the change of stress, temperature, and displacement when (τ_o = 0.02, $\tau_1 = \tau_2 = 0.02$, $v = 0.5$) under different values of the intensity of the heat source (Q_o= 1, 1.5, 2, 2.5). Under varying thermal relaxation time values ($\tau_o = 0.0, 0.01, 0.02$) In the second group, Figures 5-7 illustrate the changes in stress, temperature, and displacement at ($\tau_1 = \tau_2 = 0.02$, $v = 0.5$, $Q_o = 1$). The third, Figures 8-10, illustrates the change in stress, temperature, and displacement when ($\tau_o = 0.2$, $v = 0.5$, $Q_o = 1$) under different values of thermal relaxation time ($\tau_1 = \tau_2$ = 0.0, 0.01, 0.02). Figures 11-13 show the change of stress, temperature, and displacement when ($\tau_o = 0.02, \tau_1 = \tau_2 = 0.02$, $Q_o = 1$) under different values of velosity (v = 0.3, 0.5, 0.7).

We note from the results and through the change of physical parameters that the displacement changes along the x-axis with the change of physical parameters. We can see that the displacement starts from the lowest values on the surface of the tissue (x = 0). Before returning to zero, it progressively rises to maximum values near the surface. We also note that the temperature changes with the change of physical parameters and at its beginning it is on the surface of the tissue, where it peaks (x = 0) due to the heat flow, then it decreases and continues to decrease with the increase of the distance x, and we notice a steadily decreasing temperature. Also, through the change of physical parameters, we notice the change of stress σ_{xx} along the x-axis, where it starts from its highest negative values and ends at zero to comply with the boundary conditions. Based on the provided information, we may infer that thermal relaxation periods result in a decrease in the maximum amplitude of stress, displacement, and temperature. This suggests that thermal relaxation times are a good way to lessen the impact of mechanical thermal diffusion. An increase in the time characteristic of the pulsating heat flow also weakens the effect of thermomechanical diffusion, which is indicated by a decrease in the maximum stress, displacement, and temperature.

10. Conclusion

This study aimed to create a mathematical model for heat transfer, exploring biological heat transfer processes and the response of living skin tissues to mechanical heat stress, considering variations in heat conduction coefficients and tissue viscoelastic properties. The research examined how the velocity of a moving heat source affects changes in living tissues, including biological temperature gradients and mechanical stresses. Numerical results demonstrated that heat waves propagate slowly in skin tissues, significantly influencing thermal distributions and mechanical stresses based on various relaxation times and thermal factors. These findings highlight the impact of thermal relaxation times on temperature distributions and mechanical stresses in living skin tissues. Given that thermal therapy is among the safest methods for treating tumors, this study aimed to provide a theoretical

framework to enhance understanding of the complex mechanical and thermal processes involved in thermal treatment.

CRediT authorship contribution statement:

"Conceptualization, I.A.,A.N.A. and A.A.; methodology, I.A.; software, A.A.; validation, I.A.,A.N.A. and A.A.; formal analysis, I.A.,A.N.A. and A.A.; investigation, A.A.; resources, I.A.,A.N.A. and A.A.; data curation, A.A.; writing—original draft preparation, A.A.; writing—review and editing, I.A.,A.N.A. and A.A.; visualization, A.A.; supervision, I.A. and A.N.A; project administration, I.A.; funding acquisition, I.A. All authors have read and agreed to the published version of the manuscript."

Data availability statement

The data used to support the findings of this study are available from the corresponding author upon request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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