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SIX-PORT MODEL FOR SOUND PROPAGATION IN A POROUS MEDIA WITH APPLICATION TO DIESEL PARTICULATE FILTERS

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ABSTRACT

This work presents a 2-D field model for the study of sound propagation in a diesel particulate filter (DPF) unit. The 2-D model is formulated using the field Navier-stocks, energy, and continuity equations and retains the normal as well as transverse component of gas velocity. Temperature, pressure, density, and velocities are taken to be as plane and time harmonic variations. By substituting the differentials of these quantities with respect to both plane and time in field equations, a set of three coupled linear 2-D field variation equations for pressure, axial and transverse velocities is obtained. The obtained model is solved analytically using Fourier series approximations. The approximate solution is used to build a 2D acoustic model for the exhaust gases emission, with the existence of the diesel particulate filter, which accounts for both attenuation and phase shift defining the propagation wave constant. In addition the obtained approximate solution is used to determine the acoustics impedance of the DPF unit, comparing between different types of DPF based on sound transmission losses performance, soot loading, noise and vibration damping, in addition to calculating the noise reduction factor (NRF). In the present study, unlike previous ones, six, rather than four, roots for wave propagation constant are obtained corresponding to the obtained six port acoustic DPF model. The results obtained using the present six -port model, for selected system parameters are graphically displayed and compared with those available in the open literature using four- port models. The present model results show, in general, similar qualitative behavior and a significant quantitative improvement of the available results in the open literature obtained using a four port model.

KEY WORDS

Porous media, diesel particulate filter, acoustic model, Darcy law, wave propagation constant, acoustic transmission losses.

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INTRODUCTION

One of the leading technologies for meeting future particulate matter (PM) emission strict standards is the diesel particulate filter (DPF). These devices generally consist of a wall-flow type filter positioned in the exhaust stream of diesel engine vehicles. As the exhaust gases pass through the exhaust system, particulate emissions are collected and stored in the DPF. Because the volume of diesel particulates collected by the system will eventually fill up and even plug the DP filter, a method for controlling trapped particulate matter and regenerating the filter is necessary. The DPF is a superior system in the reduction of particulate matters because it can reduce about 70% of the generated PM. A typical DPF system contains a large number of thin tubes or cavities with a diameter of about (1-2 mm), and (0.15-0.5 cm) length. It is available in several types such as: electric heater, burner (ceramic filter), and fuel additive type; the latter is a honey-comb ceramic. The honey-comb type constitutes an additive supply and an electronic system. In this type Fe is used as an additive whereby iron oxide is formed which reacts with carbon and then it is converted to iron. The DPF is connected at a suitable intervening location along the exhaust gases path through the main exhaust pipe. Thus the noise and vibration characteristics of exhaust system are expected to change and consequently affect the performance of the engine by developing back pressure, changing temperature and velocity of the exhaust gases...etc. Hence building an acoustic model for the DPF is valuable to the efforts of predicting its effect on the overall performance of the exhaust system and the engine.

The acoustic characteristics of the DPF systems have been the subject of many theoretical and experimental investigations, e.g [1-15]. Greevesm [9] studied theoretically the origin of hydrocarbons emission from diesel engines. His results indicate that a DPF can eliminate some of PM and is a very promising as an after-treatment technique. Yu and Shahed [15] studied the effects of injection timing and exhaust gas recirculation on emissions from a diesel engine .They classified a DPF action as filtration and regenerative processes. Konstandopoulous et al. [13] studied the DPF wall-flow, pressure drop and cooling efficiency. They used Darcy's law to describe the coupling between neighboring channels of the DPF and to predict system variable changes in the fluctuating fields between the neighboring DFP channels. Peat [14] studied sound propagation in capillary tubes using FEM solutions of simplified wave equations for a visco-thermal fluid flow. Also, Astley and Cumings [4] presented FEM solutions to the axial flow though porous medium, based on simplified wave equation in a visco-thermal fluid. They presented an analysis for the laminar flow with a parabolic velocity distribution and a quadratic flow cross-section. They simplified the governing equations by assuming that the axial gradients are significantly smaller than the gradients over the cross-section. Employing the same simplified analysis presented in [4], Dokumaci [6] obtained an exact solution for the case of a plug flow and a circular cross-section. Using this approach he presented an acoustic two-port model for a catalytic converter unit, which takes into account the presence of a mean flow , assuming a uniform velocity profile, and the presence of a mean pressure gradient . Ih et al. [10] have developed analytical solutions for sound propagation in capillary cylindrical tubes which assumes a parabolic mean axial flow, and neglects the radial component of the particle velocity. Jeong and Ih [11] presented numerical solutions of the basic governing flow field equations taking into account the radial particle velocity. Their results showed that the radial velocity has a small but noticeable effect on the DPF acoustic behavior. Dokumaci [7] extended his earlier work in [6] to the case of rectangular narrow tubes with a plug flow. His analysis was based on approximate double Fourier sine series expansions for the field variables over the channel cross-section.

Allam and Abom [3] presented an approximate 1-D, two ports, and discrete acoustic model for predicting sound transmission losses for an entire diesel particulate filter (DPF) unit. Their model was based on an approximate treatment of the viscous and thermal losses along the narrow channels of the DPF. Also in this model the steady flow resistance was used to calculate equivalent lumped acoustic impedance. To include the wave propagation effects the monolith was described using coupled wave guide model, where coupling is via the porous walls of monolith. Darcy's law was used to describe the pressure drop in the porous walls. This 1-D wave propagation model yielded a constant, frequency independent, transmission loss and agreed within 1 dB with measured data on a typical hot filter but for low frequencies (<300Hz).

Allam and Abom [1] modified their 1D model in [2] using the classical (exact) Kirchhoff solution for a plane wave propagation in a homogenous , visco-thermal fluid in a rigid narrow tube The modified model includes a more detailed account for viscous and thermal losses by solving the convective acoustic wave equations for two neighboring channels using Zwikker and Kosten theory. It also uses and modifies the analysis followed by Dokumaci [7] to account for the effect of wall permeability. They used a straightforward linearization and segmentation approach to convert the obtained 1-D model to a 4-th order (4-port) eigenvalue problem whose four eigenvalue are the wave propagation constants. The presented results which showed a fair agreement with measured ones for frequencies up to 1000 Hz for a typical filter at operating (hot) conditions. It is noted that the above Allam and Abom [1] model assumes the fields to be constant with respect to space but vary harmonically with time. In the present work, the model developed by Allam and Abom [1] is used and modified by adding the effects of transverse velocity which, to authors' knowledge, has been ignored in previous models found in the open literature. Hence, a 2-D, 6-port acoustic model for the entire DPF unit is presented by taking into accounts field variations both with respect to time and the 2-D space. The calculated results for the acoustic transmission losses in a typical hot DPF obtained using the present six -port model, for selected system parameters are compared with those presented in [1] using a 4-port model.

FORMULATION AND SOLUTION OF THE PROBLEM

Following the analysis in [1, 2], the DPF is divided into five parts, as shown in Fig. 1. These parts are: the inlet (IN), narrow pipes with impermeable walls (1) and (3), the ceramic section (2), and the outlet section (OUT). The DPF may be manufactured from different materials (Cordierite or Silicon Carbide for example) and in its most common form consists of a substrate of narrow, approximately square in the cross- section with a width about (1-2) mm., channels in which each channel is blocked at one end. Adjacent channels have this blockage at alternate ends. With this construction exhaust gas may enter at one end, but must pass through the wall of a channel before exiting and is thus termed a wall flow device. It is clear that the flow in y-direction has a considerable effect on the operation of the DPF, i.e. the transverse velocity can have a significant effect on the flow characteristics and the 1-D flow approximation may not be a realistic one. Therefore, in the present work, the flow will be considered as 2-D by taking into account the effect of the transverse velocity.

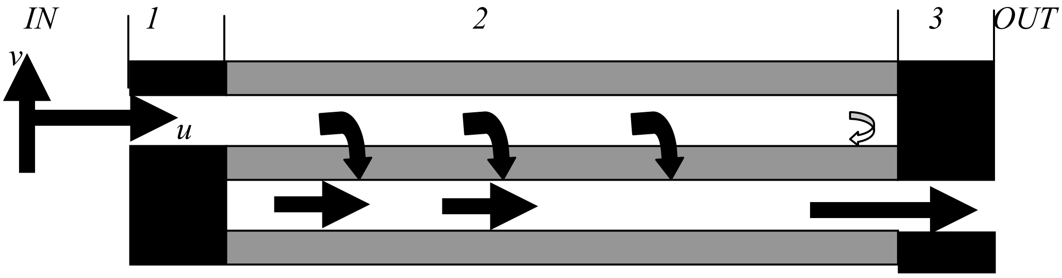

Fig. 1. DPF sections and the 2D flow of gases.

Derivation of the Governing Equations

In derive the acoustic model for the DPF unit the following assumptions, as in [1, 2], are considered: a) The DPF unit is considered partially as a porous media, b) The transverse "normal" component of velocity (v_{yj}), unlike the model in [1, 2], will not be neglected: i.e. the flow is treated as a 2-D, c) Flow is considered as viscous- thermal, incompressible, laminar, a steady and Newtonian ideal gas, d) Chemical reactions are neglected, and e) Pressure, temperature, velocities, and density are variations are considered to be harmonic in both time and the 2-D space.

By considering the field to be 2-D instead of 1-D, the describing field Navier – stocks (momentum), continuity, energy and state equations used in [1], become:

A- Navier Stocks equations:

$$\rho_{0j}\left[\frac{\partial u}{\partial t}+u.\nabla U_{0j}\right]=-\left[\nabla P\right]_i+\mu\left[\nabla^2 u_{xj}\right]_i \tag{1}$$

which, respectively, in x and y directions yields:

$$\rho_{0j}\left[\frac{\partial u_{xj}}{\partial t}+u_{xj}\frac{\partial U_{0j}}{\partial x}+v_{yj}\frac{\partial U_{0j}}{\partial y}+w_{zj}\frac{\partial U_{0j}}{\partial z}\right]+\rho_{0j}U_{0j}\frac{\partial u_{xj}}{\partial x}= -\frac{\partial P_j}{\partial x}+\mu_j\left(\frac{\partial^2 u_{xj}}{\partial x^2}+\frac{\partial^2 u_{xj}}{\partial y^2}+\frac{\partial^2 u_{xj}}{\partial z^2}\right) \tag{2}$$

$$\rho_{0j}\left[\frac{\partial v_{yj}}{\partial t}+u_{xj}\frac{\partial V_{0j}}{\partial x}+v_{yj}\frac{\partial V_{0j}}{\partial y}+w_{zj}\frac{\partial V_{0j}}{\partial z}\right]+\rho_{0j}V_{0j}\frac{\partial v_{yj}}{\partial y}=-\frac{\partial P_j}{\partial y}+\mu_j\left(\frac{\partial^2 v_{yj}}{\partial x^2}+\frac{\partial^2 v_{yj}}{\partial y^2}+\frac{\partial^2 v_{yj}}{\partial z^2}\right) \tag{3}$$

B- Continuity equation**:**

$$\frac{\partial \rho_j}{\partial t}+u_{xj}\frac{\partial \rho_{0j}}{\partial x}+U_{0J}\frac{\partial \rho_j}{\partial x}+\rho_j\frac{\partial U_{0j}}{\partial x}+\rho_{0j}\nabla.u_j=0 \tag{4}$$

C- Energy equation:

$$\rho_{0j} Cp_j \left[\frac{\partial T_j}{\partial t} + U_{0j} \frac{\partial T_j}{\partial x} + u_{xj} \frac{\partial T_{0j}}{\partial x} \right] + \rho_j Cp_j U_{0j} \frac{\partial T_{0J}}{\partial x} = -\frac{\partial P_j}{\partial t} + U_{0j} \frac{\partial P_j}{\partial x} + \frac{\partial P_{0j}}{\partial x} u_{xj}$$
$$+ K_{thj} \nabla_s^2 T_j \tag{5}$$

D-State equation:
With the assumption of ideal gas, the linearized sate equation (linearized ideal gas law) takes the form:

$$\rho_j = \left(\frac{p_j}{R_J T_{0j}} \right) - \left(\frac{\rho_{0j} T_j}{T_{oj}} \right), \quad . \tag{6}$$

where

$$\rho_j = \rho_j (x, y, z, t)$$

$$T_j = T_j (x, y, z, t)$$
$$P_j = P_j (x, y, t), \tag{7}$$

x, *y* denotes the channel axis, u, v are the acoustic particle velocities in , respectively, the x an y directions , j = 1, 2 represent the inlet and outlet pipes, respectively. Also *p, T* and *ρ* are the acoustic pressure, temperature and density, respectively, μ is the shear viscosity coefficient, k_{thj} is the thermal conductivity of the fluid, R is the gas constant, *Cp* is the specific heat coefficient at constant pressure, P_0, T_0 and ρ_0 denote the ambient pressure, temperature and density, respectively, U_0, V_0 denotes the axial mean flow velocity and transverse velocity respectively, and ∇_s^2 denotes the Laplacian over the channel cross-section.

To describe the coupling between neighboring channels (which describes the porosity of diesel particulate filter) Darcy's law is applied to the fluctuating fields [1]:

$$p_1 - p_2 = R_w u_w \tag{8}$$

where a subscript *w* refers to wall, u_w is the acoustic velocity through the wall, R_w is the wall resistance, which is given by $R_w = \mu_w h_t / \sigma_w$, μ_w is the dynamic viscosity, h_t is the wall thickness, and σ_w is the wall permeability.

In order to convert the above nonlinear model into an analytically tractable one, the linearization and segmentation approach presented in [1] is closely followed. Accordingly, and noting that the present model is a 2-D one, the following time and 2-D space harmonic variations for the fields are assumed:

$$u_{xj} = A_0 e^{iwt} \tag{9}$$

$$v_j = B_0 e^{iwt} \tag{10}$$

$$\rho_j = \rho_0 e^{iwt},$$
$$P_j = P_0 e^{-iwt}, \tag{11}$$
$$T_j = T_0 e^{iwt}$$

$$u_{xj} = -D'(A_0 e^{\Gamma k_1 x} - B_0 e^{-\Gamma k_1 x}) \tag{12}$$

$$v_{yj} = -D'(A_0 e^{\Gamma k_1 y} - B_0 e^{-\Gamma k_1 y}) \tag{13}$$

$$P_{jy} = A_0 e^{\Gamma k_1 y} + B_0 e^{-\Gamma k_1 y} \tag{14}$$

$$P_{jx} = A_0 e^{\Gamma k_1 x} + B_0 e^{-\Gamma k_1 x} \tag{15}$$

$$\rho_{jx} = A_0 e^{\Gamma k_1 x} + B_0 e^{-\Gamma k_1 x} \tag{16}$$

$$\rho_{jy} = A_0 e^{\Gamma k_1 y} + B_0 e^{-\Gamma k_1 y} \tag{17}$$

$$T_{jx} = A_0 e^{\Gamma k_1 x} + B_0 e^{-\Gamma k_1 x} \tag{18}$$

$$T_{jy} = A_0 e^{\Gamma k_1 y} + B_0 e^{-\Gamma k_1 y} \tag{19}$$

Upon substituting equations (9)-(19) and their required differentials into equations (1)-(3), and using equations (4) and (5), the governing field equations (1)-(3) take the linear form

$$\rho_{0j}\left[iw + \frac{\partial U_{0j}}{\partial x}\right]u_{xj} + \rho_{0j}\frac{\partial U_{0j}}{\partial y}v_{yj} + (D_{0j}U_{0j} + \frac{\partial}{\partial x})P_{jx} = \mu_j(\frac{\partial^2 u_{xj}}{\partial x^2} + \frac{\partial^2 u_{xj}}{\partial y^2} + \frac{\partial^2 u_{xj}}{\partial z^2}) \tag{20}$$

$$\rho_{0j}\left[iw + \frac{\partial V_{0j}}{\partial y}\right]v_{yj} + \rho_{0j}\frac{\partial V_{0j}}{\partial x}u_{xj} + (D_{0j}V_{0j} + \frac{\partial}{\partial x})P_{yj} = \mu_j(\frac{\partial^2 v_{yj}}{\partial x^2} + \frac{\partial^2 v_{yjj}}{\partial y^2} + \frac{\partial^2 v_{yjj}}{\partial z^2}) \tag{21}$$

$$(iw + \frac{\partial U_{0j}}{\partial x})\rho_j + (\frac{\partial \rho_{0j}}{\partial x} + D_{0jj}U_{0j})u_{xj} + \rho_{0j}\nabla . u_j = 0 \tag{22}$$

$$\begin{aligned}(\rho_{0j}Cp_j iw - K_{thj}\nabla_s^2)T_j + \rho_{0j}Cp_j u_{xj}\frac{\partial T_{0j}}{\partial x} + Cp_j U_{0j}\frac{\partial T_{0j}}{\partial x}\rho_{0j} = \\ (iw + U_{0j}\frac{\partial}{\partial x})P_j + (\frac{\partial P_{0j}}{\partial x} - U_{0j}D_{0jj}\rho_{0j})u_{xj}\end{aligned} \tag{23}$$

It is noted that by setting to zero V_{0j}, v_{yj} in the above equations (e.g by neglecting the transverse velocity) the above model in Eqns (21)-(24) reduces to the 1-D developed by Allam and Abom [1].

To get a description to the acoustic model of DPF unit, and to find acoustic impedance, transmission losses, and other parameters needed such as noise reduction factor, equations (21) to (23) must be solved for the variables P_j, T_j, u_{xj}, and $v_{yj.}$ These four, homogenous, coupled linear equations with four variables constitute an eigenvalue problem. The condition of a non- trivial solution to these equations leads to a characteristic frequency equation and associated eigenvectors. The eigenvalue problem is obtained as follows. First one assumes:

$$\begin{aligned} P_j &= A_J \exp(-i\Gamma k_1 x), \\ u_{xj} &= H_j(x,y,z)P_j, \\ T_j &= F_j(x,y,z)P_j, \\ v_{yj} &= B_j \exp(-i\Gamma k_1 y) \end{aligned} \tag{24}$$

where A_j, H_j, F_j and B_j are constant coefficients. Next substituting the expressions in equation (24) into equations (21)-(23) one obtains

$$\frac{\partial^2 H_j}{\partial x^2} + \frac{\partial^2 H_j}{\partial y^2} + \frac{\partial^2 H_j}{\partial z^2} - i\beta_{xj}^2 H_j = \frac{D_{0j} CM_j}{\mu_j} - \frac{-i\Gamma k_1}{\mu_j} \tag{25}$$

$$\frac{\partial^2 H_j}{\partial x^2} + \frac{\partial^2 H_j}{\partial y^2} + \frac{\partial^2 H_j}{\partial z^2} - i\beta_{jy}^2 H_j = \frac{D_{0j} CM_j}{\mu_j} - \frac{-i\Gamma k_1}{\mu_j} \tag{26}$$

$$\frac{\partial^2 F_j}{\partial x^2} + \frac{\partial^2 F_j}{\partial y^2} + \frac{\partial^2 F_j}{\partial z^2} - \sigma_{jx}^2 F_j = \sigma_{0j}^2 + \sigma_{1j}^2 H_j \tag{27}$$

where

$$\beta_{xj}^2 = (-\Gamma k_1 + \frac{1}{w}\frac{\partial U_{0j}}{\partial x} + \frac{1}{w}\frac{\partial U_{0j}}{\partial y})S_j^2 \ ,$$

$$\beta_{jy}^2 = (-\Gamma k_1 + \frac{1}{w}\frac{\partial V_{0j}}{\partial x} + \frac{1}{w}\frac{\partial V_{0j}}{\partial y})S_j^2$$

$$\sigma_{jx}^2 = S_j^2 \Pr(i - \frac{U_{0j}}{wT_{0j}}\frac{\partial T_{0j}}{\partial x}),$$

$$\sigma_{0jx}^2 = (\frac{\Pr S_j^2}{w}\frac{\partial T_{0j}}{\partial x} - \frac{1}{K_{thj}}\frac{\partial P_{0j}}{\partial x} + \frac{S_j^2 MU_{0j}}{wK_{thj}} D),$$

$$\sigma_{jx}^2 = \frac{1}{K_{thj}}(iw - iU_{0j}\Gamma Mw - \frac{Cp_j U_{0j}}{R_j T_{0j}}\frac{\partial T_{0j}}{\partial x}),$$

$$\Pr = \sqrt{\frac{\mu_j Cp_j}{K_{thj}}}$$

Next, following Allam and Abom [1], the fields in equations are expanded as Fourier sinus series as follows:

$$H(x,y,z) = \sum_{m,n} a_{mn} \sin\frac{m\Pi y}{2a_j} \sin\frac{n\Pi z}{2a_j} \tag{28}$$

$$F(x,y,z) = \sum_{m,n} b_{mn} \sin\frac{m\Pi y}{2a_j} \sin\frac{n\Pi z}{2a_j} \tag{29}$$

Then upon substituting equations (28) and (29) into equations (25)-(27), and, as in [1], averaging the mass conservation equation (22), one obtains the following eigenvalue problem:

$$\left[\frac{1}{R_j T_{0j}}(iw+\frac{\partial U_{0j}}{\partial x})(1-\rho_{0j}R_j < F_j >)+(\frac{\partial \rho_{0j}}{\partial x}+DU_{0j}-2ik_1\Gamma\rho_{0j}) < H_j >\right]A_j$$
$$+(-1)^{j-1} * \frac{2\rho_{0j}(A_1-A_2)}{a_j R_w}=0 \qquad (30)$$

which can be written in matrix form as follows

$$\begin{bmatrix} K_{11}+K_{21} & -K_{12} \\ K_{21} & K_{12}-K_{22} \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (31)$$

where

$$K_{1j}=\left[\frac{R_w a_j}{2\rho_{0j}R_j T_{0j}}(iw+\frac{\partial U_{0j}}{\partial x})(1-\rho_{0j}R_j < F_j >)+\frac{R_w a_j}{2\rho_{0j}}(\frac{\partial \rho_{0j}}{\partial x}+DU_{0j}-2ik_1\Gamma\rho_{0j}) < H_j >\right] \qquad (32)$$

and

$$K_{2j}=(-1)^{j-1} \qquad (33)$$

For a non trivial solution the determinant of the coefficients matrix in equation (31):

$$\begin{bmatrix} K_{11}+K_{21} & -K_{12} \\ K_{21} & K_{12}-K_{22} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (34)$$

is set to zero, whereby one obtains the following system characteristic frequency equation.

$$(K_{11}+K_{21})(K_{12}-K_{22})+K_{12}K_{21}=0 \qquad (35)$$

The above frequency equation is a 6th order equation for the propagation constant "Γ which can only be solved numerically. A specially constructed Matlab program was used to solve for the six roots of equation (35) for selected values of system parameters .Samples of the obtained numerical results are shown in Table 1.

Table 1. Calculated wave propagation constants $\Gamma_{j,} \quad j=1,.....,6.$

a- Cold conditions (T=293˚K, w=200-300 Hz)

Γ_1=-0.6808-0.5580i;	*Γ_2=0.6808+0.5580i;*
Γ_3=-0.6185+0.1188i;	*Γ_4=-0.6185+0.1188i;*
Γ_5=-0.0623-0.0748i;	*Γ_6=-0.0623+0.0748i;*

b- Hot conditions (T=773˚K, w=400-1000 Hz);

Γ_1=-0.3407-0.1522i;	*Γ_2=-0.3407+0.1522i;*
Γ_3=-0.2719+0.2250i;	*Γ_4=-0.2719+0.2250i;*
Γ_5=-0.0688-0.0391i;	*Γ_6=-0.0688+0.0391i;*

Note that the roots (eigenvalues) of equation (35) are complex conjugate pairs, e.g the equation has three complex conjugate pairs. The corresponding three complex eigenvectors are given by:

$$\begin{pmatrix} \hat{p}_1(x) \\ \hat{p}_2(x) \\ \hat{p}_3(x) \end{pmatrix} = \sum_{n=1}^{6} \hat{a}_n e^{-ik_1\Gamma_n x} e_{j,n} \tag{36}$$

where $\hat{p}\ (x)$ is the acoustics pressure. , $\hat{a}_n$ is the modal amplitude, Γ_n is the calculated wave propagation constant (eigenvalue) and $e_{j,n}$, $j=1,2$, are the corresponding components of the 2-D eigenvector, with j=1 corresponds to the inlet and j=2 corresponds to outlet. Note that equation (36) represents the sound field (pressure fluctuation) in filter section. In addition to equation (36) one needs the field volume pressure velocities to calculate the sound transmission losses in the filter. The corresponding volume pressure velocities $\hat{q}_n(x)$ are obtained, as in [1], by dividing equation (36) by the characteristic wave impedance Z, where $Z = \frac{\rho c}{A}$, c= speed of sound, $\rho =$ mass density of the medium, and A is the cross sectional area of the DPF part. This leads to the following expression for the volume pressure velocities $\hat{q}_n(x)$:

$$\begin{pmatrix} \hat{q}_1(x) \\ \hat{q}_2(x) \\ \hat{q}_3(x) \end{pmatrix} = \sum_{n=1}^{6} \hat{a}_n e^{-ik_2\Gamma_n x} e'_{j,n} \tag{37}$$

where $e'_{j,n} = 4a_j^2 < H_{j,n} > e_{j,n}$, $k_2 = k_1\sqrt{1-8i\beta/k_1}$, $\beta = C_j\rho_w / dh_j R_w$ $<H_j>=4a_{11}/\Pi^2$ **.** Finally, to calculate the acoustic transmission losses in the DPF unit, one uses equation (36) and (37) to formulate the so called the transformation matrix T_{DPF} ,

$$[\hat{q}] = [T_{DPF}][\hat{P}] \tag{38}$$

where the T_{DPF} is a 6×6 matrix, which is formed as a product of the I port matrices for the five segments of the DPF, and takes the form [1]:

$$T_{DPF} = T_{IN}\ T_1\ T_2\ T_3\ T_{out} \tag{39}$$

The acoustic transmission losses *TL* are then calculated using the relation [1]:

$$TL = 20\ log\ |T_{DPF}/2| \tag{40}$$

In addition, the noise reduction factor *NRF* can be calculated by using the following equation:

$$NRF = LP_2 - LP_1 = 20\log\frac{P_2}{P_1} \tag{41}$$

RESULTS AND CONCLUSIONS

The wave propagation constant Γ_n, transmission losses TL and the nose reduction factor NRF were calculated, for selected system parameters, using equations (35), (40) and (41). A Matlab program was used to carry out all the needed calculations. The results obtained were for the cases: hot and cold. For the hot case, the temperature was taken as $500^0 C$. While for the cold case the temperature was assumed to be $25^0 C$. For both cases the frequency was varied over the range $300-1000$ Hz. Also results were obtained for both cases:: one with soot layer and the other with no soot layer. Samples of the obtained for different types of DPF are displayed in figurers (2) – (10). From these figures the following points are made:

1. Both transmission loses and noise reduction factor for the typical filter and other types of DPFs tend to increase as frequency increases.
2. Wave propagation through the DPF unit suffers of both attenuation and phasing shift, and both attenuation and phase shift damped as shear wave number increases.
3. Transmission losses for the case of soot layer formation are significantly higher than those with no soot layer formation.
4. The EX80:200/14 DPF type has the best characteristics of transmission losses, while the EX80:100/17 DPF type has the lowest transmission losses performance.
5. The present 6-port model show similar tends concerning transmission losses as those presented in [1] using a 4-port model. However the present 6-port model result, which takes into account the effect of the field transverse velocity , are in closer agreement with the experimental ones presented in [1] than the 4-port model obtained in [1] which ignore this effect. Therefore one may conclude that ignoring the transverse velocity can have a significant effect on the evaluation of acoustic transmission losses for a DPF unit.

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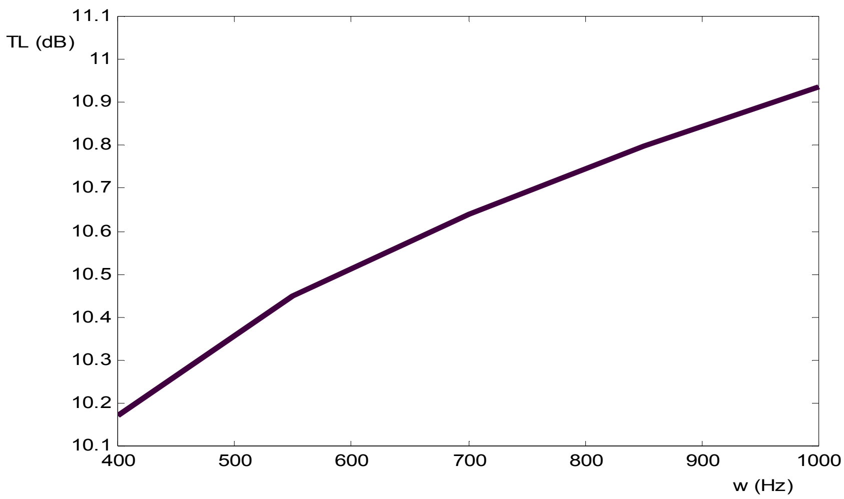

Figure 2. Transmission losses vs. frequency in the case of cold conditions, with no soot layer, Mach=0.02, space and time variation case, and for Typical filter.

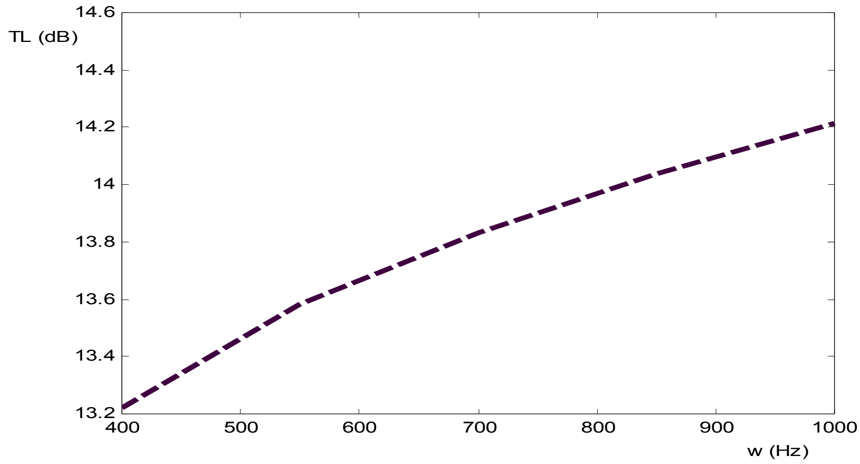

Figure 3. Transmission losses vs. frequency in the case of cold conditions, with soot layer, Mach=0.02, Plane and time variation case, and for Typical filter.

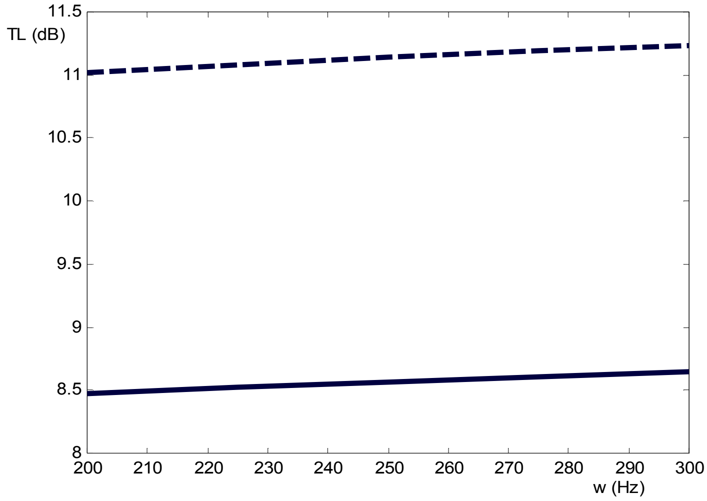

Figure 4. Transmission losses vs. frequency in the case of cold conditions for RC 200/12 filter type, ----- for with soot layer, —— is for the case of no soot layer, Mach=0.02, and for space and time variation case.

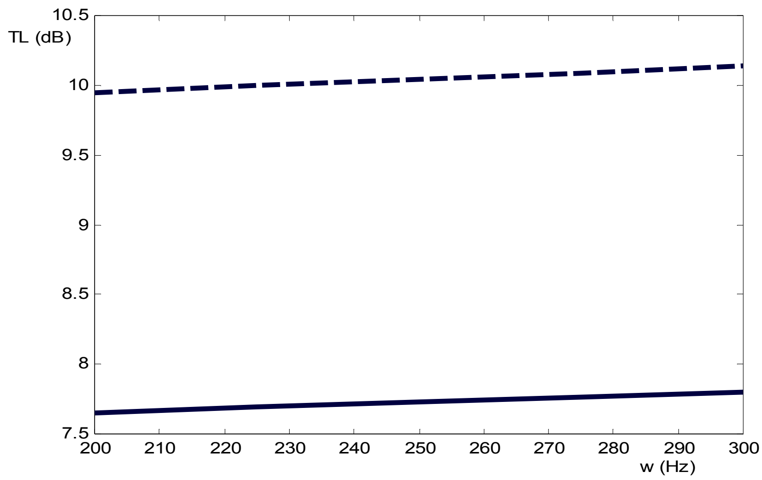

Figure 5. Transmission losses vs. frequency in the case of cold conditions for RC 200/20 filter type, ----- for with soot layer, —— is for the case of no soot layer, Mach=0.02, and Plane and time variation case.

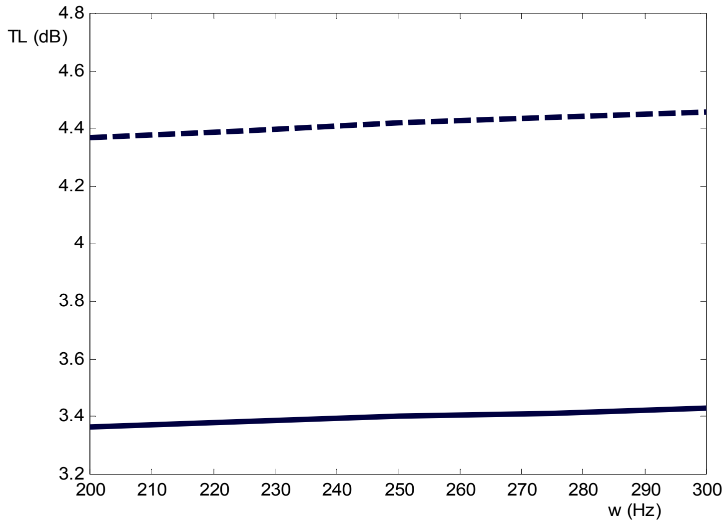

Figure 6. Transmission losses vs. frequency in the case of cold conditions for EX: 100/17 filter type, ----- for with soot layer, —— is for the case of no soot layer, Mach=0.02, and Plane and time variation case.

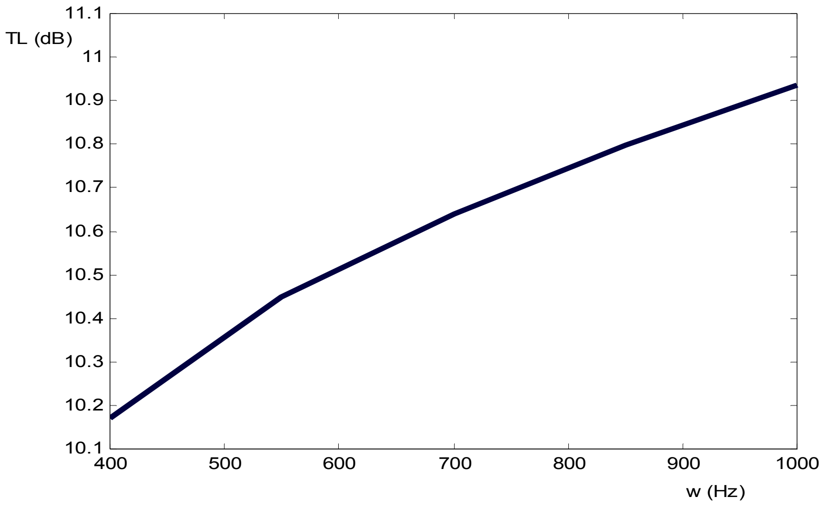

Figure 7. Transmission losses vs. frequency in the case of hot conditions for typical filter for the case of no soot layer, Mach=0.02 and space and time variation case.

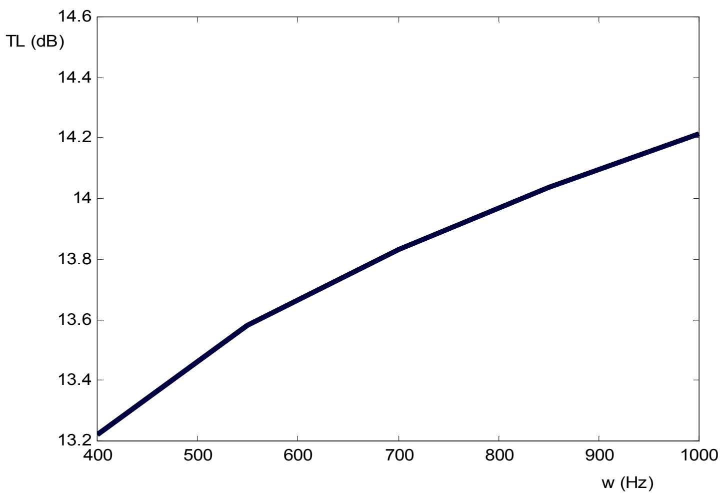

Figure 8. Transmission losses vs. frequency in the case of hot conditions for typical filter for the case of with soot layer, Mach=0.02, and Plane and time variation case.

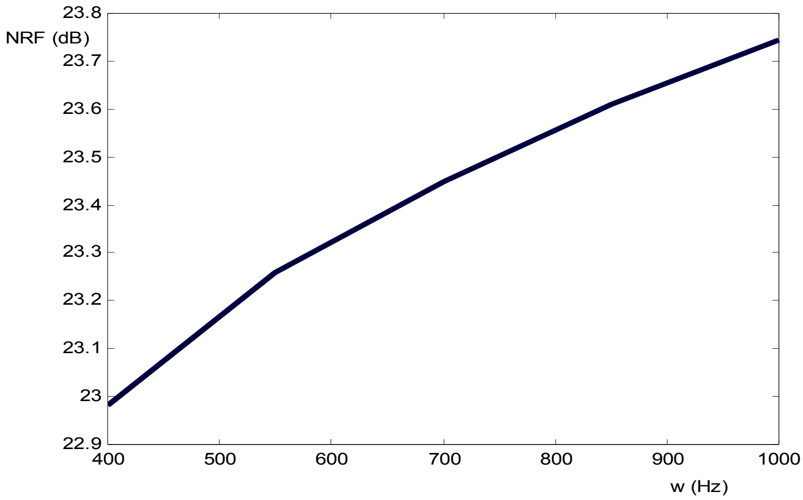

Figure 9. NRF vs. frequency for typical DPF in the case of hot conditions, (With no soot layer). Mach=0.02, in the case of plane and time variation.

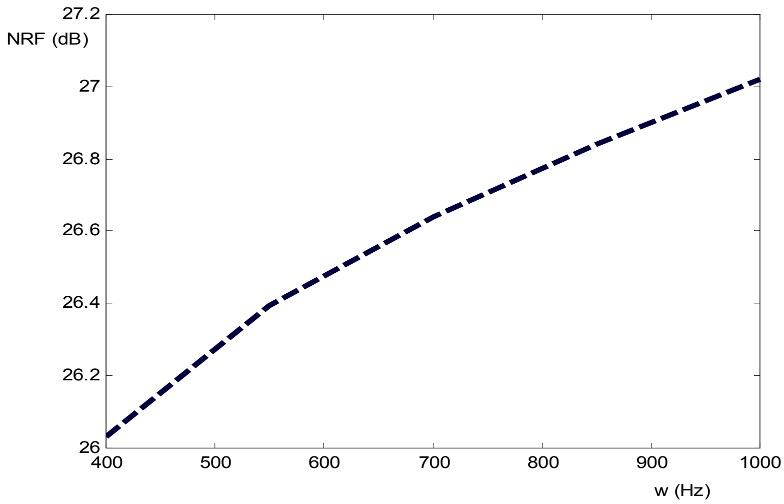

Figure 10. NRF vs. frequency for typical DPF in the case of hot conditions, (With soot layer). Mach=0.02, in the case of plane and time variation.