Military Technical College
Kobry El-Kobbah,
Cairo, Egypt.

13th International Conference
on Applied Mechanics and
Mechanical Engineering.

VIBRATION CHARACTERISTICS OF PERIODIC SANDWICH BEAMS

BADRAN[*] H., TAWFIK[**] M. and NEGM[***] H.

ABSTRACT

Periodic Structures have been in the focus of research for their useful characteristics and ability to attenuate vibration in frequency bands called “stop-bands”. In the previous studies, periodic changes in the geometry of beams were introduced to create the periodicity; then the vibration characteristics were studied accordingly. In this study, for the first time, we are analyzing the vibration characteristics of a sandwich beam with the periodic change in the sandwich material. The new technique preserves the external geometry of the beam structure and depends on changing the material of the sandwich material. The periodic analysis and the vibration response characteristics of the model are investigated using finite element model for a sandwich beam. The response to bending excitation has shown promising results for periodic sandwich beams, that may encourage further study of the problem with more practical configurations.

KEY WORDS

Vibration, Sandwich beam, Finite element, Periodic core, Stop bands, Kinetic energy, Strain energy, Hamilton’s principle and Transfer matrix.

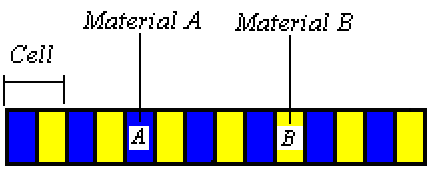
[*] Egyptian Air-force, Cairo, Egypt.
[**] Mechanical Engineering Department, British University in Egypt, Cairo, Egypt.
[***] Aerospace Engineering Department, Cairo University, Cairo, Egypt.

1. INTRODUCTION

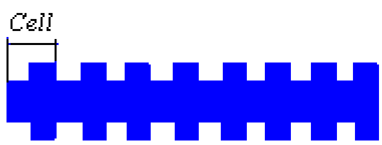
Sandwich structures have a lot of different applications in the industry, which attracts researchers to study them in different forms, such as beams and plates. The three-layer sandwich beam has been considered by a number of investigators. Raville et. al.[1] analyzed the problem of determination of the natural frequency of vibration of a fixed sandwich beam with an elastic core. DiTaranto [2] and with Blasingame [3] investigated the vibration characteristics, natural frequency and the associated composite loss factor of a sandwich beam with a viscoelastic core having a complex shear modulus. Mead and Markus [4] analyzed the forced vibration of a three-layer sandwich beam with a linearly viscoelastic core having a complex shear modulus and arbitrary boundary conditions. Ahmed [5], [6] analyzed the flexural vibration characteristics of a curved sandwich beam with an elastic core. Mead and Yaman [7] presented an exact analytical method for the vibration response of a finite, three-layered, rectangular sandwich plate with a viscoelastic core. Sakiyama et. al.[8] presented an analytical method for the free vibration of a three-layer sandwich beam with an elastic or linearly viscoelastic core and arbitrary boundary conditions. Khatua and Cheung [9] presented a finite element formulation for the bending and vibration of multilayer beams and plates with constrained cores. Rao and Nakra [9] carried out an analysis of vibration of unsymmetric sandwich beams and plates with viscoelastic cores. Rao [9] derived the complete set of equations of motion and boundary conditions governing the vibration of sandwich beams using energy approach. Sismore and Darvennes [11] considered the effect of compression energy of the core on damping, in addition to the conventional approach which uses shear deformation of the core for the estimation of damping.

Recently, Banerjee [12] used the dynamic stiffness method for the free vibration analysis of three-layer sandwich beams. Ganapathi et. al. [13] developed a finite element approach based on high-order shear deformation theories for laminated beams, which analyze sandwich beams experiencing both elastic bending, torsion and traction in small displacements. The element is fully free of shear locking, and is based on a refined shear deformation theory. Ha [14] developed a procedure for the exact buckling analysis of sandwich beams and frame structures subjected to arbitrary mechanical loading. Lan et. al.[15] discussed the thermal buckling of bi-modular sandwich beams having thick facings and moderately stiff cores. Ganesan and Pradeep [16] found that the effect of core thickness on the buckling temperature is greater when the value of the shear modulus of the core is high. Kerwin [17] presented an analysis for a simply supported sandwich beam using a complex modulus to account for damping and stiffness of the viscoelastic core. DiTaranto [18] extended his work and derived a sixth-order partial differential equation of motion for sandwich beams in terms of the longitudinal displacement (u). Mead and Marcus [19] analyzed three-layered sandwich beams with a viscoelastic core using sixth order differential equations of motion in terms of the transverse displacement (w). Also, they examined the form of boundary conditions for many end constraints encountered in practice. Rao and Nakra [20] used energy methods to develop equations of motion including the inertia effects of transverse, longitudinal and rotary motion. Lu et al. [21] developed a finite element model and presented experimental data for sandwich plates under free boundary conditions. Cupial and Niziol [22] used variational methods to model sandwich plates with anisotropic face-plates, and presented simplified forms of the equations for symmetric plates and plates with orthotropic face layers. Mead and Markus [19] presented the basic theory of vibration of a three-layer beam system. The theory assumes that: (1) the constraining layer bends in the transverse direction exactly as the base layer, (2) The viscoelastic layer undergoes pure shear, and (3) the viscoelastic layer does not change its thickness during deformation. These assumptions are valid if the viscoelastic material (VEM) is relatively thin, and the stiffness of the two surface layers is on the same order of the core stiffness. Dwivedy et. al. [23] studied parametric instability of a three-layered, soft-cored, symmetric sandwich beam subjected to a periodic axial load. The displacements of the top and bottom skins are allowed to be different, and instead of using the classical theory of displacement, a higher-order theory is used. Applying extended Hamilton's principle and using beam theory for the skins and a two-dimensional theory for the core, the governing equations of motion and boundary conditions are derived. Lin and Chen [24] studied the dynamic stability of a rotating sandwich beam with a constrained damping layer using the finite element method. They found that the system becomes more stable as the rotating speed, hub radius and thickness of the (VEM) layer increases. Larger setting angles reduce the stiffness of the rotating sandwich beam causing the rotating sandwich beam to be more unstable.

In his paper, Mead [25] defined a periodic structure as a structure that consists fundamentally of a number of identical structural components (cells) that are joined together to form a continuous structure. This introduces sudden changes in the properties of the structure between cells. There are two main types of discontinuities: (1) Geometric discontinuity and (2) Material discontinuity. Fig.1 shows the two different types of discontinuities.



A- Material discontinuity



B- Geometric discontinuity

Fig. 1 Types of discontinuities

Ungar [26] presented a derivation of an expression that could describe the steady state vibration of an infinite beam uniformly supported on impedances. Later, Gupta [27] presented an analysis for periodically-supported beams that introduced the concepts of the cell and the associated transfer matrix. He presented the propagation and attenuation parameter plots which form the foundation for further studies of one-dimensional periodic structures. Faulkner and Hong [28] presented a study of general mono-coupled periodic systems. Their study analyzed the free vibration of spring-mass systems as well as point-supported beams using analytical and finite element methods. Mead and Yaman [29] presented a study for the response of one-dimensional periodic structures subject to periodic loading. Their study involved the generalization of the support condition to involve rotational and displacement springs as well as impedances. The effects of the excitation point as well as the elastic support characteristics on the pass and stop characteristics of the beam are presented. Tawfik et. al.[30] studied the wave attenuation in a periodic helicopter blade, and concluded that the periodic beam characteristics have promising features regarding the attenuation of vibration transmission in rotating as well as non-rotating beam applications. Alaa El-Din and Tawfik [31] studied the vibration attenuation in rotating beams with periodically distributed piezoelectric controllers, and proved the ability of piezoelectric pairs to damp vibrations of certain frequency band. Moreover, the periodic nature of the structure was found to introduce vibration attenuation in other frequency ranges.

In this study we will analyze the vibration characteristics of a sandwich beam with the periodic change in the sandwich material using finite element model.

2. FINITE ELEMENT MODEL

2.1 Displacement Trial Functions (three node elements)

The axial displacement function for 3-nodes, a beam could be presented as follows

$$u_k = \left\lfloor 1-\frac{3x}{L}+\frac{2x^2}{L^2} \quad \frac{4x}{L}-\frac{4x^2}{L^2} \quad \frac{-x}{L}+\frac{2x^2}{L^2} \right\rfloor \begin{bmatrix} u_{ki} \\ u_{kj} \\ u_{kn} \end{bmatrix} = \lfloor H \rfloor \lfloor \delta_k \rfloor \qquad (1)$$

where k denotes (t) for top layer and (b) for bottom layer.

On the other hand, the transverse displacement may be written as

$$w(x) = \lfloor N_1 \quad N_2 \quad N_3 \quad N_4 \quad N_5 \quad N_6 \rfloor [\delta\ ] = \lfloor N \rfloor [\delta\ ] \qquad (2)$$

where [31]:

$$N_1 = 1-23\frac{x^2}{L^2}+66\frac{x^3}{L^3}-68\frac{x^4}{L^4}+24\frac{x^5}{L^5}, \quad N_2 = x-6\frac{x^2}{L}+13\frac{x^3}{L^2}-12\frac{x^4}{L^3}+4\frac{x^5}{L^4}$$

$$N_3 = 16\frac{x^2}{L^2}-32\frac{x^3}{L^3}+16\frac{x^4}{L^4}, \quad N_4 = -8\frac{x^2}{L}+32\frac{x^3}{L^2}-40\frac{x^4}{L^3}+16\frac{x^5}{L^4}$$

$$N_5 = 7\frac{x^2}{L^2}-34\frac{x^3}{L^3}+52\frac{x^4}{L^4}-24\frac{x^5}{L^5}, \quad N_6 = -\frac{x^2}{L}+5\frac{x^3}{L^2}-8\frac{x^4}{L^3}+4\frac{x^5}{L^4}$$

2.2 Kinematics of the Core

The type of sandwich structures considered in this study consists of two thin but relatively stiff sheets bonded to each side of a thick periodic core Fig.2. A typical setup could be two aluminum sheets glued to foam, viscoelastic, piezoelectric core or any combination. The purpose of the core is to maintain the distance between the face-sheets and to resist shear deformation, thus ideally maintaining pure bending of the beam around the neutral axis.

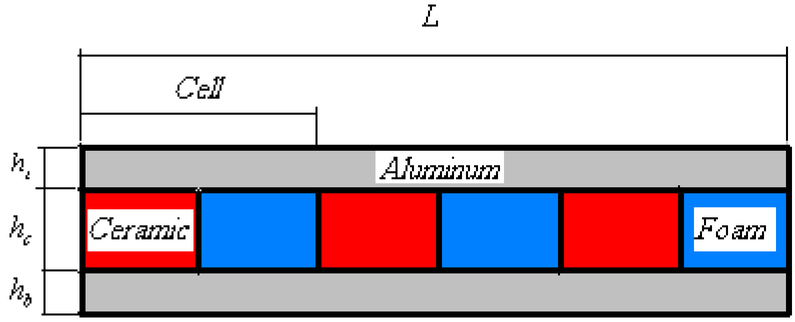


Fig. 2 Periodic sandwich beam

In this model, the axial displacement and shear are taken into consideration while developing the structural model. The deviation from normality of the cross-section is produced by transverse shear which is assumed to be constant over the cross section. The total slope of the beam in that model consists of two parts, one due to bending and the other due to shear Figure 3.

All mechanical quantities (displacements, strains, energies ...) can be written in terms of the transverse deflection (w) and the axial displacements of the top (u_t) and bottom (u_b) layers. A fundamental assumption, which is used for modeling the core, is that line B-C in the core remains straight after deformation, as shown by line $B'-C'$ in Fig.3. This defines the axial deformation of any material position x inside the core as a linear interpolation of the displacements u'_b and u'_t on the surfaces of the face-sheets (the top and bottom layers) given by:

$$u'_b = u_b - \left(\frac{h_b}{2}\right)\frac{\partial w}{\partial x} \tag{3}$$

$$u'_t = u_t + \left(\frac{h_t}{2}\right)\frac{\partial w}{\partial x} \tag{4}$$

The resulting displacement field within the core is thus given by

$$u_P(x,z) = \frac{1}{2}(u'_t + u'_b) + \frac{z}{h_p}(u'_t - u'_b) \tag{5}$$

Substituting from (3) and (4)

$$u_P(x,z) = \frac{u_t + u_b}{2} + \frac{(h_t - h_b)}{4}\frac{\partial w}{\partial x} \tag{6}$$

$$w_P(x,z) = w(x) \tag{7}$$

The corresponding linear strains are

$$\varepsilon_x = \frac{1}{2}\left(\frac{\partial u_t'}{\partial x} + \frac{\partial u_b'}{\partial x}\right) + \frac{z}{h_p}\left(\frac{\partial u_t'}{\partial x} - \frac{\partial u_b'}{\partial x}\right) \tag{8}$$

$$\varepsilon_z = \left(\frac{\partial w_P}{\partial z}\right) = 0 \tag{9}$$

The corresponding shear strain is

$$\gamma_{zx} = 2\varepsilon_{zx} = \frac{\partial w_P}{\partial x} + \frac{\partial u_P}{\partial z} \tag{10}$$

From equation (5) by differentiation

$$\frac{\partial u_P}{\partial z} = \frac{1}{h_p}\left[u_t' - u_b'\right] \tag{11}$$

Substituting in equation (10)

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{1}{h_p}\left[u_t' - u_b'\right] \tag{12}$$

Substituting equations (3) and (4) into equation (12)

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{1}{h_p}\left[u_t - u_b + \frac{\partial w}{\partial x}\left(\frac{h_t}{2} + \frac{h_c}{2}\right)\right]$$

By re-arranging

$$\gamma_{zx} = \frac{u_t - u_b}{h_p} + \frac{d}{h_p}\left(\frac{\partial w}{\partial x}\right) \tag{13}$$

where (d) is the distance between the reference lines of the un-deformed face-sheets given by $d = \left(\frac{h_b}{2} + h_p + \frac{h_t}{2}\right)$, where h_b is the bottom layer thickness, h_t is the top layer thickness, and h_p is the core layer thickness.

The zero transverse normal strain in equation (9) results from the assumption that all layers have a common transverse displacement.

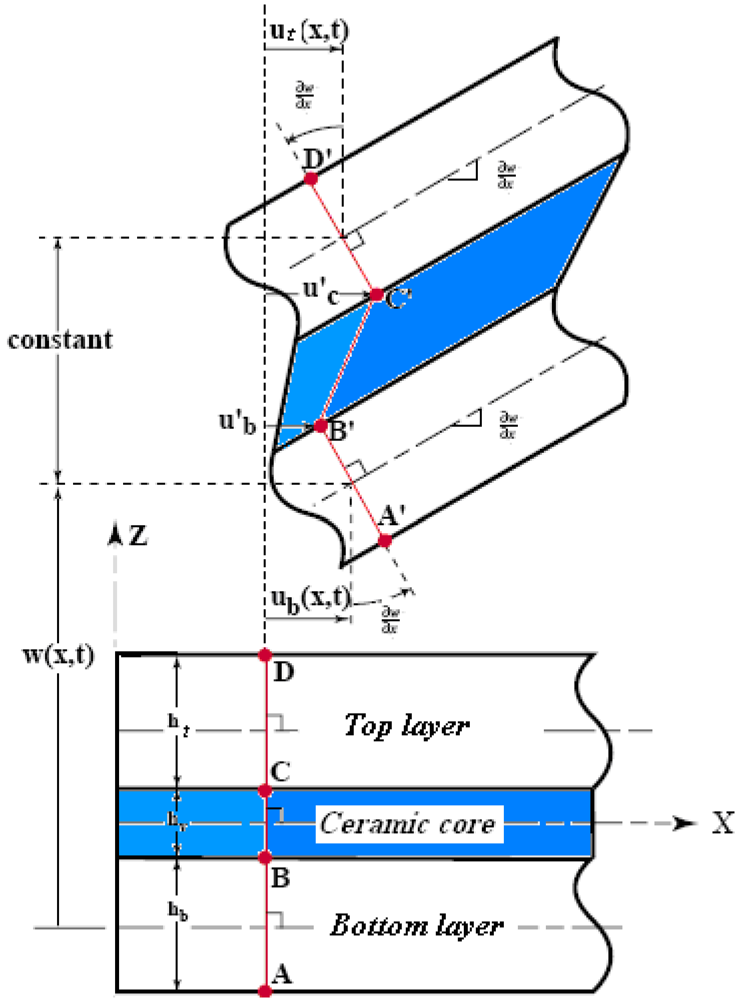


Fig. 3 Core displacement fields

2.3 Development of the equations of motion

The equations of motion in this investigation are developed using Hamilton's principle:

$$\delta\left(\int_{t1}^{t2}(T-U-V)dt\right)=0 \tag{14}$$

where T is the kinetic energy, U is the strain energy, and V is the work done by the external forces.

By taking the first variation of the strain energy, kinetic energy and potential energy of external forces, then integrating by parts with respect to time ($t_1 and\, t_2$ are arbitrary) we get the weak form of the equation of motion, which is used for deriving the finite element model of the system.

2.3.1 The face-sheets energy

Total strain energy for top and bottom layers

$$U_k = \frac{1}{2}\int_0^L E_k \, A_t \left(\frac{\partial u_k}{\partial x}\right)^2 dx + \frac{1}{2}\int_0^L E_k \, I_k \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \qquad (15)$$

Total kinetic energy for top and bottom layers

$$T_k = \frac{1}{2}\int_0^L \rho_k \, A_k \left(\frac{\partial u_k}{\partial t}\right)^2 d\,x + \frac{1}{2}\int_0^L \rho_k \, A_k \left(\frac{\partial w}{\partial t}\right)^2 dx \qquad (16)$$

where u_k, is the axial displacement of the face-sheets (top and bottom layers) and w, is the transverse displacement of the core centre line.

2.3.2 The core energy

Total potential energy of the core layer due to shear strain and bending

$$U_m = \frac{1}{2}\int_0^L G_m \, A_m (\gamma_{zx})^2 dx + \frac{1}{2} E_m \, I_m \int_0^L \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \qquad (17)$$

Total kinetic energy of the core layer due to axial and transverse displacements

$$T_m = \frac{1}{2}\int_0^L \rho_m \, A_m \left(\frac{\partial u_m}{\partial t}\right)^2 d\,x + \frac{1}{2}\int_0^L \rho_m \, A_m \left(\frac{\partial w}{\partial t}\right)^2 dx \qquad (18)$$

where m is related to the material of core

2.4 Element matrices in terms of nodal displacements

The element model presented has three nodes, and is obtained by substituting the energies in terms of nodal displacements. Substituting equations (1) and (2) into equations (15) we get the strain energy stored in the element's face sheets.

2.4.1 Face sheets strain energy

For the bottom layer

$$U_b = \frac{1}{2}\int_0^L E_b \, A_b \left(\frac{\partial u_b}{\partial x}\right)^2 dx + \frac{1}{2}\int_0^L E_b \, I_b \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx$$

$$U_b = \frac{1}{2}\{\delta\}^T E_b A_b \int_0^L \lfloor N_b' \rfloor^T \lfloor N_b' \rfloor dx \{\delta\} + \frac{1}{2}\{\delta\}^T E_b I_b \int_0^L \lfloor N_w'' \rfloor^T \lfloor N_w'' \rfloor dx \{\delta\}$$

$$U_b = \frac{1}{2}\{\delta\}^T \left[K_b^u\right]\{\delta\} + \frac{1}{2}\{\delta\}^T \left[K_b^w\right]\{\delta\} \qquad (19)$$

Similarly for the top layer

$$U_t = \frac{1}{2}\{\delta\}^T E_t A_t \int_0^L \lfloor N_t' \rfloor^T \lfloor N_t' \rfloor dx \{\delta\} + \frac{1}{2}\{\delta\}^T E_t I_t \int_0^L \lfloor N_w'' \rfloor^T \lfloor N_w'' \rfloor dx \{\delta\}$$

$$U_t = \frac{1}{2}\{\delta\}^T \left[K_t^u\right]\{\delta\} + \frac{1}{2}\{\delta\}^T \left[K_t^w\right]\{\delta\} \qquad (20)$$

2.4.2 Face-sheets kinetic energy

Substituting equations (1) and (2) into equations (16) we get the kinetic energy stored in the element's face-sheets.

For the bottom layer

$$T_b = \frac{1}{2}\int_0^L \rho_b A_b \left(\frac{\partial u_b}{\partial t}\right)^2 dx + \frac{1}{2}\int_0^L \rho_b A_b \left(\frac{\partial w}{\partial t}\right)^2 dx$$

$$T_b = \frac{1}{2}\left\{\dot{\delta}\right\}^T \rho_b A_b \int_0^L \lfloor N_b \rfloor^T \lfloor N_b \rfloor dx \left\{\dot{\delta}\right\} + \frac{1}{2}\left\{\dot{\delta}\right\}^T \rho_b A_b \int_0^L \lfloor N_w \rfloor^T \lfloor N_w \rfloor dx \left\{\dot{\delta}\right\}$$

$$T_b = \frac{1}{2}\left\{\dot{\delta}\right\}^T \rho_b A_b \left[M_b^u\right]\left\{\dot{\delta}\right\} + \frac{1}{2}\left\{\dot{\delta}\right\}^T \left[M_b^w\right]\left\{\dot{\delta}\right\} \qquad (21)$$

Similarly for the top layer

$$T_t = \frac{1}{2}\left\{\dot{\delta}\right\}^T \rho_t A_t \int_0^L \lfloor N_t \rfloor^T \lfloor N_t \rfloor dx \left\{\dot{\delta}\right\} + \frac{1}{2}\left\{\dot{\delta}\right\}^T \rho_t A_t \int_0^L \lfloor N_w \rfloor^T \lfloor N_w \rfloor dx \left\{\dot{\delta}\right\}$$

$$T_t = \frac{1}{2}\left\{\dot{\delta}\right\}^T \rho_t A_t \left[M_t^u\right]\left\{\dot{\delta}\right\} + \frac{1}{2}\left\{\dot{\delta}\right\}^T \left[M_t^w\right]\left\{\dot{\delta}\right\} \qquad (22)$$

2.4.3 Core layer strain energy

First we have to express γ_{zx} and u_m in terms of nodal displacements and element shape functions. From equations (6) and (13):

$$u_m = \frac{u_t + u_b}{2} + \frac{(h_t - h_b)}{4} \frac{\partial w}{\partial x}$$

$$\gamma_{zx} = \frac{u_t - u_b}{h_p} + \frac{d}{h_p} \left(\frac{\partial w}{\partial x} \right)$$

Substituting equations (1) and (2) we get

$$\gamma_{zx} = \left[\frac{\lfloor N_t \rfloor - \lfloor N_b \rfloor}{h_p} + \frac{d}{h_p} \lfloor N_w \rfloor' \right] \{\delta\} = \lfloor N_\gamma \rfloor \{\delta\} \tag{23}$$

$$u_m = \left[\frac{\lfloor N_t \rfloor \lfloor N_b \rfloor}{2} + \frac{(h_t - h_b)}{4} \lfloor N_w \rfloor' \right] \{\delta\} = \lfloor N_m \rfloor \{\delta\} \tag{24}$$

where: $\lfloor N_\gamma \rfloor = \left[\frac{\lfloor N_t \rfloor - \lfloor N_b \rfloor}{h_p} + \frac{d}{h_p} \lfloor N_w \rfloor' \right]$ (25)

$$\lfloor N_m \rfloor = \left[\frac{\lfloor N_t \rfloor \lfloor N_b \rfloor}{2} + \frac{(h_t - h_b)}{4} \lfloor N_w \rfloor' \right] \tag{26}$$

By substituting (25) into (17) we get:

$$U_m = \frac{1}{2} \{\delta\}^T G_m A_m \int_0^L \lfloor N_\gamma \rfloor^T \lfloor N_\gamma \rfloor dx \{\delta\} + \frac{1}{2} \{\delta\}^T E_m I_m \int_0^L \lfloor N''_w \rfloor^T \lfloor N''_w \rfloor dx \{\delta\}$$

$$U_m = \frac{1}{2} \{\delta\}^T \left[K_m^\gamma \right] \{\delta\} + \frac{1}{2} \{\delta\}^T \left[K_m^w \right] \{\delta\} \tag{27}$$

2.4.4 Core layer kinetic energy

By substituting (26) into (18) we get:

$$T_m = \frac{1}{2} \left\{ \dot{\delta} \right\}^T \left[\rho_m A_m \int_0^L \lfloor N_m \rfloor^T \lfloor N_m \rfloor dx + \rho_m A_m \int_0^L \lfloor N_w \rfloor^T \lfloor N_w \rfloor dx \right] \left\{ \dot{\delta} \right\}$$

$$T_m = \frac{1}{2} \left\{ \dot{\delta} \right\}^T \left[M_m^u \right] \{\delta\} + \frac{1}{2} \left\{ \dot{\delta} \right\}^T \left[M_m^w \right] \{\delta\} \tag{28}$$

2.4.5 Element total stiffness matrix $[K]^e$

Using equations (19), (20) and (27) we can calculate the overall element stiffness matrix as follows:

$$\left[K\right]^{e}=\left[K_{b}^{u}\right]+\left[K_{b}^{w}\right]+\left[K_{t}^{u}\right]+\left[K_{t}^{w}\right]+\left[K_{m}^{\gamma}\right]+\left[K_{m}^{w}\right] \qquad (29)$$

where:

$$\left[K_{b}^{u}\right]=E_{b}A_{b}\int_{0}^{L}\lfloor N_{b}'\rfloor^{T}\lfloor N_{b}'\rfloor dx\ ,\ \left[K_{t}^{u}\right]=E_{t}A_{t}\int_{0}^{L}\lfloor N_{t}'\rfloor^{T}\lfloor N_{t}'\rfloor dx\ ,$$

$$\left[K_{b}^{w}\right]=E_{b}I_{b}\int_{0}^{L}\lfloor N_{w}''\rfloor^{T}\lfloor N_{w}''\rfloor dx\ ,\ \left[K_{t}^{w}\right]=E_{t}I_{t}\int_{0}^{L}\lfloor N_{w}''\rfloor^{T}\lfloor N_{w}''\rfloor dx\ ,$$

$$\left[K_{m}^{\gamma}\right]=G_{m}A_{m}\int_{0}^{L}\lfloor N_{\gamma}\rfloor^{T}\lfloor N_{\gamma}\rfloor dx\ ,\ \left[K_{m}^{w}\right]=E_{m}I_{m}\int_{0}^{L}\lfloor N_{w}''\rfloor^{T}\lfloor N_{w}''\rfloor dx$$

2.4.6 Element total mass matrix $[M]^{e}$

Using equation (21), (22) and (28) we can calculate the overall element mass matrix as follows:

$$\left[M\right]^{e}=\left[M_{b}^{u}\right]+\left[M_{b}^{w}\right]+\left[M_{t}^{u}\right]+\left[M_{t}^{w}\right]+\left[M_{m}^{u}\right]+\left[M_{m}^{w}\right] \qquad (30)$$

where:

$$\left[M_{b}^{u}\right]=\rho_{b}A_{b}\int_{0}^{L}\lfloor N_{b}\rfloor^{T}\lfloor N_{b}\rfloor dx\ ,\ \left[M_{c}^{u}\right]=\rho_{c}A_{c}\int_{0}^{L}\lfloor N_{c}\rfloor^{T}\lfloor N_{c}\rfloor dx\ ,$$

$$\left[M_{b}^{w}\right]=\rho_{b}A_{b}\int_{0}^{L}\lfloor N_{w}\rfloor^{T}\lfloor N_{w}\rfloor dx\ ,\ \left[M_{c}^{w}\right]=\rho_{c}A_{c}\int_{0}^{L}\lfloor N_{w}\rfloor^{T}\lfloor N_{w}\rfloor dx\ ,$$

$$\left[M_{m}^{u}\right]=\rho_{m}A_{m}\int_{0}^{L}\lfloor N_{u}\rfloor^{T}\lfloor N_{u}\rfloor dx\ ,\ \left[M_{m}^{w}\right]=\rho_{m}A_{m}\int_{0}^{L}\lfloor N_{w}\rfloor^{T}\lfloor N_{w}\rfloor dx$$

In the present work, the transfer matrix is calculated using the procedure mentioned in reference [31].

3. NUMERICAL RESULTS

A code was developed using MATLAB for a periodic sandwich beam which has its core as Ceramic and Foam in order to study the effect of material discontinuity for attenuating the vibration of the beam. The following figures show the results obtained by the developed code for six cells. These results are introduced for the first time since no similar model could be found in the literature.

Top and bottom layers are aluminum with E (modulus of elasticity) =73 GPa and ρ (density) = 2790 Kg/m^{3}. The core layer is composed of a hard core (Ceramic) with

E_p (modulus of elasticity) =86.8 GPa, G_p (shear modulus) =21.1 GPa, ρ_p (density) = 7700 Kg/m^3, and a soft core (Foam) with E_f (modulus of elasticity) =35.3 GPa, G_f (shear modulus) =12 MPa, ρ_f (density) = 32 Kg/m^3. The main dimensions of the beam are: thickness of top layer h_t = 0.5 *mm*, thickness of bottom layer h_b = 0.5 *mm*, thickness of core layer h_c = 2*mm*, length of soft core per cell L_f = 15 *cm*, length of hard core per cell L_p =15 *cm*, and beam width b =5 *cm*.

From Fig. 4, it appears that there are three stop bands extending in the ranges 450-500 Hz, 1500-2200 Hz and 7000-8000 Hz. These stop bands in parallel with Fig. 5 correspond to the drops in the frequency response.

4. DISCUSSION AND CONCLUSIONS

In this study, the analysis of a sandwich beam with periodic core is presented. The analysis involved a finite element model based on a first order shear deformation theory for the core material and thin beam theory for the face-sheets. The finite element model is then reduced using dynamic condensation to obtain the transfer matrix of the cell including the core shear deformation. The results obtained indicate the existence of the stop bands similar to the cases of beams in pure bending conditions. The result of this study are encouraging towards further study of the effect of periodic, active or passive, controllers in the shear mode.

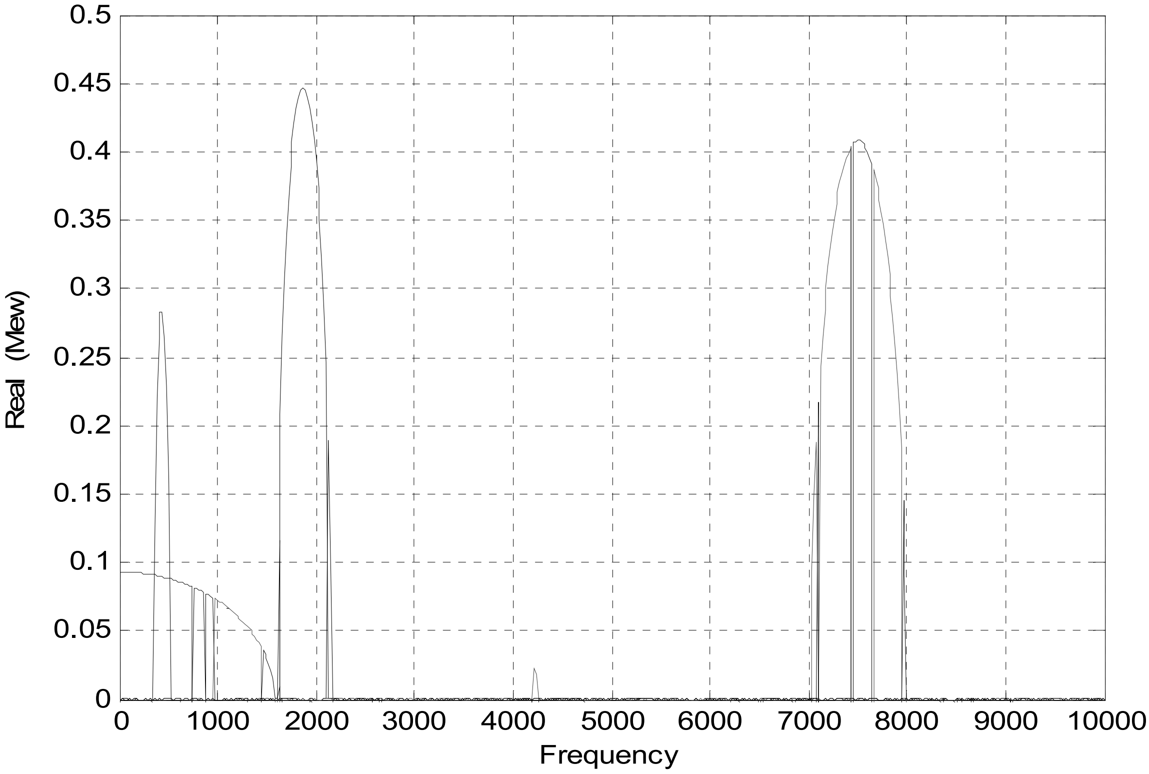


Fig. 4 Stop bands obtained from the transfer matrix eigenvalues

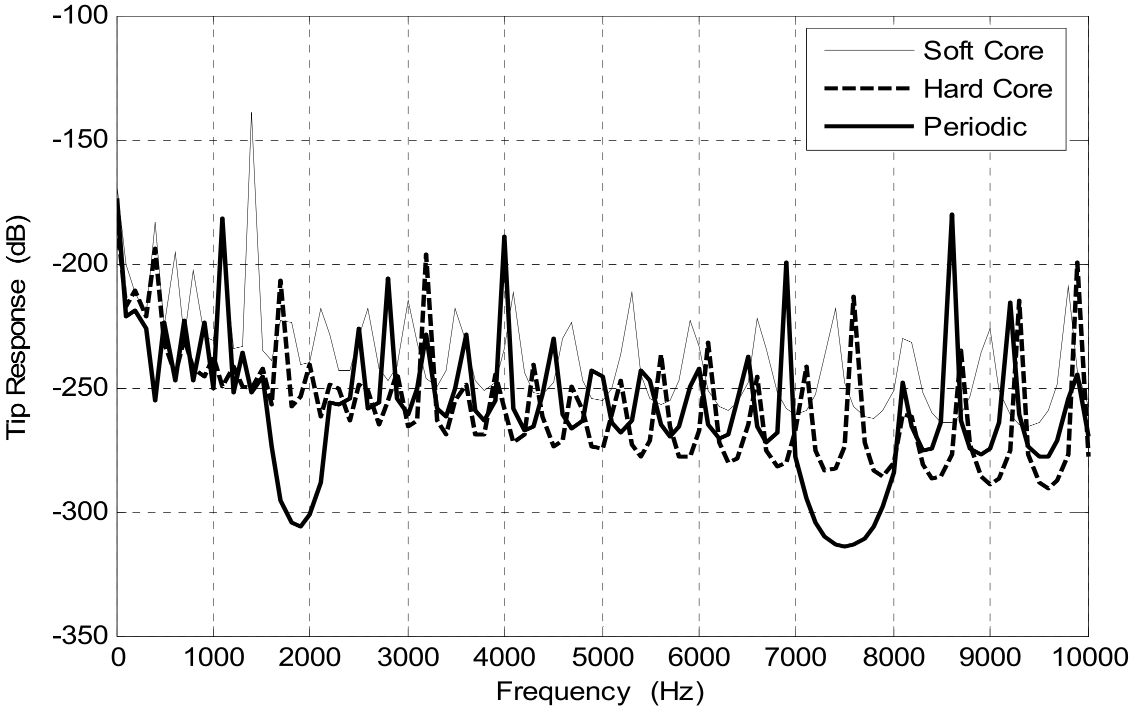


Fig. 5 Frequency response of t he beam with different core configurations

REFERENCES

[1] M.E.Raville, E.S.Ueng and M.M.Lei "Natural frequencies of vibration of fixed-fixed sandwich beams" Journal of Applied Mechanics, Transactions ASME 83, 367-371,1961.

[2] R.A. DiTaranto "Theory of vibratory bending for elastic and viscoelastic layered finite-length beams" Journal of Applied Mechanics, Transactions ASME 32, 881-886, 1965.

[3] R.A. DiTaranto and W.Blasingame "Composite damping of vibrating sandwich beams" Journal of Engineering for Industry, Transactions ASME 89, 633-638,1967

[4] D.J.Mead and S. Markus "The forced vibration of a three-layer, damped sandwich beam with arbitrary boundary conditions" Journal of Sound and Vibration 10,163-175,1969.

[5] K.M.Ahmed "Free vibration of curved sandwich beams by the method of finite elements" Journal of Sound and Vibration 18, 61- 74, 1972.

[6] K.M.Ahmed "Dynamic analysis of sandwich beams" Journal of Sound and Vibration 21, 263-276, 1972.

[7] D.J.Mead and Y. Yaman "The harmonic response of rectangular sandwich plates with multiple stiffening: a flexural wave analysis" Journal of Sound and Vibration 145,409-428, 1991.

[8] T. Sakiyama, H.Matsuda and C. Morita Yaman "Free vibration analysis of sandwich beams with elastic or viscoelastic core by applying the discrete Green function" Journal of Sound and Vibration 191,189-206, 1996.
[9] T.P. Khatua, Y.K. Cheung, Bending and vibration of sandwich beams and plates, International Journal for Numerical Methods in Engineering 6, 11–24, 1973.
[10] Y.V.K. Sadasiva Rao, B.C. Nakra, Vibrations of unsymmetrical sandwich beams and plates with viscoelastic cores, Journal of Sound and Vibration 34 (3), 309–326, 1974.
[11] C.L. Sisemore, C.M. Darvennes, Transverse vibration of elastic–viscoelastic–elastic sandwich beams: compression experimental and analytical study, Journal of Sound and Vibration 252 (1), 155–167, 2002.
[12] J.R. Banerjee, Free vibration of sandwich beams using dynamic stiffness method, Computers and Structures 81 (2003) 1915–1922.
[13] M. Ganapathi, B. P. Pateli, O. Polit, and M. Touratier "A C1 Finite element including transverse shear and torsion warping for rectangular sandwich beams" Int. J. Numer. Math. Eng. 45, 47(75) 1999.
[14] K.H. Ha, Exact analysis of bending and overall buckling of sandwich beam systems, Computers and Structures 45 (1) (1992) 31–40.
[15] T. Lan, P.D. Lin, L.W. Chen, Thermal buckling of bimodular sandwich beams, Composite Structures 25, 1993.
[16] N. Ganesan and V. Pradeep "Buckling and vibration of sandwich beams with viscoelastic core under thermal environments" Journal of Sound and Vibration 286 (2005) 1067–1074
[17] Jr. Kerwin, E.M. ''Damping of Flexural Waves by a Constrained viscoelastic Layer'' Journal of the Acoustical Society of America 31 (7), 952–962, 1959.
[18] R. A. DiTaranto, ''Theory of Vibratory Bending for Elastic and Viscoelastic Layered Finite Length Beams,'' Journal of Applied Mechanics Transactions of the American Society Series E (87) 881–887, 1965.
[19] , D. J. Mead and S. Marcus, ''The Forced Vibration of a Three-layer, Damped Sandwich Beam with Arbitrary Boundary Conditions,'' Journal of Sound and Vibration, 10 (2), pp. 163–175, 1969.
[20] Y. V. K. S Rao and B. C. Nakra, ''Vibrations of Unsymmetrical Sandwich Beams and Plates with Viscoelastic Cores, Journal of Sound and Vibration, 34, No.3, pp. 309–326,1974.
[21] Y. P. Lu, J. W. Killian, and G. C. Everstine, ''Vibrations of Three Layered Damped Sandwich Plate Composites,'' J. Sound Vib., 64, No. 1, pp. 63–71, 1979.
[22] P. Cupial and J. Niziol, ''Vibration and Damping Analysis of a Three layered Composite Plate with a Viscoelastic Mid-layer,'' J. Sound Vib. 183,No. 1, pp. 99–114 ,1995.
[23] S.K. Dwivedy , K.C. Sahu and Sk. Babu "Parametric instability regions of three-layered soft-cored sandwich beam using higher-order theory" Journal of Sound and Vibration (304) 326–344,2007.
[24] Y.L. Chung- and W. C. Lien "Dynamic stability of a rotating beam with a constrained damping layer" Journal of Sound and Vibration (267) 209–225,2003.
[25] D.J. Mead, "Wave Propagation in Continuous Periodic Structures: Research Contributions From Southampton", Journal of Sound and Vibration, Vol. 190, No. 3, pp. 495-524, 1996.

[26] E.E. Ungar, "Steady-State Response of One-Dimensional Periodic Flexural Systems" The Journal of The Acoustic Society of America, Vol. 39, No. 5, pp. 887-894, 1966.
[27] G.S. Gupta, "Natural Flexural Waves and The Normal Modes of Periodically-Supported Beams and Plates" Journal of Sound and Vibration, Vol. 13, No. 1, pp.89-101, 1970.
[28] M.G. Faulkner and D.P. Hong, "Free Vibration of Mono-Coupled Periodic System" Journal of Sound and Vibration, Vol. 99, No. 1, pp. 29-42, 1985.
[29] D. J. Mead, and Y. Yaman, "The Response of Infinite Periodic Beams to Point Harmonic Forces: A Flexural Wave Analysis" Journal of Sound and Vibration, Vol. 144, No. 3, pp.507-530, 1991.
[30] M. Tawfik, J. Chung, and A. Baz, "Wave Attenuation in Periodic Helicopter Blade," 5th Jordan International Mechanical Engineering Conference, Amman, Jordan, 26-28 April ,2004.
[31] M.M. Alaa El-Din, and M. Tawfik, "Vibration attenuation in rotating beams with periodically distributed piezoelectric controllers" The Thirteenth International Congress on Sound and Vibration, Vienna, Austria, July 2006