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THE USE OF BASELINE MEASUREMENTS FOR IMPROVED DAMAGE DETECTION USING DAMAGE LOCATION VECTORS

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ABSTRACT

Vibration-based techniques are increasingly being recognized as effective non-destructive structural damage identification tools. One promising technique relies on combining a finite element model (FEM) of the structure under investigation with a set of experimental frequency response functions (FRFs) to construct a so-called damage location vector (DLV). Emphasis in this paper is placed on investigating, both theoretically and experimentally, damage detection using DLVs while attempting to achieve an enhanced sensitivity through comparisons with baseline measurements of an initially damaged structure. To this end, the method is first studied theoretically on a space truss using simulated damage to illustrate its capability. The method is then improved to handle randomly assigned initial damage that is not predicted by the FEM through subtraction and normalization of the DLV with respect to baseline data. The improved method is finally tested experimentally on cantilever beams provided with damage of various sizes. The proposed technique is effective in identifying damage that would otherwise be concealed within an initially damaged structure, and provides useful insight into the location and severity of damage.

KEY WORDS

Vibration-based damage detection, damage location vector

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INTRODUCTION

The ability to detect damage in structures has been of concern to the engineering community for several years. The problem is encountered in many applications, such as detecting damage in steel bridges and performing non-destructive tests on mechanical components. While various damage identification techniques have been proposed, those relying on vibration analysis are appealing in many aspects. As the existence of a localized damage is likely to affect the global structural dynamic behavior, the use of modal parameters in deriving damage indicators has received significant attention. For a comprehensive review of such techniques, the reader is referred to the work of Doebling *et al*. [1] and Alvandi and Cremona [2]. Among the viable methods are those based on changes in the dynamic flexibility matrix to investigate the existence of damage. One promising technique in this context relies on combining a finite element model (FEM) of the structure under investigation in its intact state with a set of experimental frequency response functions of the actual structure to construct a so-called Damage Location Vector (DLV). Elements of the DLV represent a qualitative measure of the damage at each degree of freedom (DOF) of the structure at a certain frequency. Accordingly, the DLV can either be plotted in 3-D as a function of frequency and DOFs, or can be shown more conveniently as a 2-D projection of cumulative DLV versus DOFs over a specified frequency range.

The DLV approach has been employed by Huynh *et a*l. [3] theoretically and experimentally with random noise being applied to the theoretical FRFs to investigate the effect of measurement noise on the accuracy of the method. Gao and Spencer [4] and Duan *et al.* [5] improved the DLV technique by considering structural excitations by ambient vibrations. While the use of DLVs in structural damage identification has been studied, most of the DLV approaches reported in the literature thus far are based on structures that are considered free of any initial damage. If, however, that is not the case, the FEM will fail to simulate the intact state of the structure and the DLV would likely give a false indication of further damage induced in the structure. The aim of this work is hence to investigate, both theoretically and experimentally, the use of DLVs in structural damage detection and to attempt to overcome the difficulties of extracting the damage indicators that would otherwise be concealed in an initially damaged structure through the use of baseline data.

The remainder of this paper is organized into four sections. Section 2 outlines the brief theory and section 3 presents an analytical assessment of the proposed method. The method is verified experimentally in section 4, and finally conclusions and recommendations are given in section 5.

THEORY

While the formulation of the damage location vectors for the purpose of damage detection has previously been treated in the literature, the method will be briefly outlined herein for completeness. Under linear conditions, an undamped multi-degree-of-freedom structure has an equation of motion in the form:

$$[M]\{\ddot{x}\}+[K]\{x\}=\{F\} \tag{1}$$

where $[K]$ and $[M]$ are the stiffness and mass matrices, respectively, $\{F\}$ is a vector of externally applied loads and $\{x\}$ is the vector of degrees of freedom. The use of DLVs in vibration-based damage detection relies upon the construction of the DLV in the form:

$$\{d(\Omega)\} = \left[Z(\Omega)\right]_{UD} \{\Delta\alpha(\Omega)\}_k \tag{2}$$

where $\left[Z(\Omega)\right]$ is the dynamic stiffness matrix given by:

$$\left[Z(\Omega)\right] = \left[[K] - \Omega^2[M]\right] \tag{3}$$

Ω being the frequency, and $\{\Delta\alpha(\Omega)\}_k$ is the k^{th} column of the matrix $\left[\Delta\alpha(\Omega)\right]$, where:

$$\left[\Delta\alpha(\Omega)\right] = \left[\alpha_D(\Omega)\right] - \left[\alpha_{UD}(\Omega)\right] \tag{4}$$

$\{\alpha(\Omega)\}$ being the receptance frequency response function given by:

$$\left[\alpha(\Omega)\right] = \left[[K] - \Omega^2[M]\right]^{-1} \tag{5}$$

and the subscripts UD and D denote the undamaged and damaged states of the structure, respectively. In the above treatment, it is assumed that damage, such as the presence of a crack, only affects the stiffness matrix, and not the mass matrix, of the structure under investigation. It is also assumed that damping is negligible. The presence of a non-zero element in the DLV suggested by equation (2) indicates a damaged structural degree of freedom.

To this end, the construction of a DLV can conveniently be performed by developing a numerical (finite element) model of the undamaged (intact) structure, which can be used in collaboration with a set of experimentally-determined receptance frequency response functions, to build up the DLV suggested by equation (2).

One of the difficulties encountered in this approach is spatial incompleteness in the sense that not all structural degrees of freedom can usually be measured due to practical considerations and sensor limitations. Furthermore, experimental testing points can possibly be fewer than the number of elements used in the FE model, which can pose an incompatibility problem. To overcome these difficulties, model expansion techniques have been addressed in the literature. In this work, the dynamic expansion method [6] will be used to compensate for the unmeasured degrees of freedom. In this regard, the measured degrees of freedom, $\{x_m\}$, relate to the unmeasured degrees of freedom, $\{x_s\}$, through

$$\begin{Bmatrix} \{x_m\} \\ \{x_s\} \end{Bmatrix} = [T_s]\{x_m\} \tag{6}$$

where $[T_s]$ is a transformation matrix having the form:

$$[T_s] = \begin{bmatrix} I \\ -\left[[K_{ss}] - \Omega^2[M_{ss}]\right]^{-1}\left[[K_{sm}] - \Omega^2[M_{sm}]\right] \end{bmatrix} \tag{7}$$

and subscripts *ss* and *sm* pertain to the partitioned stiffness and mass matrices into measured and unmeasured degrees of freedom, in accordance with equation (6). Hence, the unmeasured degrees of freedom can be estimated from the measured ones, together with the numerical model mass and stiffness matrices, at a certain frequency.

ANALYTICAL ASSESSMENT

In this section, the use of DLVs in damage identification is investigated numerically through simulated damage in a truss structure, with emphasis on improving the method to handle structures suffering initial damage. The eight-bay truss shown in Fig. 1 is studied for this purpose. Each bay is assumed to have a length of 1 m. The structural members are assumed to have a modulus of elasticity of 210 GPa, density of 7850 kg/m^3 and cross sectional area of 10^{-4} m^2. The nodes are numbered as indicated in Fig. 1, with the first four nodes all grounded. Behavior of the intact structure is obtained by a finite element analysis utilizing three-dimensional truss elements.

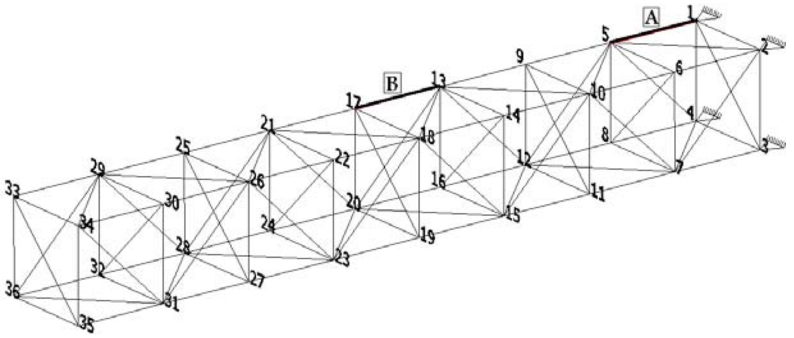


Figure 1. Eight-bay truss.

Initial damage was simulated by randomly assigning variations in the elastic modulus of all members within ± 5% of the original value. To emphasize that such a random damage is assigned to simulate a seemingly intact structure, this case is denoted as the intact, as in reality a structure may appear to be intact while it actually contains slight damage leading to discrepancies between its actual response and that predicted by numerical models. Additional damage was assigned to longerons A and B by reducing their elastic modulus further by 10% (case 1) and 20% (case 2). In this way, the truly damaged elements are concealed within an initially-damaged structure and the aim to attempt to distinguish those particular elements in an already damaged structure.

Figure 2 shows the cumulative DLV (CDLV) of the intact truss, revealing that all structural degrees of freedom exhibit damage. This is because the behavior of the initially damaged structure is expected to depart from that predicted by the FEM, resulting in non-zero entries of the DLV as shown yielding the baseline performance.

Figure 3 shows the CDLV of the damaged truss, cases 1 and 2. The top plots refers to damage applied at longeron A only, whereas the bottom plot is for damage applied at longerons A and B. The degrees of freedom affected by those damages, namely numbers 1, 25 and 37, are indicated by arrows on all plots. Comparison of the CDLV of the intact (randomly damaged) truss with that of the damaged cases reveals that the CDLV can successfully detect the damage at the expected degrees of freedom, with increasing values for increased damage severities, however the identification of

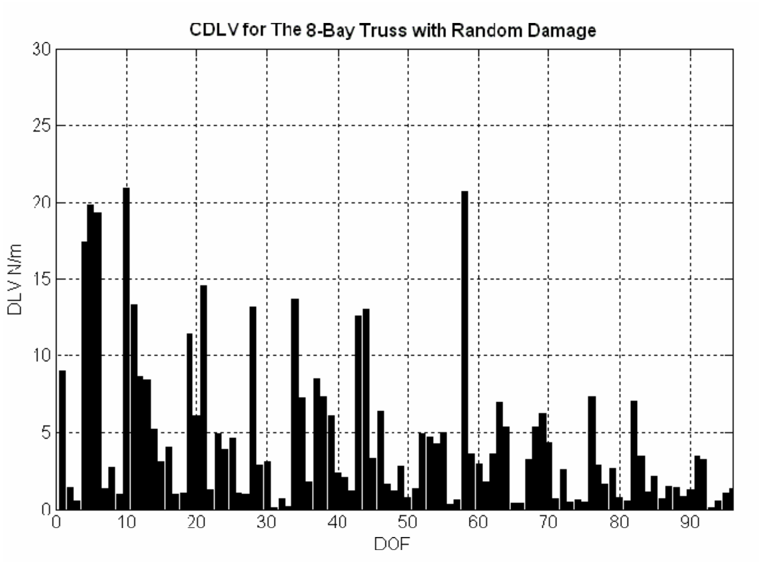
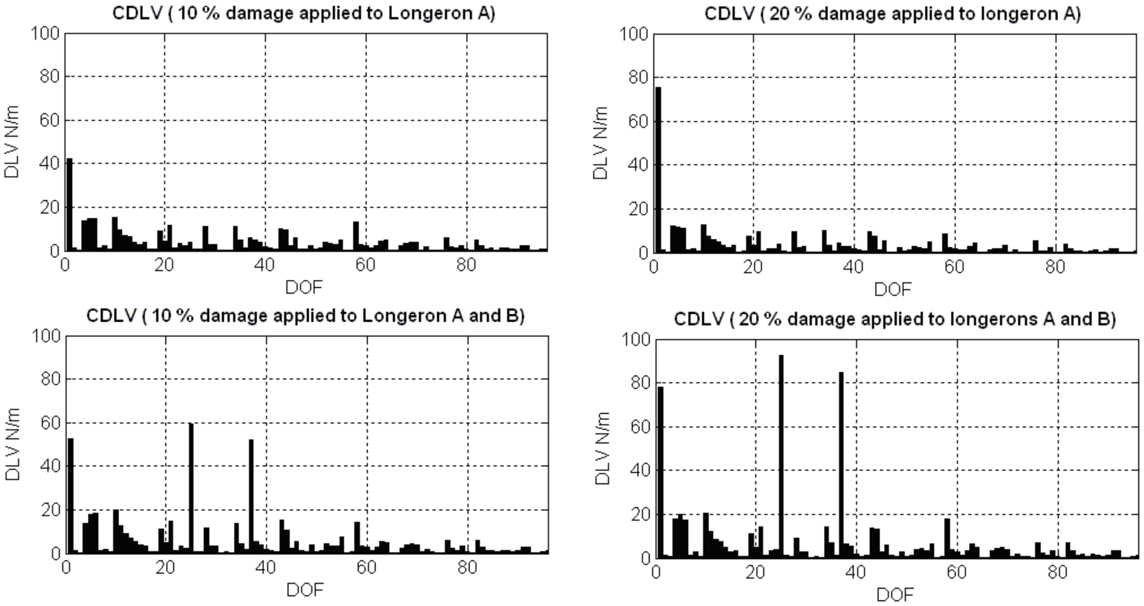


Figure 2. CDLV of intact truss.

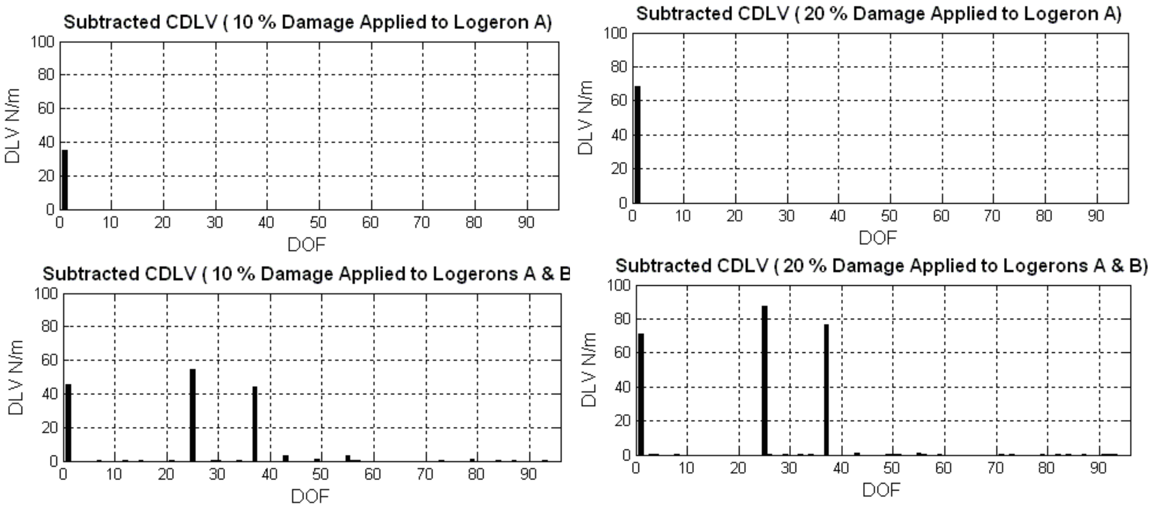


(a) (b)

Figure 3. CDLV of damaged truss: (a) case 1, (b) case 2.

damage becomes difficult when damage is not high enough to stand out among the damage originally exiting in the structure. The one-by-one inspection of each degree of freedom, before and after damage, is not a practical approach, especially for structures having large number of degrees of freedom. This has prompted further research into the use of baseline data to improve this visualization.

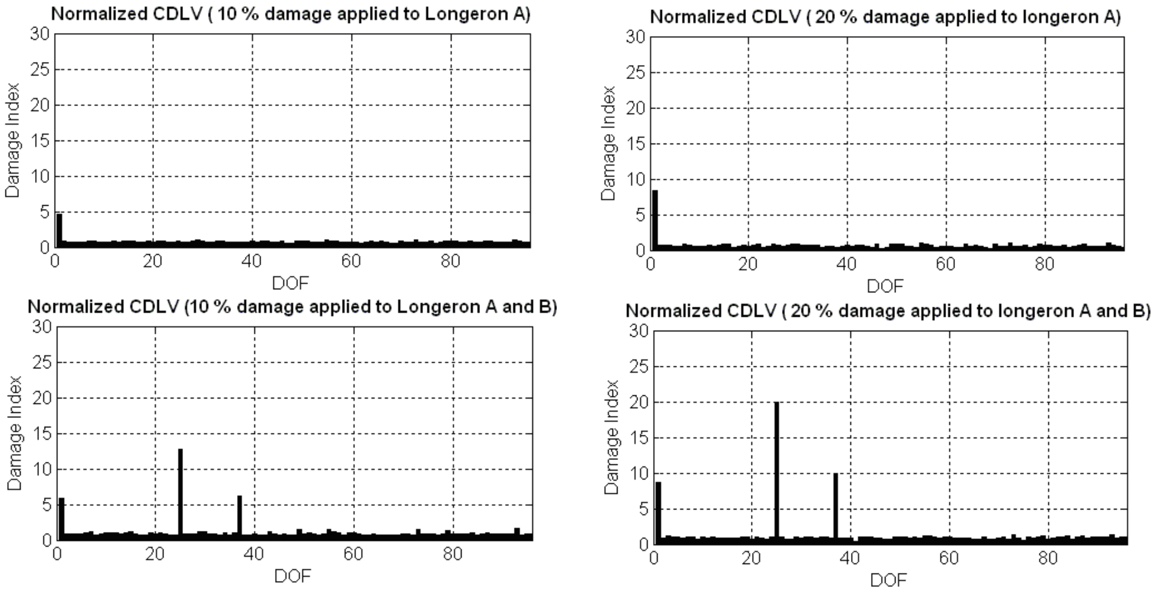
Subtraction of the CDLV of the intact case from that of the damaged one can reveal departures from baseline data to detect concealed damage. Figure 4 shows the subtracted CDLV plots revealing clear identification of damage locations for the various cases considered and with generally increasing CDLV amplitudes for higher damage levels.



(a) (b)

Figure 4. Subtracted CDLV of truss: (a) case 1, (b) case 2.

Another approach is to normalize the CDLV through division by the intact CDLV, resulting in a damage index that is close to unity for unaffected (intact) degrees of freedom. Inspection of the normalized CDLV shown in Fig. 5 indicates that damage can also be identified successfully. In this way, the use of subtracted and normalized CDLV is regarded an improvement of current use of CDLV. The choice between these two approaches can be affected by measurement noise, and hence these approaches are tested experimentally in the following section.

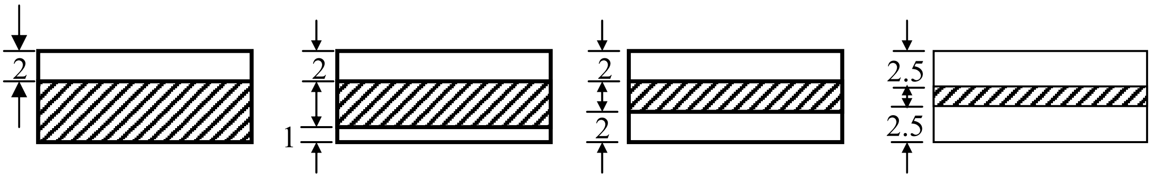


(a) (b)

Figure 5. Normalized CDLV of truss: (a) case 1, (b) case 2.

EXPERIMENTAL INVESTIGATION

For experimental testing, a steel cantilever beam measuring 38 x 500 x 6 mm was studied. The beam was discretized into one dimensional 20 finite elements, based on the Euler-Bernoulli beam theory, with the nodes sequentially numbered from 0 to 20, running from the support to the free end. Preliminary inspection of the beam revealed minor variations of the width and thickness of the beam due to manufacturing errors. These deviations can be regarded as initial random damage present in the beam. Damage was induced in the cantilever beam by slots using a hand saw. The four damage cases studied herein consist of transverse cuts of various depths ranging from 2 mm to 5 mm, as illustrated in Fig. 6, induced at node 1 of the cantilever beam. The intact beam was tested prior to inducing damage.



2 mm 3 mm 4 mm 5 mm

Figure 6. Cross sections at damage locations.

Impact hammer testing with a roving hammer (PCB 085C03) was conducted. Hitting points were chosen along the beam centerline to avoid exciting torsional modes. An accelerometer (Brüel and Kjaer 2626) was mounted in the midspan of the beam using a magnetic base. Data was acquired and processed using a multi-channel signal analyzer (LMS Pimento). In each beam test, twenty FRFs were measured. Further calculations to convert accelerance to receptance, estimate unmeasured (rotational) response using dynamic expansion and build the DLV were undertaken in accordance with the procedures outlined in Section 2. All CDLVs were based on a frequency range from 50 to 150 Hz, which contains one resonance frequency of the beam. This working range was selected because it contained the least noise and distortions among the different FRFs. Since the damage is induced at node 1, which is connecting elements 1 and 2, the newly induced damage is expected to affect degrees of freedom numbers 1 through 4.

Figure 7 shows the CDLV of the intact beam. It is clear that deviations between the finite element predictions and experimental measurements give non-zero entries in the CDLV. Such deviations are likely due to initial damage present in the beam in the form of thickness and width deviations that are not predicted by the FEM. While one approach is to attempt to pre-tune the FEM to minimize such deviations through mode updating techniques, the approach adopted in this work relies on using this baseline behavior to predict further induced damage.

Figure 8 shows a sample of the FRFs of the damaged beam cases. There are noticeable changes in the resonant frequencies due to induced damage.

Figure 9 (a) shows the experimental CDLV of the damaged beam for the different cases studied. Inspection of the plots reveals that damage cannot be identified for the case of a 2 mm cut. For a 3 mm cut, the damage is not clearly identified, even though the CDLV at DOFs 2 and 4 show an increase. The method gave a fair prediction of the 4 and 5 mm cuts. The high CDLV values observed at DOFs 35 to 40 for 2 mm and 3 mm cuts is a false indication of damage. These DOFs correspond to nodes lying close to the tip of the beam, and such an error can be attributed to practical errors encountered acquiring the FRFs of tip nodes during the experimental testing. Values of the CDLV at

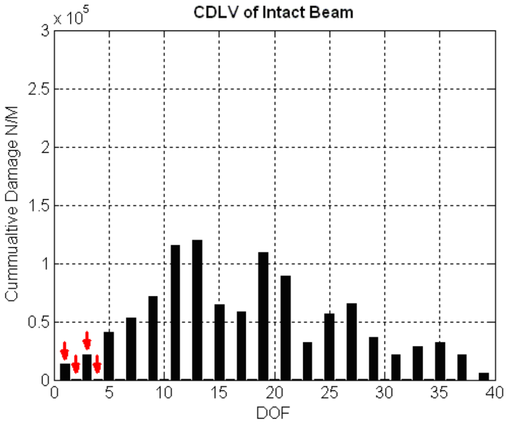


Figure 7. CLDV of the intact beam.

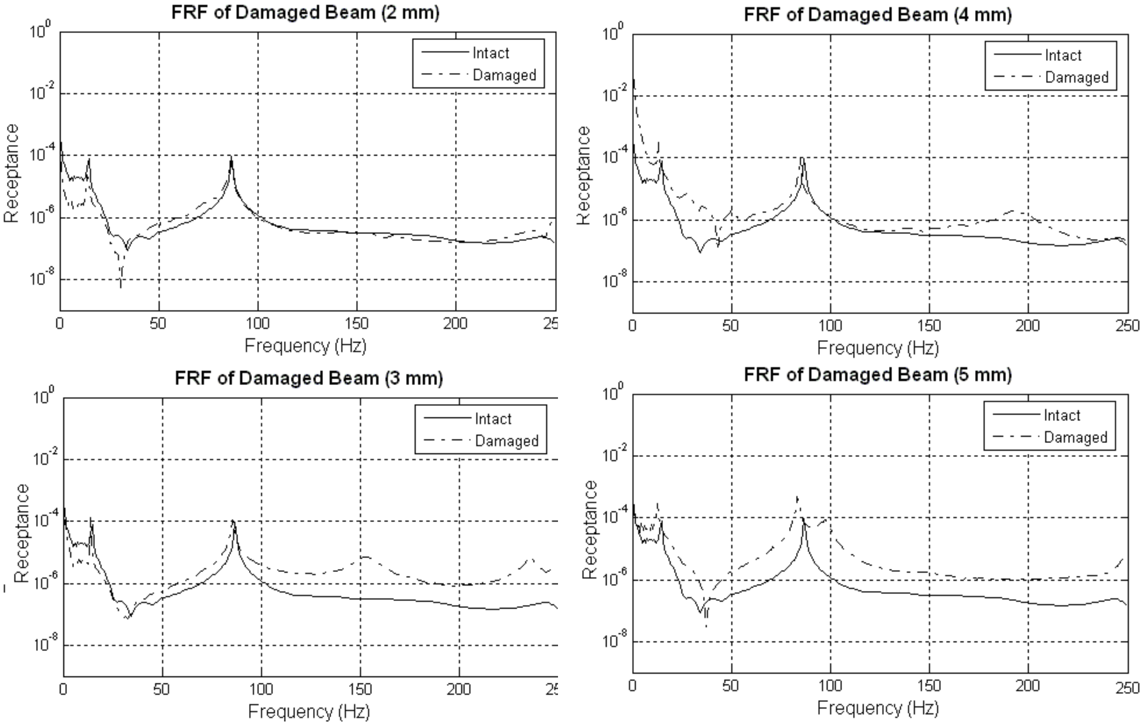
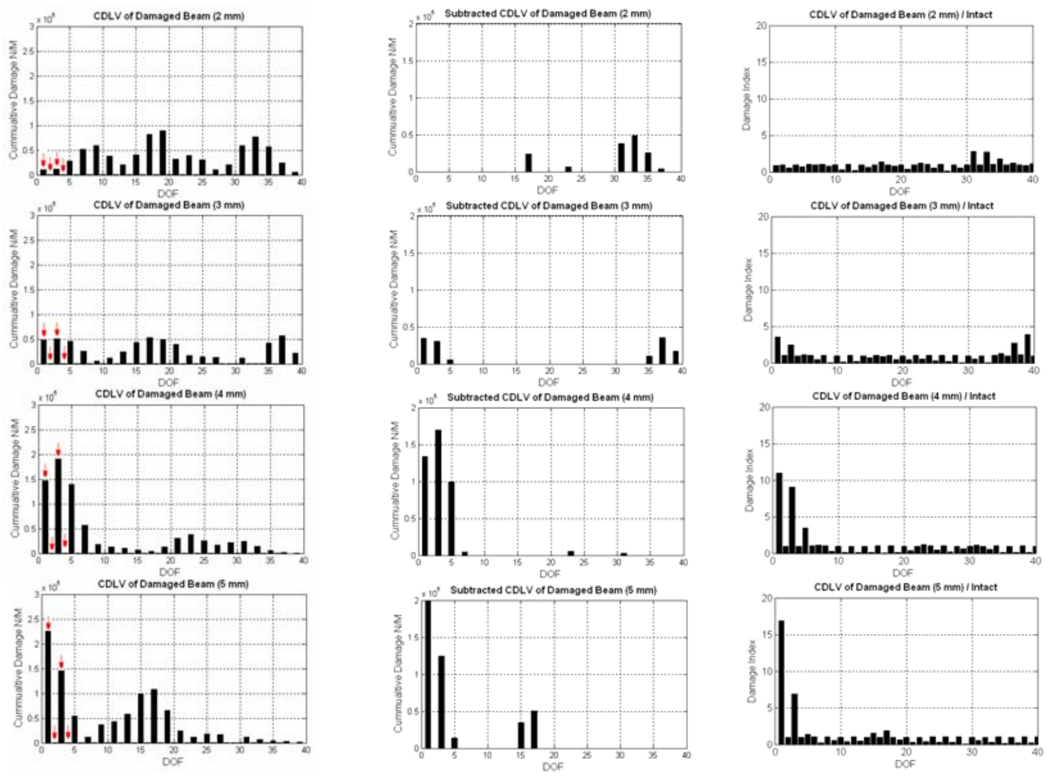


Figure 8. FRFs of intact and damaged beams.

rotational DOFs are substantially lower than translational DOFs because these estimated values are expected to be close to the numerical predictions.

The subtracted CDLV of the experimental beam is shown in Fig 9 (b). While this approach shows enhanced damage detection at the correct DOFs for the 3 mm, 4 mm and 5 mm cuts, the method fails to detect the 2 mm cut. In addition, some erroneous predictions are suggested in the 3 mm and 5 mm cuts at DOFs other than the expected ones due to experimental errors. Figure 9 (c) shows the normalized CDLV for the same damage cases. While the normalization could not improve the ability of the DLV method to detect the 2 mm damage case, it resulted in an improved identification for the other cases. It is noted that each experimental damage case corresponds to a different beam, and due to the random nature of initial damage, the use of the same intact (baseline) case data for comparisons with all cases seems to overshadow the merit gained by the suggested methodologies, especially for the 2 mm and 3 mm damage cases of Figs. 9 (b) and (c). In practical situations, such as structural health monitoring applications, one and the same structure would normally be tested at several time instants. An early inspection would then define the intact or baseline state and subsequent data taken on the same structure would reveal deviations from that pristine state. It is thought that this would yield an enhanced prediction of the existence of damage.



(a) (b) (c)

Figure 9. Experimental CDLV of beam cases: (a) CDLV, (b) subtracted CDLV, (c) normalized CDLV.

CONCLUSION

Emphasis in this paper was placed on assessing, both theoretically and experimentally, the use of DLVs in vibration-based damage identification. The method was first studied theoretically on a space truss to illustrate its capability in identifying the location of damage. The method was also improved to handle randomly assigned initial damage that is not predicted by the finite element model through comparisons with baseline DLVs. The proposed method was then tested experimentally on a cantilever beam provided with several damage sizes. Based on the analytical and experimental assessment it can be concluded that it is hard to identify relatively small damage that can be hidden within a pre-damaged structure. The use of baseline data through subtraction and normalization of the CDLV improved the ability of the method to detect smaller damage and gave better visualization of damage by isolating damaged DOFs from undamaged ones. Accordingly, the proposed technique was found to be viable and provided useful insight into the location and severity of damage. Further improvements to handle smaller damage sizes can be obtained by integrating model updating techniques to the approaches presented in this work. The method can also be improved by including structural damping and its change due to the presence of damage.

REFERENCES

[1] Doebling, S.W., Farrar, C.R., Prime, M.B. and Shevitz, D.W., *Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review*, Research Report, LA-13070-MS, ESA-EA, Los Alamos National Laboratory, Los Alamos, N.M., 1996.

[2] Alvandi, A. and Cremona, C., “Assessment of vibration-based damage identification techniques”, *Journal of Sound and Vibration* 292 (2006) 179-202.

[3] Huynh, D., He, J. and Tran, D. ,“Damage location vector: A non-destructive structural damage detection technique” *Computers & Structures*, 83 (2005) 2353-2367.

[4] Gao, Y. and Spencer, B.F., “Damage localization under ambient vibration using changes in flexibility”, *Earthquake Engineering and Engineering Vibration* 1 (1) 2002 136-144.

[5] Duan, Z., Yan, G., Ou J., and Spencer B.F. “Damage detection in ambient vibrations using proportional flexibility matrix with incomplete measured DOFs”, *Structural Control and Health Monitoring*, 14 (2) (2007) 186-196.

[6] Pilkey, D.F., *Computation of a damping matrix for finite element model updating*, Ph.D thesis, Virginia Polytechnic Institute and State University, 1998.