

Exact Solutions For some nonlinear Partial Differential Equations by A Variation of (G'/G)-Expansion Method:

A. R. Shehata^{1*}, F. Abdel Basser¹, Safaa S. M. abu-amra^{1,2}

¹ Department of Mathematics, Faculty of Science, Minia University, 61519 Minia, Egypt

² Department of Mathematics Faculty of Science, Omar Al-Mukhtar University, Al-Bayda. Libya

* Correspondence: shehata1433@yahoo.com; Tel: +20 862092239; Fax: +20 862363011

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Abstract

In this article, we study exact solutions of nonlinear of Burger's Equation, Symmetric Regularized Long Wave (SRLW) equation, and Whitham-Broer-Kaup equations by using the variation of (G'/G)-expansion method, respectively. With the aid of mathematical software Maple, we can obtain the exact solutions for the above equations. Here we use the variation of (G'/G)-expansion method by applying it to solve the above mentioned equations, some new exact traveling wave solutions are obtained which include solitary wave solutions. When the arbitrary constants are taken some special values, the periodic and soliton solutions are obtained from the travelling wave solutions. The obtained solutions are new and not found elsewhere. It is shown that the methods are effective and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics. This methods is effectual, uncomplicated and can also be used to tackle a number of other differential equations related to mathematical physics

1. Introduction

As we know, many phenomena's of physics and biology can be represented by Nonlinear PDEs [1-6]. We extended the known (G'/G)-expansion method [7-9] to it, then we find here its exact solutions.

A. R. Shehata, used the modified (G'/G) -expansion method [10], (w/g) -expansion method [11], Geometrical Properties and Exact Solutions of Three (3+1)- Dimensional Nonlinear Evolution Equations in Mathematical Physics Using Different Expansion Methods [12].

Here we use the variation of (G'/G) -expansion method by applying it to solve the above mentioned equations; some new exact traveling wave solutions are obtained which include solitary wave solutions. It is shown that the methods are effective and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics.

2. Description of the variation of (G'/G)-expansion method

We use the variation of (G'/G)-expansion method [13,14] to solve the above equations.

Let

$$P(u, u_t, u_x, u_y, u_{tt}, u_{tx}, u_{xx}, u_{xy}, u_{yy}, \dots) = 0, \quad (2.1)$$

Where P is a polynomial of u and its partial derivatives.

Step 1. Use the solitary wave variable and transformation

$$u(x, y, t) = U(\xi), \quad (2.2)$$

$$\xi = \tau x + \delta y + \lambda t, \quad (2.3)$$

where τ, δ and λ are nonzero arbitrary constants, we can rewrite Eq.(2.1) as the following nonlinear ODE:

$$Q(U, U', U'', U''', \dots) = 0, \quad \text{where } (') = \frac{d}{d\xi}. \quad (2.4)$$

Step2. Suppose

$$u(\xi) = \sum_{i=1}^m a_i (G'/G)^i + \sum_{i=1}^m b_i (G'/G)^{i-1} (F'/F), \quad (2.5)$$

where $G = G(\xi)$ and $F = F(\xi)$ expresses the solution of the coupled Riccati equation,

$$G'(\xi) = -G(\xi) \cdot F(\xi), \quad (2.6)$$

$$F'(\xi) = 1 - F(\xi)^2, \quad (2.7)$$

$a_i (i = 0, 1, \dots, m), b_i (i = 1, 2, \dots, m)$ are constants to be determined later.

These governing equations lead us two types of general solutions:

$$G(\xi) = \pm \operatorname{sech}(\xi), F(\xi) = \tanh(\xi), \quad (2.8)$$

$$G(\xi) = \pm \operatorname{csch}(\xi), F(\xi) = \coth(\xi). \quad (2.9)$$

Step 3. By considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (2.4), we can find the positive integer m as follows:

$$D \left[U^r \left(\frac{d^q U}{d\xi^q} \right)^s \right] = mr + s(q + m), \quad \text{where } D \text{ denotes the degree of the expression.}$$

Step 4. Substituting Eq.(2.5) into Eq.(2.4) and using Eq.(2.6) and Eq.(2.7), collecting all terms with the same order of (G'/G) or (F) together, left-hand side of Eq.(2.4) is converted into another polynomial in (G'/G)

or (F). Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for a_i, b_i, r_i and τ, δ and λ .

Step 5. Determining the constants a_i, b_i, r_i and τ, δ and λ by solving the algebraic equations in step 5. As the general solutions of Eq.(2.6) and Eq.(2.7) are already known to us, then substituting $a_i, b_i, r_i, \tau, \delta, \lambda$ and the general solutions of Eq.(2.6) and Eq.(2.7), we obtain the exact solutions of Eq.(2.1).

3. Applications of the method:

3.1. Example 1: The Burgers' Equation [15]:

$$u_t + auu_x + b u_{xx} = 0, \quad (3.1.1)$$

It can be observed that the transform:

$$u(x, t) = U(\xi), \quad \xi = \tau x + \lambda t, \quad (3.1.2)$$

where τ and λ are constants, permits us to reduce the Equation (3.1.1) into an ODE. After integrating once, we have the following form:

$$C + \lambda U + \frac{a}{2} \tau U^2 + b \tau^2 U' = 0, \quad (3.1.3)$$

where C is a constant of integration. Now by considering the homogeneous balance between the order of U' and U^2 in Eq.(3.1.3), we obtain $m = 1$.

By using step 2 the solution of Eq. (3.1.3), can be written as,

$$U(\xi) = a_0 + a_1(G'/G) + b_1(F'/F) \\ = a_0 - a_1 F + b_1(F^{-1} - F), \quad (3.1.4)$$

Substituting Eq. (3.1.4) into Eq. (3.1.3), collecting the coefficients of $(F)^i (i = 0, \pm 1, \pm 2)$, and letting it be zero, yields a set of simultaneous algebraic equations for a_0, a_1, b_1, τ and λ as follows:

$$F^0: C + \lambda a_0 + \frac{1}{2} \tau a a_0^2 - \tau a a_1 b_1 - \tau a b_1^2 - \tau^2 b a_1 = 0, \quad (3.1.5)$$

$$F^1: -a a_0 a_1 \tau - a a_0 b_1 \tau - \lambda a_1 - \lambda b_1 = 0, \quad (3.1.6)$$

$$F^2: \frac{1}{2} \tau a a_1^2 + \tau a a_1 b_1 + \frac{1}{2} \tau a b_1^2 + \tau^2 b a_1 + \tau^2 b b_1 = 0, \quad (3.1.7)$$

$$F^{-1}: a a_0 b_1 \tau + \lambda b_1 = 0, \quad (3.1.8)$$

$$F^{-2}: \frac{1}{2} \tau a b_1^2 - \tau^2 b b_1 = 0. \quad (3.1.9)$$

After solving these algebraic equations for a_0, a_1, b_1, τ and λ with the help of software Maple, yields the following results.

Case 1:

$$C = \frac{1}{2} \frac{(a^2 a_0^2 - 4 b^2 \tau^2)}{a}, \quad \lambda = -a a_0 \tau, \quad a_1 = -\frac{2 \tau b}{a}, \quad b_1 = 0, \quad (3.1.10)$$

where a_0, τ are arbitrary.

Case 2:

$$C = \frac{1}{4} \frac{a^2 b_1 (a_0^2 - b_1^2)}{b}, \quad \lambda = -\frac{1}{2} \frac{a^2 a_0 b_1}{b}, \quad a_1 = -b_1, \quad \tau = \frac{1}{2} \frac{a b_1}{b}, \quad (3.1.11)$$

where b_1, a_0 is arbitrary.

Substituting Eqs. (3.1.10), (3.1.11) into Eq.(3.1.4) we get two types of the exact solutions of Eq.(3.1.1) as follows:

According to case 1.

Type 1:

$$\text{Class I: } U_{11}(x, t) = a_0 + \frac{2 \tau b}{a} \tanh(\tau x - a a_0 \tau t). \quad (3.1.12)$$

$$\text{Class II: } U_{12}(x, t) = a_0 + \frac{2 \tau b}{a} \coth(\tau x - a a_0 \tau t). \quad (3.1.13)$$

According to case 2.

Type 2:

$$\text{Class I: } U_{21}(x, t) = a_0 + b_1 \coth\left(\frac{a b_1}{2 b} x - \frac{a^2 a_0 b_1}{2 b} t\right). \quad (3.1.14)$$

$$\text{Class II: } U_{22}(x, t) = a_0 + b_1 \tanh\left(\frac{a b_1}{2 b} x - \frac{a^2 a_0 b_1}{2 b} t\right). \quad (3.1.15)$$

3.2. Example 2: The SRLW equation:

$$u_{tt} + u_{xx} + uu_{xt} + u_x u_t + u_{xxtt} = 0, \quad (3.2.1)$$

which arises in several physical applications including ion sound waves in plasma. It arises in many nonlinear problems of mathematical physics and applied mathematics. Periodic wave solutions of SRLW have been given by using the Exp function method [16], and (G'/G) -expansion method [17].

It can be observed that the transform:

$$u(x, t) = U(\xi), \quad \xi = \tau x + \lambda t, \quad (3.2.2)$$

where τ and λ are constants, permits to reduce the Equation (3.2.1) into an ODE. After integrating twice, we have the following form:

$$(\lambda^2 + \tau^2)U + \frac{1}{2} \lambda \tau U^2 + \lambda^2 \tau^2 U'' + C = 0, \quad (3.2.3)$$

Balancing the order of U'' and U^2 in Eq.(3.2.3), we obtain $m = 2$.

By using step 2 the solution of Eq. (3.2.3), can be written as,

$$U(\xi) = a_0 + a_1(G'/G) + a_2(G'/G)^2 + b_1(F'/F) \\ + b_2(G'/G)(F'/F) \\ = a_0 - a_1 F + a_2 F^2 + b_1(F^{-1} - F) - b_2(1 - F^2) \quad (3.2.4)$$

Substituting Eq. (3.2.4) into Eq. (3.2.3), collecting the coefficients of $(F)^i (i = 0, \pm 1, \pm 2, \pm 3, \pm 4)$, and letting it be zero, yields a set of simultaneous algebraic equations for $a_0, a_1, a_2, b_1, b_2, \tau$ and λ as follows:

$$F^4: \frac{1}{2} \tau \lambda a_2^2 + \frac{1}{2} \tau \lambda b_2^2 + 6 \tau^2 \lambda^2 a_2 + 6 \tau^2 \lambda^2 b_2 \\ + \tau \lambda a_2 b_2 = 0, \quad (3.2.5)$$

$$F^3: -2 \tau^2 \lambda^2 a_1 - 2 \tau^2 \lambda^2 b_1 - \tau \lambda a_1 a_2 - \tau \lambda a_1 b_2 \\ - \tau \lambda a_2 b_1 - \tau \lambda b_1 b_2 = 0, \quad (3.2.6)$$

$$F^2: \frac{1}{2} \tau \lambda a_1^2 + \frac{1}{2} \tau \lambda b_1^2 - \tau \lambda b_2^2 - 8 \tau^2 \lambda^2 a_2 - \\ 8 \tau^2 \lambda^2 b_2 + \tau \lambda a_1 b_1 + \tau \lambda a_0 b_2 + \tau \lambda a_0 a_2 - \\ \tau \lambda a_2 b_2 + a_2 \tau^2 + b_2 \lambda^2 + b_2 \tau^2 + a_2 \lambda^2 = 0, \quad (3.2.7)$$

$$F^1: 2 \tau^2 \lambda^2 a_1 + 2 \tau^2 \lambda^2 b_1 - \tau \lambda a_0 a_1 - \tau \lambda a_0 b_1 + \\ \tau \lambda a_1 b_2 + \tau \lambda a_2 b_1 + 2 \tau \lambda b_1 b_2 - \tau^2 a_1 - \tau^2 b_1 - \\ \lambda^2 a_1 - \lambda^2 b_1 = 0, \quad (3.2.8)$$

$$F^0: -\tau \lambda a_1 b_1 - \tau \lambda a_0 b_2 + C + \frac{1}{2} \tau \lambda b_2^2 - \tau \lambda b_1^2 + 2\tau^2 \lambda^2 a_2 + 2\tau^2 \lambda^2 b_2 + \frac{1}{2} \tau \lambda a_0^2 + a_0 \tau^2 + a_0 \lambda^2 - b_2 \tau^2 - b_2 \lambda^2 = 0, \quad (3.2.9)$$

$$F^{-1}: -2\tau^2 \lambda^2 b_1 + \tau \lambda a_0 b_1 - \tau \lambda b_1 b_2 + \tau^2 b_1 + \lambda^2 b_1 = 0, \quad (3.2.10)$$

$$F^{-2}: \frac{1}{2} \tau \lambda b_1^2 = 0, \quad (3.2.11)$$

$$F^{-3}: 2\tau^2 \lambda^2 b_1 = 0. \quad (3.2.12)$$

After solving these algebraic equations for $a_0, a_1, a_2, b_1, b_2, \tau, \lambda$ and C with the help of software Maple, yields the following results.

Case 1:

$$C = -\frac{1}{2} \tau \lambda a_0^2 + \tau \lambda a_0 b_2 - \frac{1}{2} \tau \lambda b_2^2 - a_0 \tau^2 + b_2 \tau^2 - a_0 \lambda^2 - b_2 \lambda^2, a_1 = 0, a_2 = -b_2, b_1 = 0, \quad (3.2.13)$$

where τ, λ, a_0, b_2 are arbitrary.

Case 2:

$$C = -\frac{1}{2} \frac{16\tau^4 \lambda^4 - \tau^4 - 2\tau^2 \lambda^2 - \lambda^4}{\tau \lambda}, \quad (3.2.14)$$

$$a_0 = \frac{8\tau^2 \lambda^2 + \tau \lambda b_2 - \tau^2 - \lambda^2}{\tau \lambda}, a_1 = 0, a_2 = -12\tau \lambda - b_2, b_1 = 0,$$

where τ, λ, b_2 are arbitrary.

Substituting Eqs.(3.2.13),(3.2.14) into Eq.(3.2.4) we get two types of the exact solutions of Eq.(3.2.1) as follows:

According to case 1.

Type 1:

$$\text{Class I: } U_{11}(x, t) = a_0 - b_2. \quad (3.2.15)$$

$$\text{Class II: } U_{12}(x, t) = a_0 - b_2. \quad (3.2.16)$$

According to case 2.

Type 2:

Class I:

$$U_{21} = 8\tau \lambda - \frac{\tau}{\lambda} - \frac{\lambda}{\tau} - 12\tau \lambda \tanh^2(\tau x + \lambda t). \quad (3.2.17)$$

Class II:

$$U_{22}(x, t) = 8\tau \lambda - \frac{\tau}{\lambda} - \frac{\lambda}{\tau} - 12\tau \lambda \coth^2(\tau x + \lambda t). \quad (3.2.18)$$

3.3. Example 3: The Whitham-Broer-Kaup equations [18,19]:

$$\begin{cases} u_t + uu_x + v_x + pu_{xx} = 0, \\ v_t + (uv)_x - pv_{xx} + qu_{xxx} = 0, \end{cases} \quad (3.3.1)$$

where $u = u(x, t)$ is the field of horizontal velocity, $v = v(x, t)$ is the height deviating from the equilibrium position of liquid, p and q are real constants that represent different diffusion powers.

For our purpose, we use the following transformation:

$$\begin{cases} u(x, t) = U(\xi), & \xi = \tau x - \lambda t, \\ v(x, t) = V(\xi), & \xi = \tau x - \lambda t, \end{cases} \quad (3.3.2)$$

where $\lambda \neq 0$ and $\tau \neq 0$ constants. Then by using Eq.(3.3.2), Eq.(3.3.1) can be turned into an ODE:

$$\begin{cases} -\lambda U' + \tau U U' + \tau V' + \tau^2 p U'' = 0, \\ -\lambda V' + \tau(UV)' - \tau^2 p V'' + \tau^3 q U''' = 0, \end{cases} \quad (3.3.3)$$

where $U' = dU/d\xi$. By integrating once and setting the constants of integration to be zero, we obtain

$$\begin{cases} -\lambda U + \tau \frac{U^2}{2} + \tau V + \tau^2 p U' = 0, \\ -\lambda V + \tau UV - \tau^2 p V' + \tau^3 q U'' = 0, \end{cases} \quad (3.3.4)$$

balancing the order of U^2 and U', UV and U'' in Eq.(3.3.4), we obtain:

$$m = 1, n = 2. \quad (3.3.5)$$

By using step 2 the solutions of Eqs. (3.3.4) can be written as,

$$\begin{aligned} U(\xi) &= a_0 + a_1 \left(\frac{G'}{G}\right) + b_1 \left(\frac{F'}{F}\right), \\ &= a_0 - a_1 F + b_1 (F^{-1} - F), \end{aligned} \quad (3.3.6)$$

$$\begin{aligned} V(\xi) &= c_0 + c_1 \left(\frac{G'}{G}\right) + c_2 \left(\frac{G'}{G}\right)^2 + d_1 \left(\frac{F'}{F}\right) + d_2 \left(\frac{G'}{G}\right) \left(\frac{F'}{F}\right), \\ &= c_0 - c_1 F + c_2 F^2 + d_1 (F^{-1} - F) - d_2 (1 - F^2), \end{aligned} \quad (3.3.7)$$

substituting Eqs. (3.3.6),(3.3.7) into Eqs.(3.3.4), collecting the coefficients of $(F)^i (i = 0, \pm 1, \pm 2, \pm 3)$, and letting it be zero, yields a set of simultaneous algebraic equations for $a_0, a_1, b_1, c_0, c_1, c_2, d_1, d_2, \tau$ and λ as follows:

$$F^0: -\lambda a_0 + \frac{1}{2} \tau a_0^2 - \tau b_1^2 + \tau c_0 - \tau d_2 - \tau a_1 b_1 - \tau^2 p a_1 = 0 \quad (3.3.8)$$

$$F^1: -a_0 a_1 \tau - a_0 b_1 \tau + a_1 \lambda + b_1 \lambda - c_1 \tau - d_1 \tau = 0 \quad (3.3.9)$$

$$\begin{aligned} F^2: \tau a_1 b_1 + \tau^2 p a_1 + \tau^2 p b_1 + \tau d_2 + \frac{1}{2} \tau b_1^2 + \tau c_2 \\ + \frac{1}{2} \tau a_1^2 = 0, \end{aligned} \quad (3.3.10)$$

$$F^{-1}: a_0 b_1 \tau - b_1 \lambda + d_1 \tau = 0, \quad (3.3.11)$$

$$F^{-2}: -\tau^2 p b_1 + \frac{1}{2} \tau b_1^2 = 0, \quad (3.3.12)$$

$$F^0: \tau^2 p c_1 + \tau a_0 c_0 - \tau a_0 d_2 - \tau a_1 d_1 - \tau b_1 c_1 - 2\tau b_1 d_1 - \lambda c_0 + \lambda d_2 = 0, \quad (3.3.13)$$

$$\begin{aligned} F^1: 2a_1 \tau^3 q + 2b_1 \tau^3 q - 2c_2 \tau^2 p - 2d_2 \tau^2 p - a_0 c_1 \tau - \\ a_0 d_1 \tau - a_1 c_0 \tau + a_1 d_2 \tau - b_1 c_0 \tau + b_1 c_2 \tau + \\ 2b_1 d_2 \tau + \lambda c_1 + \lambda d_1 = 0, \end{aligned} \quad (3.3.14)$$

$$\begin{aligned} F^2: -c_1 \tau^2 p - d_1 \tau^2 p + a_0 c_2 \tau + a_0 d_2 \tau + a_1 c_1 \tau + a_1 d_1 \tau \\ + b_1 c_1 \tau + b_1 d_1 \tau - \lambda c_2 - \lambda d_2 = 0, \end{aligned} \quad (3.3.15)$$

$$F^3: -2a_1 \tau^3 q - 2b_1 \tau^3 q + 2c_2 \tau^2 p + 2d_2 \tau^2 p - a_1 c_2 \tau - a_1 d_2 \tau - b_1 c_2 \tau - b_1 d_2 \tau = 0, \quad (3.3.16)$$

$$F^{-1}: -2b_1 \tau^3 q + a_0 d_1 \tau + b_1 c_0 \tau - b_1 d_2 \tau - \lambda d_1 = 0, \quad (3.3.17)$$

$$F^{-2}: d_1 \tau^2 p + b_1 d_2 \tau = 0, \quad (3.3.18)$$

$$F^{-3}: 2\tau^3 q b_1 = 0. \quad (3.3.19)$$

After solving these algebraic equations for $a_0, a_1, b_1, c_0, c_1, c_2, d_1, d_2, \tau$ and λ with the help of software Maple, yields the following results.

Case 1:

$$\begin{aligned}
 a_0 &= \frac{\lambda}{\tau}, a_1 = -\frac{\lambda}{\tau}, b_1 = 0, c_0 \\
 &= \frac{1-2\lambda\tau^2p+2d_2\tau^2+\lambda^2}{2\tau^2}, c_1 = 0, c_2 \\
 &= -\frac{1-2\lambda\tau^2p+2d_2\tau^2+\lambda^2}{2\tau^2}, d_1 \\
 &= 0,
 \end{aligned} \quad (3.3.20)$$

$$q = \frac{1-4\tau^4p^2+\lambda^2}{4\tau^4},$$

where λ, τ, d_2, p are arbitrary.

Case 2:

$$\begin{aligned}
 a_0 &= \frac{\lambda}{\tau}, a_1 = 0, b_1 = \pm \frac{\lambda}{\sqrt{2}\tau}i, c_0 = d_2, c_1 = 0, c_2 = \\
 &\frac{1-2d_2\tau^2+\lambda^2}{2\tau^2}, d_1 = 0, p = \pm \frac{\lambda}{2\sqrt{2}\tau^2}i, q = 0,
 \end{aligned} \quad (3.3.21)$$

where λ, d_2, τ are arbitrary.

substituting Eqs.(3.3.20),(3.3.21) into

Eqs.(3.3.6),(3.3.7) we get two types of the exact solutions of Eq.(3.3.1) as follows:

According to case 1.

Type 1:

$$\text{Class I: } U_{11}(x, t) = \frac{\lambda}{\tau} + \frac{\lambda}{\tau} \tanh(\tau x - \lambda t), \quad (3.3.22)$$

$$V_{11}(x, t) = \left(\frac{-2\lambda\tau^2p+2d_2\tau^2+\lambda^2}{2\tau^2} - d_2 \right) (1 - \tanh^2(\tau x + \lambda t)). \quad (3.3.23)$$

$$\text{Class II: } U_{12}(x, t) = \frac{\lambda}{\tau} + \frac{\lambda}{\tau} \coth(\tau x - \lambda t), \quad (3.3.24)$$

$$V_{12}(x, t) = \left(\frac{-2\lambda\tau^2p+2d_2\tau^2+\lambda^2}{2\tau^2} - d_2 \right) (1 - \coth^2(\tau x + \lambda t)). \quad (3.3.25)$$

According to case 2.

Type 2:

Class I:

$$U_{21} = \frac{\lambda}{\tau} \pm \frac{\lambda}{\sqrt{2}\tau} i (\coth(\tau x - \lambda t) - \tanh(\tau x - \lambda t)), \quad (3.3.26)$$

$$V_{21} = \left(\frac{-2d_2\tau^2+\lambda^2}{2\tau^2} + d_2 \right) \tanh^2(\tau x + \lambda t). \quad (3.3.27)$$

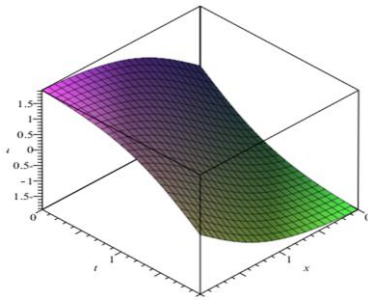
Class II:

$$U_{22} = \frac{\lambda}{\tau} \pm \frac{\lambda}{\sqrt{2}\tau} i (\tanh(\tau x - \lambda t) - \coth(\tau x - \lambda t)), \quad (3.3.28)$$

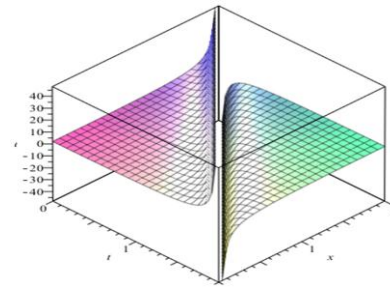
$$V_{22} = \left(\frac{-2d_2\tau^2+\lambda^2}{2\tau^2} + d_2 \right) \coth^2(\tau x + \lambda t). \quad (3.3.29)$$

4. Numerical solutions for the exact solutions of the above NPD equations:

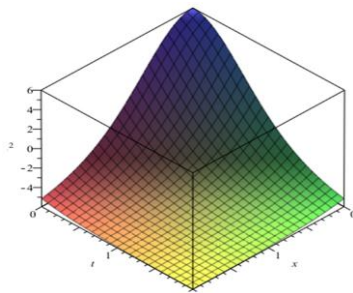
We can study the behavior of the travelling wave solutions which obtained above by illustrating the following figures:



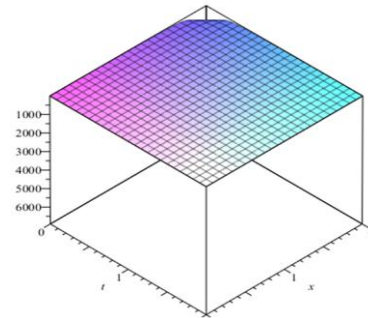
(Figure 1): The exact solution of(3.1.12)
When $a_0 = 0, a = b = \tau = 1$



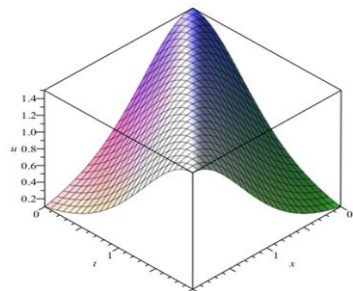
(Figure 2): The exact solution of (3.1.14)
When $a_0 = 0, b_1 = 2, a = b = \tau = 1$



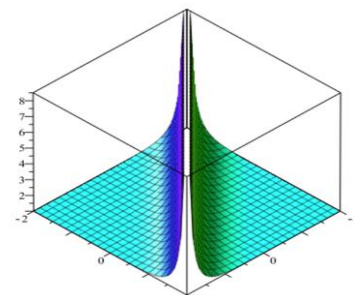
(Figure 3): The exact solution of(3.2.17)
When $\lambda = \tau = 1$



(Figure 4): The exact solution of (3.2.18)
When $\lambda = \tau = 1$



(Figure 5): The exact solution of (3.3.23)
When $\lambda = -1, \tau = p = 1, d_2 = 0$



(Figure 6): The exact solution of (3.3.26)
When $\lambda = \tau = 1$

5. Conclusion

Here we use the variation of (G'/G) -expansion method to solve some NLPDEs, namely nonlinear of Burger's Equation, Symmetric Regularized Long Wave (SRLW) equation and Whitham-Broer-Kaup equations. This method is reliable and efficient and gives new solutions.

And it can be used for many others nonlinear partial differential equations in mathematical physics.

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