Military Technical College
Kobry El-Kobbah,
Cairo, Egypt.

13th International Conference
on Applied Mechanics and
Mechanical Engineering.

INFLUENCE OF THE VISCOTHERMAL EFFECTS ON THE VIBRO-ACOUSTIC BEHAVIOR OF A DOUBLE PLATES SYSTEM

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ABSTRACT

In this work, a numerical modeling to study the vibro-acoustic behavior of a double plate system (such as double glazing system) taking into account of viscosity and the thermal conductivity of the fluid between the two plates in the acoustic equations, is presented. The dynamic equations of the coupled system are established. To solve these equations, a variational formulation for the fluid and the two plates is developed. The discretization by the finite element method of this variational formulation gives after minimization a symmetrical coupled matrix system with nonlinear aspect. An iterative procedure is derived to determine the eigenmodes of the coupled system. The modal approach is adopted to determine the vibro-acoustic system's response which numerical results show the importance of the viscothermal effects in the case of thin fluid layers.

KEY WORDS

Double plate, vibro-acoustic behaviour, viscothermal fluid layer, finite element.

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INTRODUCTION

This paper deals with the vibro-acoustic interaction between a thin fluid layer and two vibrating plates. In the case of thin fluid layer, the effects of viscosity and thermal conductivity of the fluid must be taken into account in the acoustic equations : Bruneau [1] and Morse [2]. Many applications have been considered. In the same context, Cummings [3] and Dockumaci [4] have studied the propagation of acoustic waves in narrow tubes. Moreover, Onsay [5] and Beltman [6, 7] have investigated the dynamic interaction between bending vibrations of a plate and a thin fluid layer. To overcome the acousto-elastic problems, Bruneau [8, 9], Karra [10] and Henriquez [11] have analyzed the vibro-acoustic behavior of a membrane coupled to a micro-cavity which can be used in miniaturized transducers. Bossart [12] has developed an hybrid numerical and analytical method, to analyse the acoustic boundary problems in viscothermal fluids. Moreover, Mottelet [13] has developed a new model for double glazing integrating absorption due to viscothermal air effects in a glass spacer.

This paper deals with numerical investigation on two plates vibro-acoustic behaviour enclosing a viscothermal fluid cavity. The acoustic equation is established in two dimensional form including the effects of the fluid's viscosity and thermal conductivity. In the same way, dynamic equations of plates vibrations are established in dimensionless form. To solve the equations, a total variational formulation of the coupled system is established. Its discretization gives after minimisation a nonlinear symmetrical coupled matrix system which is solved numerically using an iterative approach. The numerical results are compared to those corresponding to a perfect fluid model.

ACOUSTO-ELASTIC MODEL

The double panels system is composed of a viscothermal fluid layer of thickness h_0 enclosed between two clamped plates as sketched in Fig 1. In this case, $(\tilde{w}_1, \tilde{w}_2)$, $(\vec{n}_1, \vec{n}_2)$ and ℓ_x are respectively the plates deflections, the outward normal of plates, and the half length of plates. $\tilde{p}_s$ is the external imposed pressure applied on plate 1 and $\tilde{p}$ is the pressure in the fluid layer. The dynamic behavior of the double panels system is governed by both equations of fluid and plates.

Dynamic Equations of the Fluid Layer

The dynamic equation of the fluid layer is established by considering the following hypothesis :

- The fluid layer thickness is small compared to both plates dimensions and acoustic wavelength,
- The viscous and thermal boundary layer thickness are small compared to plates dimensions,
- The velocity in the z direction is small compared to the in-plane velocities.

In fact, the dynamic equation of the fluid, in the case of harmonic problem ($\frac{\partial}{\partial t} = -i\omega$, ω is the angular frequency) is derived by substitution Navier-Stokes equation, energy equation and the state equation of fluid supposed to be an ideal gas into continuity equation [1].

According to the hypothesis defined above, taking account for the coupling conditions, the dynamic equation of fluid cavity can be derived. In fact, the dimensionless fluid pressure is solution of the following bidimensional differential equation which is written for a viscothermal fluid area (Σ) as:

$$\left(\frac{\partial^2 p}{\partial x^2}\right)+\left(\frac{\partial^2 p}{\partial y^2}\right)+K_{eff}^2(k_V).p=-\frac{\omega^2\,\ell_0^2\,\gamma}{c_0^2\,B_1(k_V)}(w_1-w_2) \quad (1)$$

with : $w_1=\frac{\tilde{w}_1}{h_0}$ and $w_2=\frac{\tilde{w}_2}{h_0}$ are dimensionless deflections of the plates.

$x=\frac{\tilde{x}}{\ell_0}$, $y=\frac{\tilde{y}}{\ell_0}$, $z=\frac{\tilde{z}}{h_0}$ are dimensionless coordinates and ℓ_0 is a characteristic length of plates.

$p=\frac{p_a}{p_0}$ where p, p_a and p_0 are respectively the dimensionless pressure in the fluid cavity, the acoustic pressure and the physical pressure at rest.

$k_V=h_0\sqrt{\frac{\rho_0\omega}{\mu}}$ is the dimensionless shear wave number.

ρ_0 is the physical density of the fluid and μ is the coefficient of the viscosity.

$K_{eff}(k_V)=\frac{\omega\,\ell_0}{C_{eff}(k_V)}$ is a dimensionless wave number affected by viscothermal effects.

$C_{eff}(k_V)=c_0\sqrt{\frac{B_1(k_V)\;B_2(\psi\,k_V)}{\gamma}}$ is the sound speed affected by viscothermal effects.

$c_0=\sqrt{\gamma\frac{p_0}{\rho_0}}$ is the sound speed in the fluid and $\gamma=\frac{C_p}{C_V}$ is the specific heats ratio.

$B_1(k_V)=2\left[\frac{1-\cosh(k_V\sqrt{-i})}{k_V\sqrt{-i}\,\sinh(k_V\sqrt{-i})}\right]+1$ characterize the viscous effects in the fluid layer.

$B_2(\psi\,k_V)=\left[1-\left[\frac{\gamma-1}{\gamma}\right]B_1(\psi\,k_V)\right]^{-1}$ characterize the thermal effects in the fluid layer.

$\psi^2=\frac{\mu\,C_p}{\lambda}$ is the Prandtl number and λ is the fluid thermal conductivity.

Plates Dynamic Equations

The dynamic equations of plates are governed by both Kirchhoff equations and the coupling conditions which can be derived respectively, for two area (Σ_1) and (Σ_2), in dimensionless form as follow :

$$\frac{\gamma\,D_{p_1}\,h_0}{\rho_0\,c_0^2\,\ell_0^4}\Delta\Delta w_1-\omega^2\,\frac{\gamma\,\rho_{p_1}\,e_{p_1}h_0}{\rho_0\,c_0^2}\,w_1=\;p-p_s \quad (2)$$

$$\frac{\gamma\,D_{p_2}h_0}{\rho_0\,c_0^2\,\ell_0^4}\Delta\Delta w_2-\omega^2\,\frac{\gamma\,\rho_{p_2}\,e_{p_2}\,h_0}{\rho_0\,c_0^2}w_2=-\,p \quad (3)$$

where $D_{p_1} = \frac{E_{p_1} e_{p_1}^3}{12(1-\nu_{p_1}^2)}$ and $D_{p_2} = \frac{E_{p_2} e_{p_2}^3}{12(1-\nu_{p_2}^2)}$ represent respectively the bending stiffness of plate1 and plate 2. $\left(E_{p_1}, E_{p_2}\right), \left(\nu_{p_1}, \nu_{p_2}\right)$ and $\left(\rho_{p_1}, \rho_{p_2}\right)$ are respectively the Young's modulus, the Poisson's ratio and the densities of plates. e_{p_1} and e_{p_2} are respectively the thickness of plate1 and plate 2. p_s is the dimensionless external pressure applied on plate 1 defined by : $p_s = \frac{\tilde{p}_s}{p_0}$

Equations (2) and (3) represent the dynamic of plates where the coupling with fluid is realised in the seconds terms.

VARIATIONAL FORMULATION

To solve the system of equations (1), (2) and (3), a total variational formulation is established. It is obtained by a linear appropriate combination of the variational formulation of the fluid associate to the equation (1) and the two variational formulations of the plates associate to the equations (2) and (3). These variational formulations are obtained by multiplying the different equations by sufficiently regular test functions and integrating over the fluid or structure domain. The discretization of this variational formulation by finite element method, gives after minimisation the following symmetrical coupled matrix system :

$$\begin{pmatrix} [K_1^S]-\omega^2[M_1^S] & 0 & -[C_1] \\ 0 & [K_2^S]-\omega^2[M_2^S] & [C_2] \\ -[C_1]^t & [C_2]^t & \frac{1}{\omega^2}\left([H(k_v)]-\omega^2[Q(k_v)]\right) \end{pmatrix} \begin{Bmatrix} \{U_1\} \\ \{U_2\} \\ \{P\} \end{Bmatrix} = \begin{Bmatrix} \{F^{ext}\} \\ \{0\} \\ \{0\} \end{Bmatrix} \qquad (4)$$

where $\left([K_1^S],[K_2^S]\right)$ and $\left([M_1^S],[M_2^S]\right)$ are respectively the stiffness matrices and mass matrices of plates. $\left[H(k_v)\right]$ and $\left[Q(k_v)\right]$ are respectively the matrix of the inertial effects and the matrix of the acoustic effects of fluid. $\left[C_1\right]$ and $\left[C_2\right]$ are the coupling matrices. $\left(\{U_1\},\{U_2\}\right)$, $\{P\}$ and $\{F^{ext}\}$ are respectively nodal displacements vectors of plates, nodal pressure vector in the fluid cavity and nodal external force vector.

DYNAMIC ANALYSIS OF THE COUPLED SYSTEM

The modal approach is adopted in order to reduce the size of the coupled matrix system. This approach needs to determine the acousto-elastic modes [14,15] and modal projection of dynamic equations. Then, dynamic responses are determined by modal recombination. It is noted that the associate eigenmodes system has a nonlinear aspect due the two matrices $\left[H(k_v)\right]$ and $\left[Q(k_v)\right]$ which depend on frequency, so that an iterative procedure [16] is derived to determine the acousto-elastic modes.

This procedure consists of resolving the following iterative system :

$$\begin{pmatrix} [K_1^S]-\omega_{r,j}^2[M_1^S] & 0 & -[C_1] \\ 0 & [K_2^S]-\omega_{r,j}^2[M_2^S] & [C_2] \\ -[C_1]^t & [C_2]^t & \frac{1}{\omega_{r,j}^2}\left([H(k_V(\omega_{r,j-1}))]-\omega_{r,j}^2[Q(k_V(\omega_{r,j-1}))]\right) \end{pmatrix} \begin{Bmatrix} \{U_1\} \\ \{U_2\} \\ \{P\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{0\} \end{Bmatrix} \quad (5)$$

$\omega_{r,j}$ is the eigenfrequency of eigenmode number r calculated at iteration j. This procedure starts with a first estimate value for the expected eigenfrequency. In fact, the starting values for all eigenmodes correspond to the eigenfrequencies of the inviscid and adiabatic fluid with the following parameters: $B_1(k_V)=1$, $B_2(\psi k_V)=\gamma$ and $C_{eff}(k_V)=c_0$.

The convergence test needs to verify the following condition:

$$\frac{\left|\omega_{r,j}-\omega_{r,j-1}\right|}{\left|\omega_{r,j}\right|}\le\varepsilon \qquad (\varepsilon<<1) \qquad (6)$$

If the eigenmodes $[\Phi_{S1}]$ and $[\Phi_{S2}]$ of each plate alone and the eigenmodes $[\Phi_f]$ of the fluid cavity in vacuum are used, the size of the matrix system (5) can be much reduced for calculation of the eigenmodes of the coupled system. In fact, the modal matrices of each plate alone and fluid cavity in vacuum can be written as follow :

$$\left[\boldsymbol{K_i^S}\right]=\left[\Phi_{Si}\right]^t\left[K_i^{S0}\right]\left[\Phi_{Si}\right]=\begin{bmatrix}\ddots & & \\ & \omega_{r\bullet Si}^2 & \\ & & \ddots\end{bmatrix},\begin{cases}r=1...n\\ i=1,\,2\end{cases} \qquad (7)$$

$$\left[\boldsymbol{M_i^S}\right]=\left[\Phi_{Si}\right]^t\left[M_i^{S0}\right]\left[\Phi_{Si}\right]=\begin{bmatrix}\ddots & & \\ & 1 & \\ & & \ddots\end{bmatrix},\ i=1,\,2 \qquad (8)$$

$$\left[\boldsymbol{H_0}\right]=\left[\Phi_f\right]^t\left[H_0\right]\left[\Phi_f\right]=\begin{bmatrix}\ddots & & \\ & k_{r\bullet f}^2 & \\ & & \ddots\end{bmatrix} \qquad (9)$$

$$\left[\boldsymbol{Q_0}\right]=\left[\Phi_f\right]^t\left[Q_0\right]\left[\Phi_f\right]=\begin{bmatrix}\ddots & & \\ & 1 & \\ & & \ddots\end{bmatrix} \qquad (10)$$

where $\omega_{r\bullet Si}$ are the eigenfrequencies of a clamped plate (i) in vacuum. $[K_i^{S0}]$ is the classical finite element stiffness matrix of plates ($[K_i^S]=\frac{\gamma h_0}{\rho_0 c_0^2}[K_i^{S0}]$). $[M_i^{S0}]$ is the classical finite element mass matrix of plates ($[M_i^S]=\frac{\gamma h_0}{\rho_0 c_0^2}[M_i^{S0}]$). $\omega_{r\bullet f}$ is the eigenfrequency number r of the air layer. $k_{r\bullet f}$ is the modal wave number index r($k_{r\bullet f}^2=\frac{\omega_{r\bullet f}^2}{C_{eff}^2(k_V(\omega_{r\bullet f}))}\ell_0^2$). $[H_0]$ is the fluid inertial effect classical finite element matrix which is related to the inertial fluid effects matrix $[H(k_V(\omega_{r,j-1}))]$:

$$[H(k_V(\omega_{r,j-1}))]=\frac{c_0^2\,B_1(k_V(\omega_{r,j-1}))}{\ell_0^2\,\gamma}[H_0] \tag{11}$$

$[Q_0]$ is the fluid compressibility classical finite element matrix which is related to the fluid compressibility effects matrix $[Q(k_V(\omega_{r,j-1}))]$:

$$[Q(k_V(\omega_{r,j-1}))]=\frac{c_0^2\,B_1(k_V(\omega_{r,j-1}))}{\gamma\,C_{eff}^2(k_V(\omega_{r,j-1}))}[Q_0] \tag{12}$$

According to the use of eigenmodes of each plate alone and fluid cavity in vacuum, the reduced eigenvalues matrix system becomes :

$$\begin{pmatrix} \frac{\gamma h_0}{\rho_0 c_0^2}[\omega_{r\bullet S1}^2-\omega_{r,j}^2] & 0 & -[\boldsymbol{C_1}] \\ 0 & \frac{\gamma h_0}{\rho_0 c_0^2}[\omega_{r\bullet S2}^2-\omega_{r,j}^2] & [\boldsymbol{C_2}] \\ -[\boldsymbol{C_1}]^t & [\boldsymbol{C_2}]^t & \frac{c_0^2\,B_1(k_V(\omega_{r,j-1}))}{\ell_0^2\,\gamma\,\omega_{r,j}^2}[k_{r\bullet f}^2-\frac{\omega_{r,j}^2\;\ell_0^2}{C_{eff}^2(k_V(\omega_{r,j-1}))}] \end{pmatrix}\begin{Bmatrix}\{\boldsymbol{U_1}\}\\ \{\boldsymbol{U_2}\}\\ \{\boldsymbol{P}\}\end{Bmatrix}=\begin{Bmatrix}\{\boldsymbol{0}\}\\ \{\boldsymbol{0}\}\\ \{\boldsymbol{0}\}\end{Bmatrix} \tag{13}$$

where $\{\boldsymbol{P}\}$, $\{\boldsymbol{U_1}\}$ and $\{\boldsymbol{U_2}\}$ are respectively pressure in the fluid cavity and displacements of plates 1 and 2 which are expressed in the modal basis of fluid in vacuum $[\Phi_f]$ and in the modal basis of each plate alone $[\Phi_{S1}]$ and $[\Phi_{S2}]$:

$$\{\boldsymbol{P}\}=[\Phi_f]^t\{P\},\qquad \{\boldsymbol{U_1}\}=[\Phi_{S1}]^t\{U_1\},\qquad \{\boldsymbol{U_2}\}=[\Phi_{S2}]^t\{U_2\}.$$

Hence, the coupling matrices between fluid cavity and plates index 1 and 2 become :
$[\boldsymbol{C_i}]=[\Phi_{Si}]^t[C_i][\Phi_f]$, i = 1, 2

Moreover, due to the fact that the first eigenfrequency of the fluid cavity is equal to zero, the B.IRONS transformation [17] is used in order to isolate the first eigenmodes of the fluid cavity and write a classical eigenvalue system.

NUMERICAL RESULTS

In this part, the vibro-acoustic behaviour of a double plates system enclosing a viscothermal fluid cavity in which the effects of viscosity and thermal conductivity of fluid and acousto-elastic interaction are taking into account is investigated. The geometrical and physical features of the studied system are:

$\ell_0 = 1\text{m}$, $\ell_x = \ell_y = 125$ mm , $e_{p1} = e_{p2} = 1$ mm , $h_0 = 1\text{mm}$, $\rho_{p1} = \rho_{p2} = 2710\ \text{Kg/m}^3$, $\nu_{p1} = \nu_{p2} = 0.3$, $E_{p1} = E_{p2} = 7\ 10^4$ MPa , $\rho_0 = 1.2\ \text{Kg/m}^3$, $c_0 = 340$ m/s , $T_0 = 298$ K , $\lambda = 2.56\ 10^{-2}$ W/mK , $\mu = 1.82\ 10^{-5}\ \text{Ns/m}^2$, $C_p = 1004$ J/KgK , $C_V = 716$ J/KgK .

Acousto-Elastic Modes

The eigenfrequencies of the fluid cavity in vacuum, of each plate alone and of the coupled system are calculated with the same plane geometry for fluid cavity, plates and coupled system with a mesh of 15×15 linear quadrilateral elements. Table 1 regroups the eigenfrequencies of a clamped plate in vacuum. The numerical eigenfrequencies of the fluid cavity in vacuum are presented in Table 2 and compared to the analytical values calculated by the following equation [9] :

$$f_{mn} = \frac{C_{eff}(k_V)}{2} . \sqrt{\left(\frac{m}{2\ell_x}\right)^2 + \left(\frac{n}{2\ell_y}\right)^2}$$

where (m, n) are the fluid cavity indices modes.

The eigenfrequencies of the coupled system are presented in Table 3. The real eigenfrequencies represent those of each plate alone. The imaginary part of the complex eigenfrequencies traduce the viscothermal damping of the coupled system.

Figure 2 shows the deformed shape of the first plate, the modal pressure distribution in the fluid layer and the deformed shape of the second plate corresponding to the first acousto-elastic mode of the coupled system. All the deformed shapes present a maximum value in the centre. The deformed shapes of the two plates are different compared to the modal pressure distribution in the fluid layer.

Vibro-Acoustic Response

The vibro-acoustic responses are determined by modal recombination in the case of double plates system excited by an harmonic uniform normal external pressure on the first plate. In order to identify the vibro-acoustic behaviour of the double plates system, it is presented respectively in Fig. 3 the non-dimensional modulus of the displacement in the first plate centre, the non-dimensional modulus of the displacement of the second plate centre and the non-dimensional modulus of the pressure in the fluid cavity centre, versus frequency. Fig 3 shows clearly the viscothermal effects which are traduced by a considerable damping of the modulus displacements and pressure and a decrease of resonance frequencies as compared to the perfect fluid model. Moreover, in Fig. 4 is presented the non-dimensional pressure in the fluid cavity centre versus frequency for different fluid layer thickness. It is observed that, for thick fluid layer thickness (h_0 = 10 mm), the two graphics corresponding to a viscothermal fluid model and a perfect fluid model are almost similar. So, the viscothermal effects decrease with the increase of the fluid layer thickness. Furthermore, the effects of viscosity and thermal conductivity should be taken into account in the case of thin fluid layers.

CONCLUSION

In this paper, the analysis of a double plates system enclosing a thin fluid layer is presented. The acousto-elastic model is established in dimensionless form including viscothermal effects of the fluid. To solve the equations, a total variational formulation is derived. Its dicretization by finite element method gives after minimisation a nonlinear symmetrical matrix system which is resolved using an iterative procedure. The modal approach is adopted to determine the vibro-acoustic responses of the coupled system. Numerical results of a viscothermal fluid model are compared to those corresponding to a perfect fluid model. They show the importance of the viscothermal effects on both the coupled eigenmodes and dynamic responses. Indeed the viscothermal effects generate a more important damping and a shift to left of dynamic responses in the domain frequency in the case of a thin fluid layer.

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Tables

Table 1. Eigenfrequencies of a clamped plate in vacuum

Mode	Eigenfrequencies
1 1	140.05
2 1	285.06
1 2	285.06
2 2	415.75
3 1	511.00
1 3	513.79
3 2	630.72
2 3	630.72
4 1	818.35
1 4	818.35
3 3	830.04

Table 2. Eigenfrequencies of a viscothermal fluid cavity enclosed between two rigid panels

Eigenfrequencies (Hz)					
Mode	Analytical	Numerical	Mode	Analytical	Numerical
0 0	0.00	0.00	2 2	1852.40 – 73.10 i	1860.09 –73.30 i
1 0	637.99 – 44.37 i	638.64 – 44.41 i	3 0	1966.90 – 75.20 i	1985.44 –75.61 i
1 1	911.60 – 52.34 i	912.53 – 52.39 i	3 1	2075.30 – 77.20 i	2093.11 – 77.54 i
2 0	1300.40 – 61.80 i	1305.82 – 61.97 i	3 2	2371.60 – 82.30 i	2390.11 – 82.63 i
2 1	1457.50 – 65.20 i	1462.63 – 65.39 i			

Table 3. Eigenfrequencies of the coupled system

Mode	Eigenfrequencies (Hz)	Mode	Eigenfrequencies (Hz)
1	139.42 – 0.076 i	18	630.73
2	140.05	19	806.91 + 1.60 i
3	281.72 + 0.87 i	20	816.71 + 7.60 i
4	282.72 + 0.93 i	21	818.36
5	285.07	22	829.84 + 0.094 i
6	285.07	23	830.05
7	413.49 + 0.15 i	24	906.91 + 73.85 i
8	415.75	25	917.32 – 19.82 i
9	510.68 – 1.08 i	26	923.80
10	510.66 – 1.10 i	27	923.80
11	510.69 – 1.11 i	28	928.76
12	511.00	29	1108.29 + 0.31 i
13	513.36 – 0.013 i	30	1109.38 + 0.79 i
14	513.79	31	1109.42 + 0.97
15	626.77 – 8.01 i	32	1109.44
16	629.73 – 9.56 i	33	1109.44
17	630.73	34	1297.09 – 61.61 i

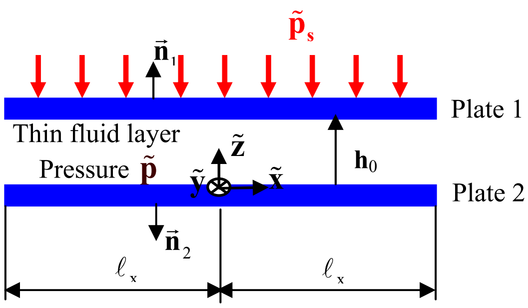


Fig.1. Double plates system (In plane (x, z) representation).

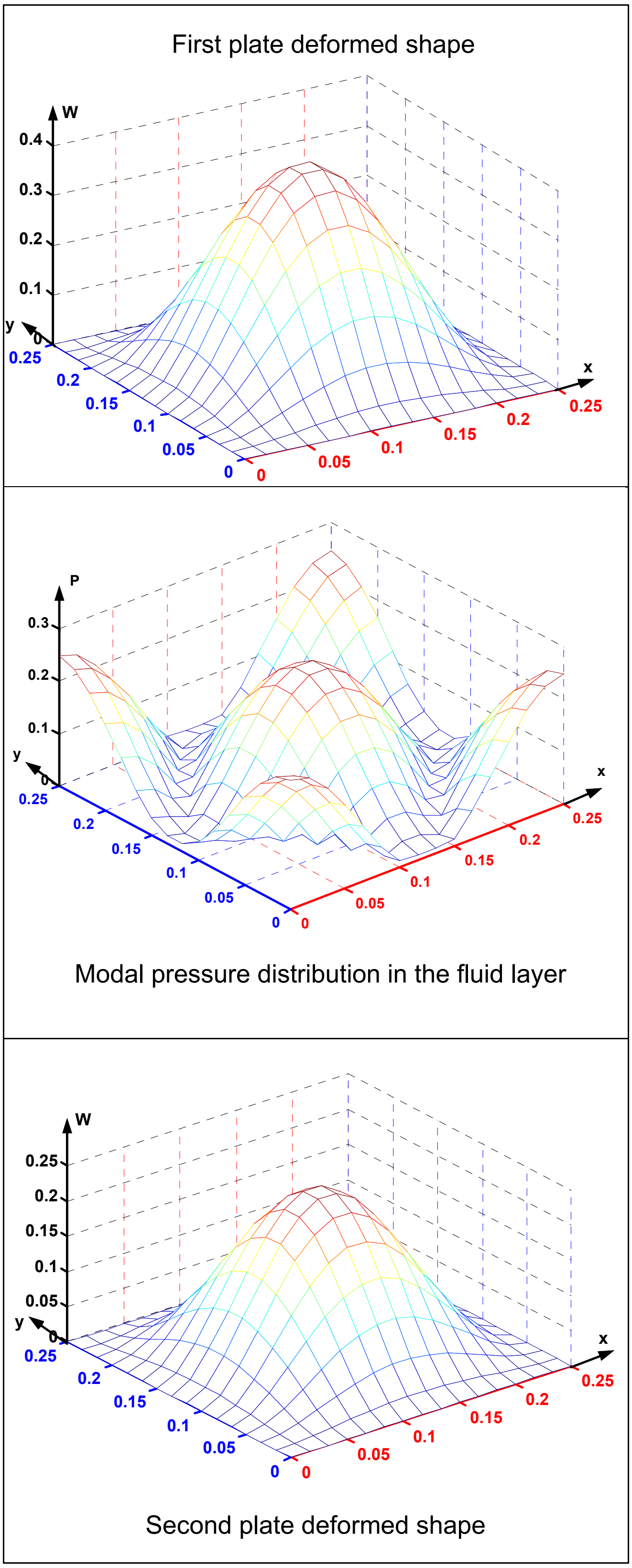
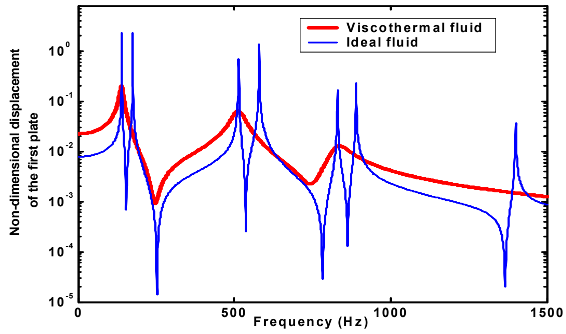
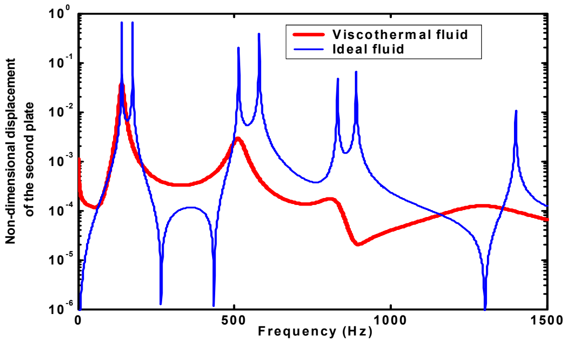


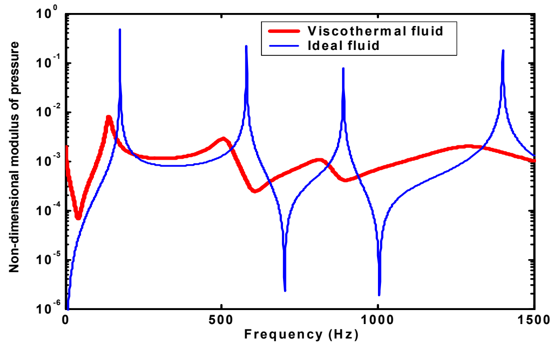
Fig.2. First eigenmode shape



Displacement modulus in the first plate centre (h_0 = 1 mm)



Displacement modulus in the Second plate centre (h_0 = 1 mm)



Pressure modulus in the fluid cavity centre (h_0 = 1 mm)

Fig.3. Vibro-acoustic response versus frequency

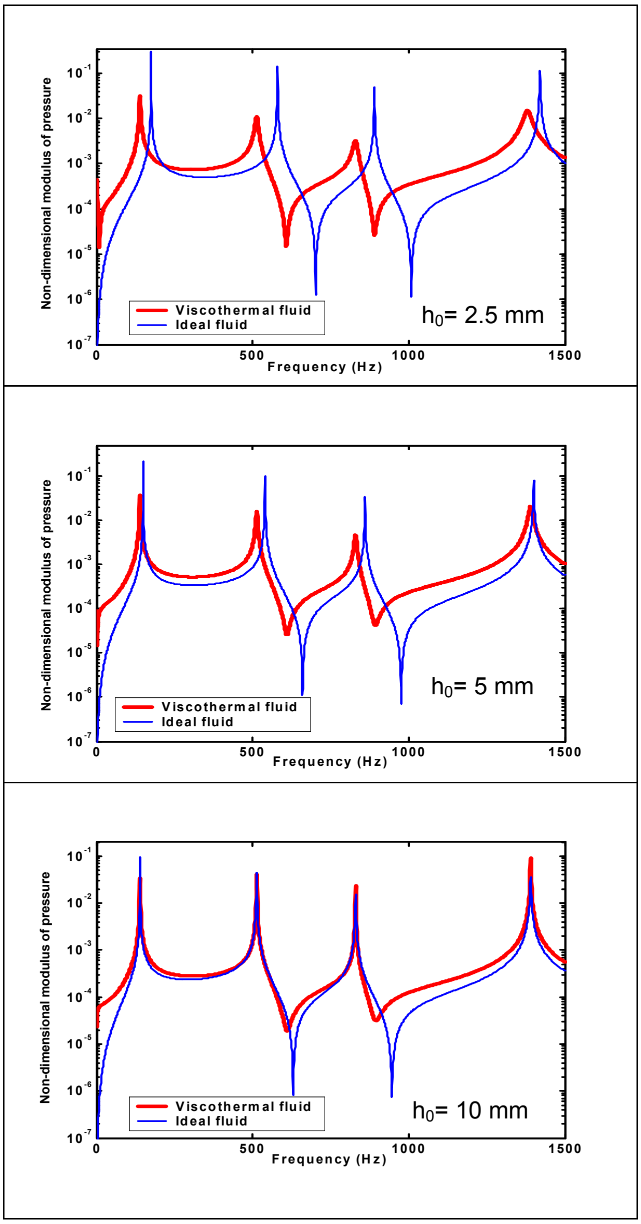


Fig.4. Vibro-acoustic response versus frequency
for different fluid layer thickness