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Vibrational Response of Tracked Vehicles with Variable Suspension Characteristics

By

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Abstract:

Evaluation of tracked vehicles vibrational response depends mainly on the design and quality of suspension system. Suspension of tracked vehicles comprises torsion bars, shock absorbers, axle arms and road wheels stroke limiters. The most effective parameters of vehicle suspension are suspension stiffness and shock absorber damping coefficient. This paper presents a theoretical evaluation of tracked vehicles response in case of forced damped vibration and predicts its performance on specific road conditions. The analytical approach estimates the vehicle response with the assumption of linear suspension elements characteristics. MATLAB program was used to predict the vibrational response of tracked vehicles with real suspension characteristics. The vibrational response of the vehicle is estimated by looking for the angular displacement of the longitudinal vibration. The suspension stiffness and damping coefficient of shock absorber are introduced in the equation of motion by their real varying values.

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Keywords:

Tracked vehicle, response, vibration, damping, rigidity

1. **Introduction:**

Tracked vehicle are cross country vehicles that operate on severe terrain conditions. While mobility is one of the main requirements, stability and maneuverability of the vehicle are of the same importance. If the road impact is transmitted directly to the vehicle body it will have a passive effect on vehicle operation. The mobility of tracked vehicles is influenced by the possible maximum speed on rough terrain. This paper aims to study the response of tracked vehicles and the effect of change of suspension parameters and road profiles on such response.

The theoretical investigation of the problem enables the prediction of a number of mobility basic parameters such as vehicle speed and suspension parameters. The analytical study evaluates the vibrational response of tracked vehicles on the basis of the assumption that the suspension stiffness and damping coefficient of shock absorber are constants. In this study the suspension stiffness and damping coefficient of shock absorber are taken variable. This will yield to precise prediction of vehicle response. The suspension stiffness and the damping coefficient change with the vertical deflection of road wheel and hence with angular displacement of vehicle hull.

In the study where the damping coefficient of shock absorber and suspension stiffness are taken constants, the solution of equation of motion of vehicle hull oscillation is simple. By taking the damping coefficient and suspension stiffness changeable with the vertical deflection of the road wheel, the equation of motion of hull oscillation becomes complicated and hence its analytical solution is not feasible. MATLAB software has been used to solve the equation of motion and to find the vehicle response.

Hereafter, the characteristics of suspension and shock absorber are determined and introduced in the equation of motion of vehicle hull oscillation as a function of road deflection.

2. **Suspension characteristics and stiffness:**

In order to investigate the tracked vehicle vibrational response, the suspension characteristics have to be identified. The suspension characteristics are defined by the relationship between the vertical force P_k acting on the road wheel and the corresponding vertical displacement f_k resulting from the acting force as shown in figure (1). These characteristics are determined on the basis of torsion bar characteristics which are the variation of twisting moment with the corresponding twist angle, expressed by the following relation [2]:

$$M = \frac{\pi d^4}{32} \frac{G}{L} \beta \qquad (1)$$

Where:

d………torsion bar diameter
G……...modulus of elasticity in shear
L………active length of the torsion bar
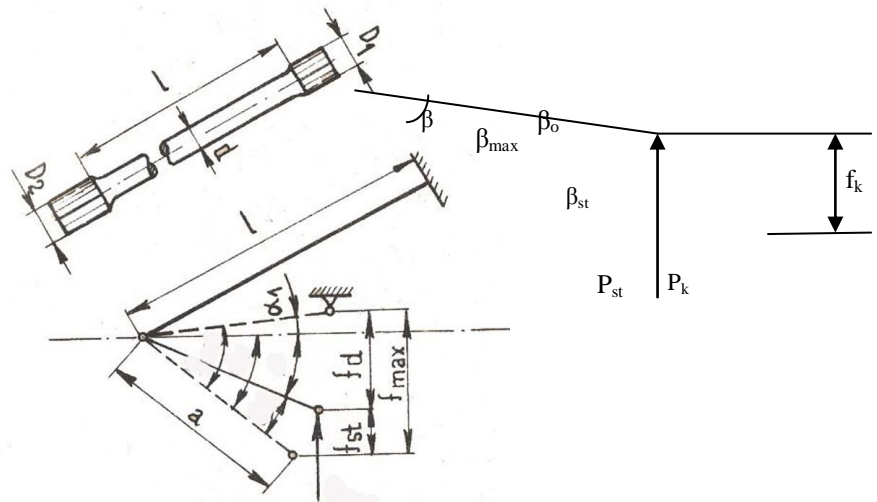*β*………angle of twist of the torsion bar in radiance

Figure (1)*: Schematic drawing of tracked vehicle suspension unit*

Referring to figure (1), the torsion bar is twisted by a moment expressed by:

$$M = P_k \, a \cos(\beta_o - \beta) \tag{2}$$

Where:

P_k ....vertical force acting on the road wheel

β_o ....angle of inclination of the axle arm with the fully released road wheel (assembly angle).

a .... axle arm length

From equations (1) and (2) the force P_k is given by the following relation:

$$P_k = \frac{\pi \, d^4}{32} \frac{G}{L} \, \beta \; \frac{1}{a \cos(\beta_o - \beta)} \tag{3}$$

The vertical displacement of road wheel f_k is given by:

$$f_k = a \sin \beta_o - a \sin(\beta_o - \beta) \tag{4}$$

For various values of angle β, it is possible to calculate from equations (3) and (4) the different values of P_k and f_k and express graphically the relationship between them. The suspension stiffness at the ith road wheel C_i is obtained by dividing the elementary change of the acting force on the road wheel ΔP_k by the corresponding change of displacement Δf_k [1], i.e:

$$C_i = \frac{\Delta P_k}{\Delta f_k}$$

Table (1) lists the parameters needed to calculate the suspension stiffness (C_i) of APC M113 while figure (2) depicts its suspension characteristics. The curve representing the

relation between the force acting on the road wheel and its vertical displacement can be fitted to the MATLAB software to get its analytical equation necessary to be introduced in the equation of motion of vehicle hull vibration.

Table (1): M113 parameters [8]

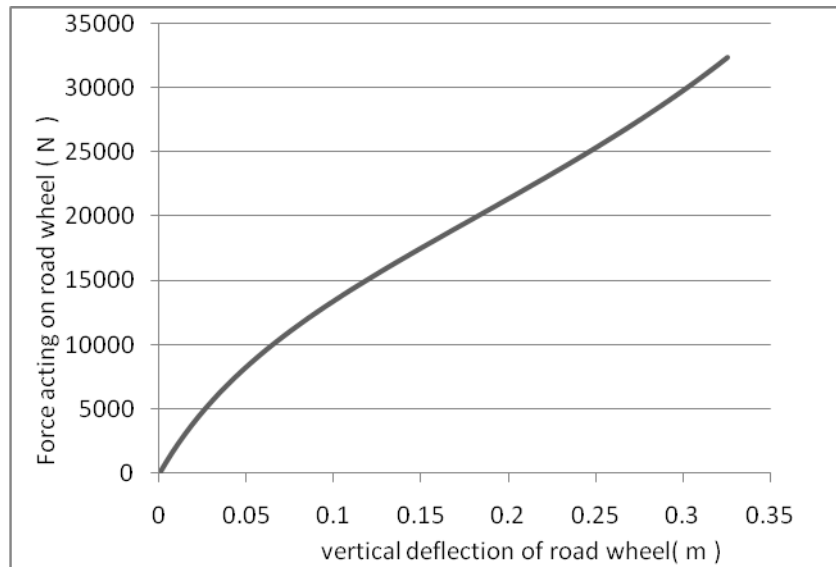| Vehicle weight (ton)	11.73
Effective length of torsion bar (m)	1.51
Axle arm length (m)	0.355
Diameter of torsion bar (m)	0.0366
Shear modulus of elasticity G (kN/m^2)	8.5×10^7
Reduced stiffness C_i (kN/m)	133
Number of torsion bars per side k	5
Assembly angle β_o	53°

Figure (2): *Characteristics of M113 suspension*

The equation obtained by MATLAB for the dependence of the vertical force on wheel F_{sp} with its vertical displacement z by a 4th degree polynomial is found as:

$$F_{sp} = a_1 z^4 + a_2 z^3 + a_3 z^2 + a_4 z + a_5 \qquad (5)$$

Where a$_1$, a$_2$, a$_3$, a$_4$, and a$_5$ are constants. These constants are determined according to the type of the curve to be fitted. For the curve of suspension characteristics, these constants have the following values:

$a_1 = -2951000,$ $a_2 = 2815000,$ $a_3 = -912400,$ $a_4 = 198700,$ $a_5 = 287.2$

Therefore, the relation representing the variation of the force acting on the ith road wheel F_{spi} with its vertical displacement z_i can be expressed by:

$$F_{spi} = a_1 z_i^4 + a_2 z_i^3 + a_3 z_i^2 + a_4 z_i + a_5 \tag{6}$$

In case of forced vibration where the vehicle is moved over irregular terrain, the vertical displacement of each road wheel is varied according to the shape of the road and given as follows:

$$z_i = \varphi l_i - \frac{h}{2} \sin\left(\frac{2\pi V}{a} t + \frac{2\pi}{a} l_i \right) \tag{7}$$

Where h and a are the road height and length [7]. The first term of equation (7) represents the deflection of the road wheel due to vehicle hull angular displacement while the second term represents the road wheel displacement due to road shape.

3. Evaluation of damping coefficient of shock absorber:

The damping coefficient of shock absorber can be determined from the condition of the required degree of decrease of amplitude of oscillation as shown in figure (3).
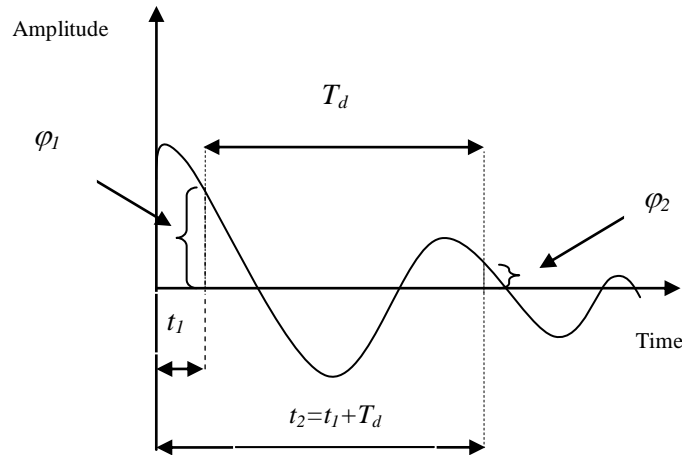

Figure (3): *Illustration of two successive amplitudes*

The shock absorber used in APC M113 is of viscous type in which its characteristic is assumed to be linear and consequently the amplitude of vibration is decreased exponentially according to the following equation [3].

$$\varphi = B_o\, e^{-nt} \cos(\omega_d t + \alpha)$$

where B_o: constant, ω_d: frequency of damped oscillation, α: phase angle

Applying the above relation for two successive oscillations:

$$\varphi_1 = B_o\, e^{-nt_1} \cos(\omega_d t_1 + \alpha), \text{ and} \tag{8}$$

$$\varphi_2 = B_o\, e^{-n(t_1 + T_d)} \cos(\omega_d (t_1 + T_d) + \alpha) \tag{9}$$

with $T_d = \dfrac{2\pi}{\omega_d}$: period of damped oscillation   and $\omega_d = \sqrt{\omega_\varphi - n^2}$

Where ω_φ is the natural frequency of angular oscillation and n is the damping constant.
Dividing equation (8) by equation (9), yields the following relation:

$$\frac{\varphi_1}{\varphi_2} = e^{nT_d} \tag{10}$$

Replacing the ratio $\dfrac{\varphi_1}{\varphi_2}$ by the constant δ and solving this equation, the damping constant
of shock absorber n can be found as:

$$n = \frac{\omega_\varphi \ln \delta}{\sqrt{4\pi^2 + (\ln \delta)^2}} \tag{11}$$

on the other hand, the damping constant n is expressed as follows [1]:

$$n = \frac{\mu \sum_1^{k_d} l_i^2}{J_y} \qquad k_d \text{ is the number of shock absorbers on one vehicle side}$$

It is clear that, the damping constant depends on the damping coefficient of shock
absorbers μ and their distribution on vehicle hull. By equating the two equations of n, the
coefficient of shock absorber μ can be found by the following relation:

$$\mu = \frac{1.41 \ln \delta}{\sum\limits_1^{k_d} l_i^2} \sqrt{\frac{J_y \sum\limits_1^{k} c_i l_i^2}{4\pi^2 + (\ln \delta)^2}} \tag{12}$$

For tracked vehicles, the value of δ is 12-15 [1]. Applying this relation for M113, the mean
value of shock absorber damping coefficient is found to be about 17 (kN.s/m). Since in
tracked vehicles and automobiles, it is usually used absorbers with different resistance in
each direction of action, hence the obtained value of μ represents the average of two
values of the absorber coefficient.

The equation of vehicle motion during forced damped vibration is as follows:

$$\ddot{\varphi} + \frac{2\sum\limits_1^{kd} \mu_i l_i^2}{J_y} \dot{\varphi} + \frac{2\sum\limits_1^{k} C_i l_i^2}{J_y} \varphi = \frac{-hq \sum\limits_1^{kd} \mu_i l_i \sin\left(\dfrac{2\pi d_i}{a}\right)}{J_y} \sin(qt) + \frac{h \sum\limits_1^{k} C_i l_i \sin\left(\dfrac{2\pi l_i}{a}\right)}{J_y} \cos(qt) \tag{13}$$

The damping coefficient of shock absorber and the suspension stiffness are introduced in
the above equation of motion in order to get the vehicle vibrational response. In case of
forced vibration, the vehicle excitation results from its motion over irregular road profile

with certain speed. The road profile is represented by the height (h) and length (a) assuming sinusoidal road profile. Figure (4) represents the vibrational response of M113 in case of damped free vibration while figure (5) represents this response in case of damped forced vibration.
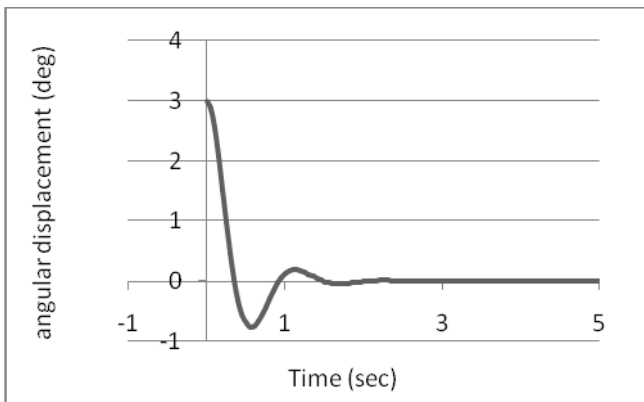

Figure (4): *vehicle response in damped free vibration*
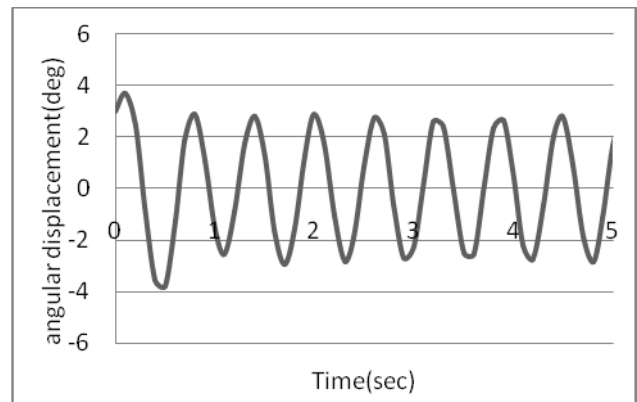

Figure (5): *vehicle response in damped forced vibration*

The damping coefficient of shock absorber of M113 is measured experimentally using MTS850 damper test system at the same road conditions where the curve shown in figure (5) is obtained. Figure (6) and figure (7) represent the variation of damping force with velocity and displacement respectively.
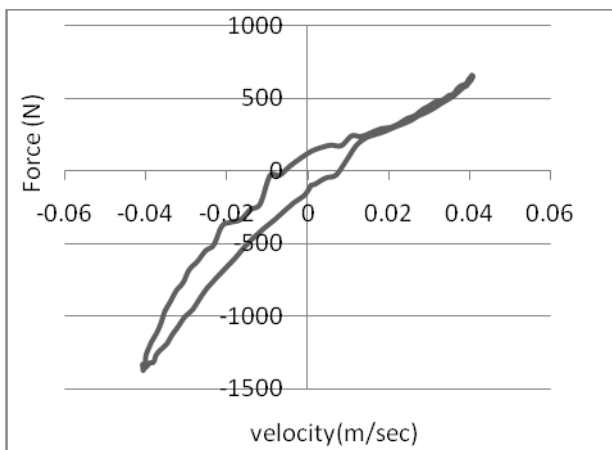
Figure (6): *Variation of damping force with velocity*
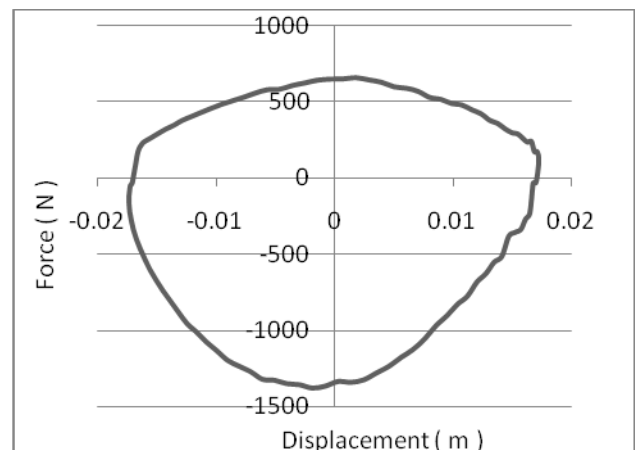
Figure (7): *Variation of damping force with displacement*

It can be observed that the damping coefficient in compression and rebound strokes has different values as mentioned before. Figure (6) shows the variation of the damping force with the vertical velocity of damper. This variation can be fitted using MATLAB where the following equation for the variation of the damping force F_d with the vertical velocity $\dot{z}$ is obtained:

$$F_d = b_1 \dot{z}^4 + b_2 \dot{z}^3 + b_3 \dot{z}^2 + b_4 \dot{z} + b_5 \tag{14}$$

With: $b_1 = 4724000$, $b_2 = 3298000$, $b_3 = -200900$, $b_4 = 18810$, $b_5 = -37.35$

Therefore, as in the case of suspension stiffness, the damping force on the ith damper is given by:

$$F_{di} = b_1\dot{z}_i^4 + b_2\dot{z}_i^3 + b_3\dot{z}_i^2 + b_4\dot{z}_i + b_5 \tag{15}$$

Where $\dot{z}_i$ is the vertical velocity of the ith road wheel given by:

$$\dot{z}_i = \dot{\varphi}l_i - \frac{h\pi V}{a}\cos\left(\frac{2\pi V}{a}t\right)\cos\left(\frac{2\pi}{a}l_i\right) + \frac{h\pi V}{a}\sin\left(\frac{2\pi V}{a}t\right)\sin\left(\frac{2\pi}{a}l_i\right) \tag{16}$$

4. Evaluation of the vehicle hull response in case of forced damped vibration:

The damped forced vibration takes place when an external excitation force is applied to the vehicle hull resulting from its drive over road unevness. This vibration is damped by vehicle shock absorbers. The characteristics of such vibrations depend on the shape of unevenness and velocity of vehicle [6]. The equation of motion of vehicle hull oscillation in case of damped forced vibration is deduced from the following relation:

$$J\ddot{\varphi} = -M_{sp} - M_d \tag{17}$$

M_{sp} and M_d are the moment of all spring forces and damper forces respectively.

$$M_{sp} = 2\sum_1^k M_{spi} = 2\sum_1^k F_{spi}l_i \tag{18}$$

$$M_d = 2\sum_1^{kd} M_{di}l_i = 2\sum_1^{kd} F_{di}l_i \tag{19}$$

Where F_{spi} and F_{di} are calculated by equation (6) and (15) respectively

From equations (17), (18) and (19) the response of vehicle in case of damped forced oscillation with variable suspension parameters can be obtained using MATLAB where the results obtained are shown in figure (8). MATLAB solved the equation of motion of the tracked vehicle by using Runge–Kutta 4. The procedures and algorithm of calculation are shown in figure (9).
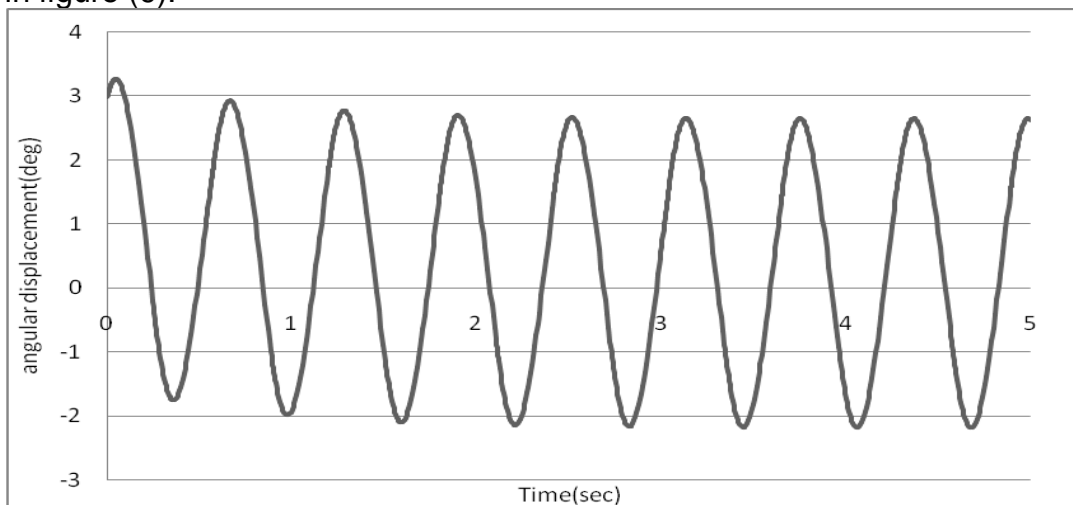

Figure (8): *Vehicle response of forced damped oscillation given by MATLAB*

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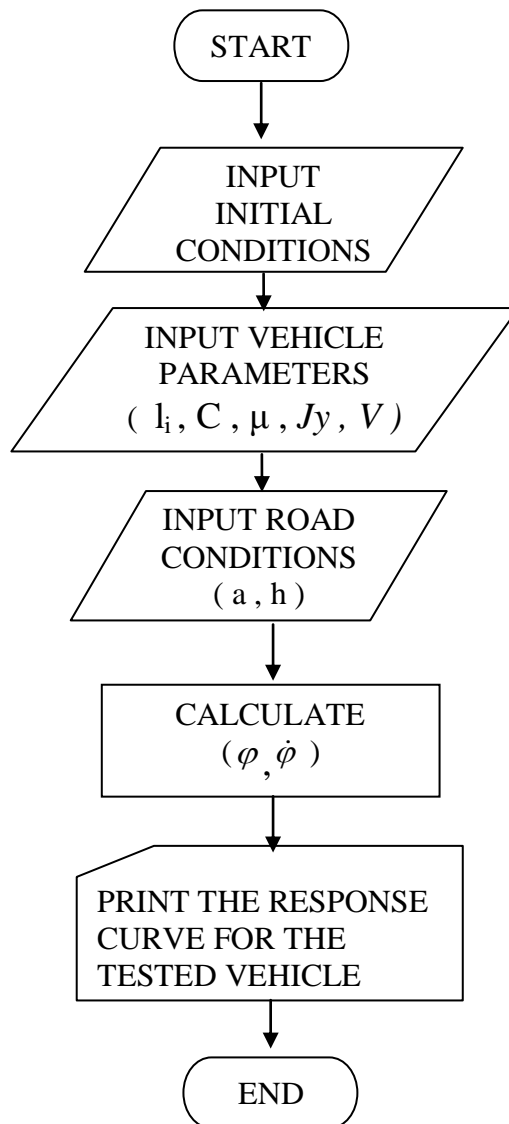
***Figure (9)**: Flow chart of the computer program*

Figure (10) compares the response obtained by MATLAB where variable stiffness of suspension and variable damping coefficient of shock absorber are used with the response obtained by analytical approach where these parameters are set constant.

The figure indicates limited difference between MATLAB and analytical results. Such difference shown in the figure is due to the real characteristics of suspension stiffness and shock absorber beside the real distribution of road wheels. Therefore during evaluation of vibrational response of tracked vehicles, it is preferable to use the real characteristics of suspension parameters.
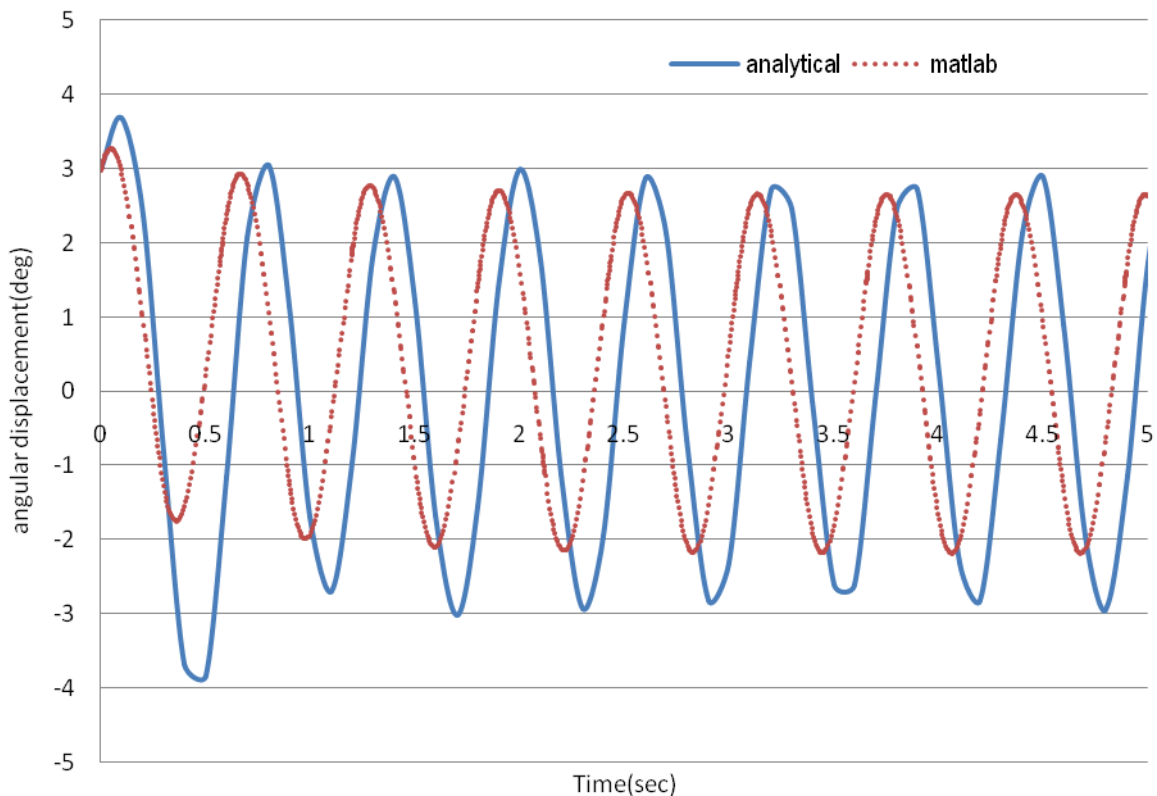
Figure (10)*: Comparison between MATLAB and analytical response*

5. CONCLUSIONS:

- Suspension elements parameters such as the suspension stiffness and shock absorber damping coefficient of tracked vehicles can be predicated satisfactory using the vehicle design parameters.
- The vibrational response of tracked vehicles was evaluated with variable values of suspension stiffness and shock absorber coefficient. This makes the prediction of the response of tracked vehicle more accurately.
- Using MATLAB, the response of the tracked vehicle on different road profiles and with different vehicle speeds can be predicted. Accordingly, the time and cost spent in the experimental tests and analytical analysis can be optimized.

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NOMENCLATURE:

a	length of sinusoidal road wave, (m)
h	height of sinusoidal road ,(m)
φ	angular displacement of vehicle hull, (rad)
$\dot{\varphi}$	angular velocity of vehicle hull, (rad/sec)
Jy	moment of inertia of suspended mass about the transverse axis passing through vehicle center of gravity,(N.m.sec^2)
k	number of road wheels in one vehicle side
k_d	Number of absorbers in one vehicle side
l_i	distance of the ith road wheel from the center of gravity of vehicle ,(m)
V	speed of the tracked vehicle,(m/sec)
C_i	stiffness of ith torsion bar, (N/m)
µ	mean damping coefficient of shock absorber, (N.sec/m)
q	excitation frequency,(rad/sec)