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ELIMINATION OF UNBALANCE IN MAGNETIC BEARING SYSTEMS WITH NEURO-FUZZY REDUCED ORDER OBSERVER

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ABSTRACT

Adaptive Neuro-Fuzzy Inference System (ANFIS) is used to design fuzzy reduced order observer for solving the problem of unbalance in magnetic bearing systems. Nonlinear model of magnetic bearing with unbalance is very complicated and coupling between various states cannot be neglected. With classical methods no one can eliminate disturbances without linearization of the system and neglecting some terms from the nonlinear model. In this paper ANFIS is used to acquire desired membership functions of the fuzzy rules for reduced order observer to achieve rejection of unbalance. Simulation results show that fuzzy observer can eliminate the effect of unbalance at any rotational speed.

KEYWORDS: ANFIS, disturbance rejection, gain scheduling, magnetic bearing, and reduced order observer.

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Nomenclature

A	Area under one magnet pole.	mm^2
a,b	Distance from center to bearings (a, b)	m
AMB	Active magnetic bearings.	
e	Eccentricity.	μm
f_j	Force produced by the $j th$ stator electromagnet.	N
I_o	Steady state Coil current.	$Ampere$
i_j	Coil current of the $j th$ stator electromagnet.	$Ampere$
I_x	Moment of inertia around x axis.	$kg.m^2$
I_y	Moment of inertia around y axis.	$kg.m^2$
k_i, k_s	Force current and position factors respectively.	
L	Coil inductance.	H
m	Rotor mass.	kg
N	Number of turns in each electromagnet coils.	$turns$
R	Electromagnet coil resistance.	ohm
S_O	Steady state air gap length.	mm
T	Sampling time.	sec
t_s	Settling time.	sec
u	Control input vector.	$volt$
Z	State vector.	

1. INTRODUCTION

Magnetic bearings are being extensively used in industrial and manufacturing applications due to their unique properties, (attain high speed of rotation, operate in environments not amenable to the use of lubrication, and non-contacting nature leading to elimination of frictional losses. Magnetic bearing systems are unstable and fast, therefore they need well designed controllers to keep its precision position and to eliminate unbalance vibrations at any time. Rotor unbalance usually generated disturbance forces and moments, the forces due to the eccentricity between the center of rigid body and center of the shaft (static unbalance), but disturbance moments due to the inclination angle between axis of symmetry of rigid body and rotation axis.

Several researches deal with unbalance in magnetic bearing system by different methods. Fumio et. all. deal with unbalance by two steps [1-4]. The first step is to compensate the unbalance forces by generating electromagnetic forces that cancel it. The second step is to make the rotor rotates around its axis of inertia (automatic balancing). Another solution is filter based automatic balancing achieved by notch filter centered at the frequency of rotation inserted in the control loop to eliminate bearing stiffness at this frequency [5]. Unbalance compensation and automatic balance using Q-parameterization theory were done by [5, 6-9], they assumed that the unbalance is a sinusoidal noise in the measured signal and choose the controller Q-parameter to reject this noise. Also this problem was solved by using gain scheduled H_∞ robust controller based on the Loop Shaping Design Procedure (LSDP) [10]. In this paper a combination of ANFIS technique and gain scheduling was proposed to derive fuzzy membership functions and fuzzy IF-THEN rules for fuzzy reduced order observer from data point to cancel disturbances at any rotational speed. Whence, the same observer was jointed to nonlinear model.

2. MATHEMATICAL MODEL

2.1 Equations of Motion

Consider the vertical shaft magnetic bearing system shown in Fig. 1, start with assuming that the rotor is rigid, the unbalance is in horizontal direction only and the gravity force is neglected.

Assume also that owing to small errors, the position of point P in which the center of gravity of the rigid body is located does not coincide with that of point G, the center of the shaft. The distance between the two points is the eccentricity e. Moreover, the axis of symmetry of the rigid body does not coincide exactly with the rotation axis. The angle between them is the angular error ε. These two errors, which are assumed to be small, cause a static unbalance and a dynamic unbalance, respectively.

The fundamental equations of motion of the rotor [11] for four radial degrees of freedom are:

$$
\begin{aligned}
& m\ddot{x} + me\omega^2 \sin\omega t = f_{ax} + f_{bx} \\
& m\ddot{y} + me\omega^2 \cos\omega t = f_{ay} + f_{by} \\
& I_x\ddot{\psi} - I_z\omega\dot{\theta} - \varepsilon\omega^2 (I_x - I_z)\sin\omega t = af_{ax} - bf_{bx} \\
& I_x\ddot{\theta} + I_z\omega\dot{\psi} + \varepsilon\omega^2 (I_x - I_z)\cos\omega t = -af_{ay} + bf_{by}
\end{aligned}
\qquad (1)
$$

where $(f_{ax}, f_{bx}, f_{ay}, f_{by})$ are forces at bearings (a & b).

2.2. Electromagnet Equations

Forces $f_B = (f_{ax}, f_{bx}, f_{ay}, f_{by})$ can be expressed in terms of coil current i and the gap length S as follows:

$$f_B = K\left(\frac{i_B{}^2}{S_{gB}^2}\right) \qquad (2)$$

where $K = \dfrac{\mu_o A_g\ N^2}{4}$.

From linearization forces as a function of current and air gap

$$
\begin{aligned}
& f_B = K_{SB} Z_B + K_{IB} i_B \\
& \textit{where} \\
& i_B = \begin{pmatrix} i_{ax} & i_{bx} & i_{ay} & i_{by} \end{pmatrix} \\
& K_{SB} = 2diag.(k_{sa}, k_{sb}, k_{sa}, k_{sb}) \\
& K_{IB} = 2diag.(k_{ia}, k_{ib}, k_{ia}, k_{ib})
\end{aligned}
\qquad (3)
$$

The voltage across the j electromagnet coils can also expressed as follows:

$$u_j = Ri_j + L\frac{d\,i_j}{dt} + k_i \dot{z}_B(j) \qquad j = 1,...4 \qquad (4)$$

where

$$z_B = \begin{bmatrix} x_a & x_b & y_a & y_b \end{bmatrix}^T$$

Let equations (1-4) be written in the form

$$M\ddot{Z} = G\dot{Z} + f + E\,w(t) \qquad (5)$$

where

$M = diag(m, I_x, m, I_x)$ G= *(gyroscopic effect) zero matrix excepted* $g_{24} = -g_{42} = \omega I_z$

$$E=\omega^2\begin{bmatrix} me & 0 \\ \varepsilon(I_x-I_z) & 0 \\ 0 & me \\ 0 & \varepsilon(I_x-I_z)\end{bmatrix} \quad ,w(t)=\begin{bmatrix}\sin\omega t \\ \cos\omega t\end{bmatrix} \tag{6}$$

Transform equation (5) to bearing coordinates

$$\begin{aligned} M_B\ddot{z}_B &= G_B\dot{z}_B + f_B + E_B w(t) \\ M_B &= T_B^T M T_B \\ G_B &= T_B^T G T_B \\ E_B &= T_B^T E \end{aligned} \tag{7}$$

T_B is the transformation matrix from principle axes to bearing axes.
If we assume that the force displacement factor and force current factor are the same for bearings (a) & (b), then

$$\ddot{z}_B = M_B^{-1}G_B\dot{z}_B + M_B^{-1}K_{SB}z_B + M_B^{-1}K_{IB}i_B + M_B^{-1}E_B\, w(t) \tag{8}$$

The state vector is composed of displacements, their derivatives and electrical currents in coils. For this model the voltage is the input variable. The final state space model is

$$\begin{aligned} \dot{x}_B &= A_B x_B(t) + B_B u(t) + \Gamma_1 w(t) \\ y &= Cx_B(t) \end{aligned} \tag{9}$$

$$x_B=\begin{bmatrix} z_B \\ \dot{z}_B \\ i_B\end{bmatrix} \qquad A_B=\begin{bmatrix} 0_{4\times4} & I_{4\times4} & 0_{4\times4} \\ ({M_B}^{-1}K_{SB})_{4\times4} & ({M_B}^{-1}G_B)_{4\times4} & ({M_B}^{-1}K_{iB})_{4\times4} \\ 0_{4\times4} & (\frac{-K_i}{L})_{4\times4} & (\frac{-R}{L})_{4\times4}\end{bmatrix} \qquad B_B=\begin{bmatrix} 0_{4\times4} \\ 0_{4\times4} \\ (1/L)_{4\times4}\end{bmatrix}$$

$$C=\begin{bmatrix} I_{4\times4} & 0_{4\times4} & 0_{4\times4}\end{bmatrix} \qquad \Gamma_1=\begin{bmatrix} 0_{4\times2} \\ (M_B^{-1}E_B)_{4\times2} \\ 0_{4\times2}\end{bmatrix}$$

Assume that $\omega = 0$ and there is no disturbance, we can divide the MIMO system to four SISO separate subsystems using decentralized method [12] and control each subsystem individually.

Discretization of each subsystem by using a zero order hold at a sampling time of (t_s sec) yields the following SISO discrete-time control system.

$$\begin{aligned} z(k+1) &= \phi z(k) + \Gamma u(k) \\ Y(k) &= C z(k) \end{aligned} \tag{10}$$

Disturbance forces in whole system obey the following equations [13]:

$$\begin{aligned} \dot{x}_d &= F_d x_d \\ w &= H_d x_d \end{aligned} \tag{11}$$

where

$$F_d = \omega^2 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad H_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The discrete model is done by zero order hold with sampling time t_s sec, as

$$\begin{aligned} x_d(k+1) &= \phi_d x_d(k) \\ w(k) &= H_d x_d(k) \end{aligned} \tag{12}$$

where

$$\phi_d = e^{F_d t_s}$$

We augmented the system model with disturbance model, so that the state space equations become

$$\begin{bmatrix} x_B(k+1)_{12\times1} \\ x_d(k+1)_{2\times1} \end{bmatrix} = \underbrace{\begin{bmatrix} \phi_{12\times12} & \Gamma_{d\,12\times2} \\ 0_{2\times12} & \phi_{d\,2\times2} \end{bmatrix}}_{\phi_w} \begin{bmatrix} x_B(k) \\ x_d(k) \end{bmatrix} + \underbrace{\begin{bmatrix} \Gamma_{12\times4} \\ 0_{2\times4} \end{bmatrix}}_{\Gamma_w} u(k)_{4\times1}$$

$$Y(k) = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C_w} \begin{bmatrix} x_B(k) \\ x_d(k) \end{bmatrix} \tag{13}$$

where

$$\Gamma_d = \begin{bmatrix} 0_{4\times2} \\ M_B^{-1} E_B H_d \\ 0_{4\times2} \end{bmatrix} \qquad x_d(k) = \begin{bmatrix} \sin\omega t \\ \cos\omega t \end{bmatrix} \qquad \phi = e^{-A_B t_s}$$

3. CONTROLLER DESIGN

The first step to design the controller is to know the feedback gain K for system in equation (10), which satisfies the control law

$$u = -Kz \tag{14}$$

In our study, pole placement method is used to obtain the gain vector K_{ki} for each subsystem. The second step is disturbance rejection where, we estimate disturbance signal by the reduced order observer for each subsystem (Fig.2) and then we use that estimate in control law to force the error to be zero.

4. NEURO-FUZZY REDUCED ORDER OBSERVER

The interest in neuro-fuzzy systems has grown tremendously over the last few years. Fuzzy systems are designed to work with knowledge in the form of linguistic rules. But the translation of these linguistic rules into the framework of fuzzy set theory depends on the choice of certain parameters, for which no formal method is known. The optimization of these parameters can be carried out by neural networks, which are designed to learn from training data [14, 15].

We present new learning strategies to derive fuzzy reduced order observer rules from data. A regular first-order Sugeno fuzzy controller uses its inputs both in the premise part to determine firing strengths and in the consequent part to determine each rule's output; this is shown in Fig.3 (a), where both the premise and consequent parts receive the same inputs. For certain applications, it suffices to use only some of the inputs for the premise part and the others for the consequent part, as shown in Fig.3 (b). Obviously, Fig.3.(b) is a special case of Fig.3.(a), but is very useful in designing fuzzy observer based on the concept of gain scheduling.

Specifically, the inputs to a fuzzy observer contain two types of variables: scheduling and state. Scheduling variables are used in the premise part to determine what mode or characteristics the plant has. Once the plant mode or characteristics have been determined, the corresponding rule is fired with an output equal to the state variables multiplied by appropriate state feedback gains and reduced order observer gain. In Fig. 3(b), z and u are the scheduling variables but x and y are the state variables [16].

A neuro-fuzzy observer has the following fuzzy IF-THEN rules:

$$\begin{aligned} &if\ u\ is\ A_1\ \ and\ \ y\ is\ B_1\ then\ \ k_{e11} = C_1\ \ and\ \ k_{e12} = D_1 .... k_{e1n} = F_{1n} \\ &if\ u\ is\ A_2\ \ and\ \ y\ is\ B_2\ then\ \ k_{e21} = C_2\ \ and\ \ k_{e22} = D_2 .... k_{e2n} = F_{2n} \end{aligned} \tag{15}$$

where

k_e *is the estimator gain at any ratational speed*

5. DESIGN APPROACH

The design of neuro-fuzzy observer procedure is as follows:
1- Determination of a set of various values of rotational speed , for each speed determine the estimator gain and the value of estimated disturbance.
2- Use of ANFIS to construct fuzzy inference system whose Membership function parameters are tuned or adjusted using the back propagation algorithm and least square method for a set of rotational speed .
3- Combination of nonlinear plant with both feed-back controller and neuro fuzzy observer Fig. 4.

6. RESULTS

Simulation with MATLAB program using data in Table .1 is done. The feedback gains are obtained via decentralized controller with pole placement method for each subsystem based on zero rotational speed. Then collect and rearrange the obtained feedback gains to the form given in gain matrix for the complete system with gyroscopic effect and imbalance.

$$\begin{bmatrix} K_{xa} & 0 & 0 & 0 \\ 0 & K_{xb} & 0 & 0 \\ 0 & 0 & K_{ya} & 0 \\ 0 & 0 & 0 & K_{yb} \end{bmatrix}_{12\times 12} \qquad (16)$$

Where $K_{xa}, K_{xb}, K_{ya}, K_{yb}$ are the controllers of the x_a, x_b, y_a, y_b subsystems respectively. The reduced order observer is designed separately for each subsystem to twenty-five different numbers of rotational speeds from $2\pi 20$ *to* $2\pi 250\, rad/\sec$. Values of static and dynamic unbalance are $e = 0.1mm, \varepsilon = 4\times 10^{-4} rad$. As similar as controller gains, we collect the estimator gains and estimated disturbances to whole system at every rotational speed. The data collected are used as training data for ANFIS technique to get membership functions and IF-THEN rules.

Fig.5. shows the membership function of estimator gains. Whence Neuro-Fuzzy reduced order observer is joined to nonlinear model. To check the performance of exact nonlinear system simulation, a MATLAB program using numerical Rung-Kutta fourth order method is obtained (see Appendix). It is shown that the motion in z-direction is necessary for describe the motion of rotor for nonlinear model, so we design a controller similar to one of radial direction and joint it with controller in gain matrix (13). Note that there is no disturbance in z-direction, and then we don't need to design an estimator. Results in Figs. 6-8 show that the Neuro-Fuzzy reduced order observer based controller gives excellent performance to the nonlinear model.

7. CONCLUSIONS

In this paper a neuro-fuzzy reduced order observer is employed to eliminate sinusoidal disturbances in magnetic bearing system at all operating speeds. A controller is designed to regulate the system and neuro-fuzzy observer to estimate the disturbance and negative feed it back to cancel the actual one. A conventional reduced order observer is designed for twenty-five values of rotational speed ranging from $\omega = 2\pi 20 \ to \ 2\pi 250 \ rad/\sec$. Then obtain estimator gain and estimated disturbance for each value to be the training data. ANFIS technique used training data to obtain membership functions and IF-THEN rules for Neuro-Fuzzy observer. The Neuro Fuzzy observer is combined with both controller and nonlinear plant of magnetic bearing. The results obtained show that we can reject the disturbance at any rotational speed in between each of the twenty-five considered intervals.

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APPENDIX

The ordinary differential equations for nonlinear model with unbalance are mentioned as follow:

$$x1 = x,\quad x2 = \psi,\quad x3 = y,\quad x4 = \theta,\ x5 = z \qquad e = eccentricity \quad epp = inclination\ \ angle$$

$$x6 = \dot{x},\quad x7 = \dot{\psi},\quad x8 = \dot{y},\quad x9 = \dot{\theta},\quad x10 = \dot{z}$$

$$x11 = i_x,\quad x12 = i_\psi,\quad x13 = i_y,\quad x14 = i_\theta,\ x15 = i_z$$

$$\mathrm{dx1 = x6}$$

$$\mathrm{dx2 = x7}$$

$$\mathrm{dx3 = x8}$$

$$\mathrm{dx4 = x9}$$

$$\mathrm{dx5 = x10}$$

$$\begin{aligned}\mathrm{dx6} &= \mathrm{m*e*(2*x9*sin(x2)*cos(x4)*x37 - x7^\wedge 2*cos(x2) + 2*x7*cos(x2)*sin(x4)*x37}\\ &\quad \mathrm{- x37^\wedge 2*cos(x2))*sin(x36) + m*e*(x7^\wedge 2*sin(x2)*sin(x4) - 2*x7*cos(x2)*cos(x4)*x9}\\ &\quad \mathrm{- 2*x7*sin(x2)*x37 + x9^\wedge 2*sin(x2)*sin(x4) + x37^\wedge 2*sin(x2)*sin(x4))*cos(x36) + fxa}\end{aligned}$$

$$
\begin{aligned}
dx7 = {} & (2*x37*sin(epp)*cos(x4)*Iz*x7*cos(epp)*sin(x4)+Iz*sin(epp)*cos(x4)*cos(epp)*x9^2 \\
& +cos(epp)*cos(x4)*Ix*sin(epp)*x37^2-sin(epp)*cos(x4)*Iz*cos(epp)*x37^2 \\
& +(Iz*sin(x4)*x9^2*cos(epp)^2+2*x37*cos(epp)^2*cos(x4)^2*Ix*x7 \\
& +m*e^2*sin(x4)*x9^2-2*x37*cos(x4)^2*Ix*x7+2*x37*cos(x4)^2*Iz*x7 \\
& +sin(x4)*Ix*x9^2-2*x37*cos(x4)^2*Iz*x7*cos(epp)^2-Iz*sin(x4)*x9^2 \\
& -Ix*cos(epp)^2*sin(x4)*x9^2-2*x37*m*e^2*x7*cos(x4)^2)*cos(x36) \\
& -2*x37*cos(epp)*cos(x4)*Ix*x7*sin(epp)*sin(x4) \\
& -Ix*sin(epp)*cos(x4)*cos(epp)*x9^2)*sin(x36) \\
& +(2*x9*Ix*cos(x4)*x7*sin(x4)*cos(epp)^2-2*x9*Iz*cos(epp)^2*cos(x4)*x7*sin(x4) \\
& -2*x9*m*e^2*x7*cos(x4)*sin(x4)-2*x37*cos(x4)*Ix*x9-2*x9*Ix*cos(x4)*x7*sin(x4) \\
& +2*x37*cos(x4)*Iz*x9-2*x9*m*e^2*cos(x4)*x37+2*x37*cos(epp)^2*cos(x4)*Ix*x9 \\
& +2*x9*Iz*sin(x4)*x7*cos(x4)-2*x9*Iz*cos(epp)^2*cos(x4)*x37)*cos(x36)^2 \\
& +(-4*x9*Iz*sin(epp)*cos(x4)^2*x7*cos(epp)+2*x9*Iz*cos(epp)*x7*sin(epp) \\
& +4*x9*Ix*sin(epp)*cos(x4)^2*x7*cos(epp)-2*x9*Ix*cos(epp)*x7*sin(epp))*cos(x36) \\
& -2*x9*Iz*cos(epp)^2*cos(x4)*x7*sin(x4)-x37*cos(x4)*Iz*x9+2*x37*cos(x4)*Ix*x9 \\
& +fxb+2*x9*Iz*cos(epp)^2*cos(x4)*x37+2*x9*m*e^2*cos(x4)*x37 \\
& -2*x37*cos(epp)^2*cos(x4)*Ix*x9+2*x9*Ix*cos(x4)*x7*sin(x4)*cos(epp)^2
\end{aligned}
$$

$$
\begin{aligned}
dx8 = {} & m*e*x9*sin(x4)*x37-m*e*(-cos(x2)*(x7*sin(x4)-x37) \\
& -sin(x2)*cos(x4)*x9)*x37)*sin(x36)+(-m*e*sin(x2)*(x7-sin(x4)*x37)*x37 \\
& +m*e*cos(x4)*x9^2)*cos(x36)+fya
\end{aligned}
$$

$$
\begin{aligned}
dx9 = {} & 2*x37*Ix*x9+2*x37*Iz*x9*cos(epp)^2+2*x9*m*e^2*x37 \\
& -2*x37*cos(epp)^2*Ix*x9-2*x37*Iz*x9)*cos(x36)*sin(x36) \\
& +(x7^2*Iz*cos(epp)^2*cos(x4)*sin(x4)-2*x37*m*e^2*cos(x4)*x7 \\
& +sin(x4)*x7^2*Ix*cos(x4)+2*x37*Iz*cos(x4)*x7-x7^2*Iz*sin(x4)*cos(x4) \\
& -sin(x4)*x7^2*Ix*cos(x4)*cos(epp)^2-2*x37*Ix*cos(x4)*x7 \\
& +m*e^2*x7^2*sin(x4)*cos(x4)-2*x37*Iz*cos(x4)*x7*cos(epp)^2 \\
& +2*x37*cos(epp)^2*Ix*x7*cos(x4))*cos(x36)^2+(cos(epp)*Ix*sin(epp)*x37^2 \\
& +2*x37*sin(epp)*Iz*x7*cos(epp)*sin(x4)+x7^2*Ix*cos(epp)*sin(epp) \\
& -x7^2*Iz*cos(epp)*sin(epp)+2*x7^2*Iz*sin(epp)*cos(x4)^2*cos(epp) \\
& -2*x7^2*Ix*sin(epp)*cos(x4)^2*cos(epp)-2*x37*cos(epp)*Ix*x7*sin(epp)*sin(x4) \\
& -sin(epp)*Iz*cos(epp)*x37^2)*cos(x36)sin(x4)*x7^2*Ix*cos(x4)*cos(epp)^2 \\
& -x37*Iz*cos(x4)*x7+fyb+x7^2*Iz*cos(epp)^2*cos(x4)*sin(x4)
\end{aligned}
$$

$$dx10 = m*e*(-\sin(x2)*(-x7*\sin(x4) + x37) - \cos(x2)*\cos(x4)*x9)*x37$$
$$+ m*e*\sin(x2)*(x7 - \sin(x4)*x37)*x7 + m*e*\cos(x2)*\cos(x4)*x37*x9)*\sin(x36)$$
$$+ (-m*e*(\cos(x2)*(-x7*\sin(x4) + x37) - \sin(x2)*\cos(x4)*x9)*x7$$
$$- m*e*\cos(x2)*(x7 - \sin(x4)*x37)*x37 - m*e*(-\sin(x2)*x7*\cos(x4)$$
$$- \cos(x2)*\sin(x4)*x9)*x9)*\cos(x36) + fz$$

$$dx11 = 2*ki/L*x6 - R/L*x11 + 1/L*uxa + 1/L*uxb$$
$$dx12 = 2*ki/L*a*x7 - R/L*x12 + 1/L*uxa - 1/L*uxb$$
$$dx13 = 2*ki/L*x8 - R/L*x13 + 1/L*uya + 1/L*uyb$$
$$dx14 = 2*ki/L*a*x9 - R/L*x14 + 1/L*uya - 1/L*uyb$$
$$dx15 = 2*ki/L*a*x10 - R/L*x15 + 1/L*uz$$

Table. 1 Data for Numerical Simulation

Parameter	symbol	value
Mass of rotor	m	14.46 kg
Mass moment of inertia in axial direction	I_z	0.0136 kg. m^2
Mass moment of inertia in radial direction	I_x	0.333 kg. m^2
Distance from center to bearing a	a	0.13 m
Distance from center to bearing b	b	0.13 m
Steady state current	i_o	0.38 Ampere
Steady state air gap	S_o	0.55 mm
Resistance of coil	R	10.7 ohm
Number of turns per coil	N	400 turns
Area of magnetic coil	A	1531 mm^2

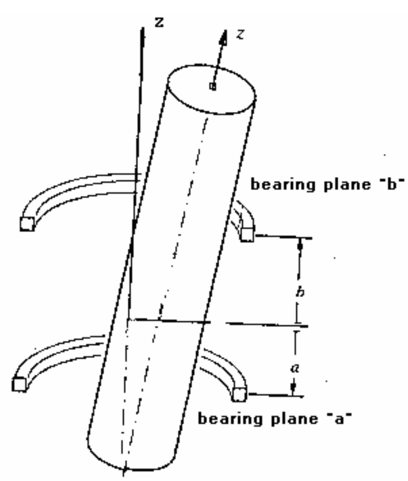


Fig.1. Rigid rotor with two radial bearings

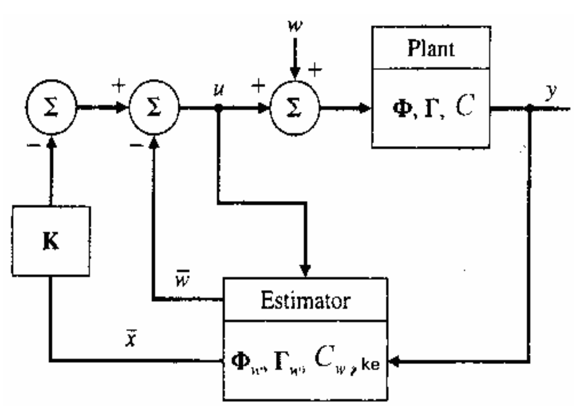
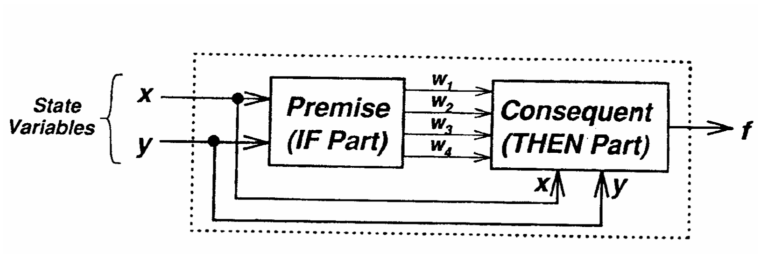
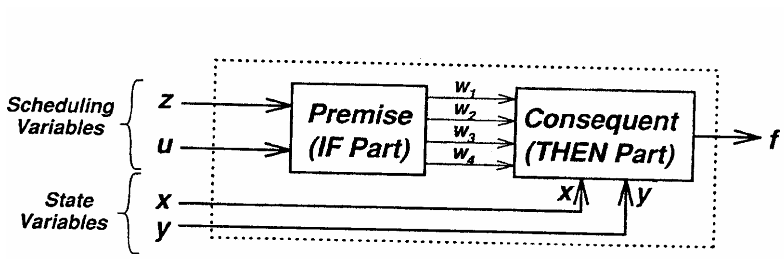


Fig. 2. Block diagram disturbance rejection



(a)



(b)

Fig. 3. Schematic diagram for (a) Sugeno fuzzy controller; (b) Fuzzy gain scheduling controller

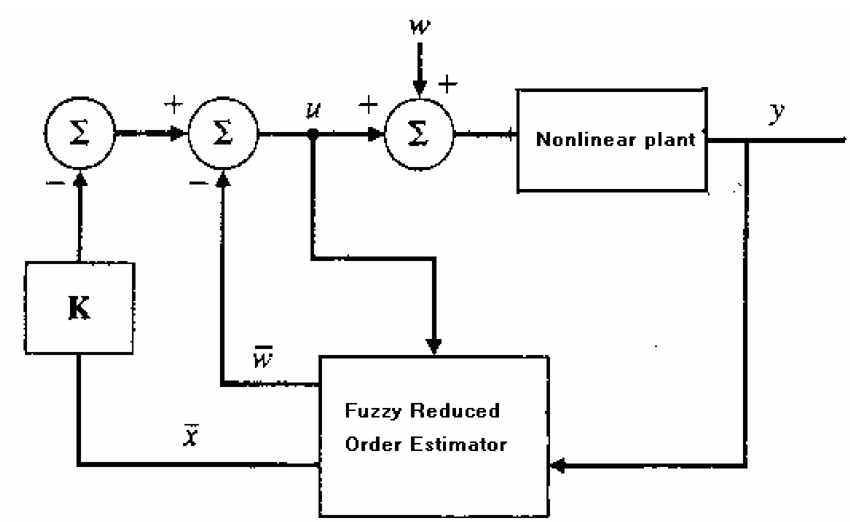


Fig. 4. Block diagram for controlled magnetic bearing system

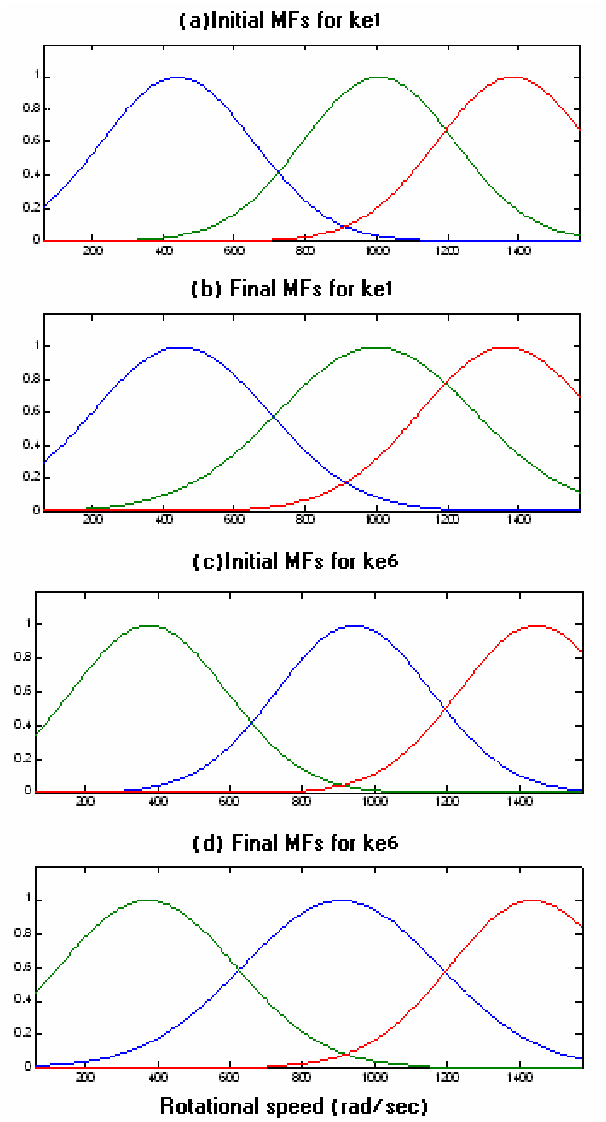


Fig.5 MFs for gain estimator

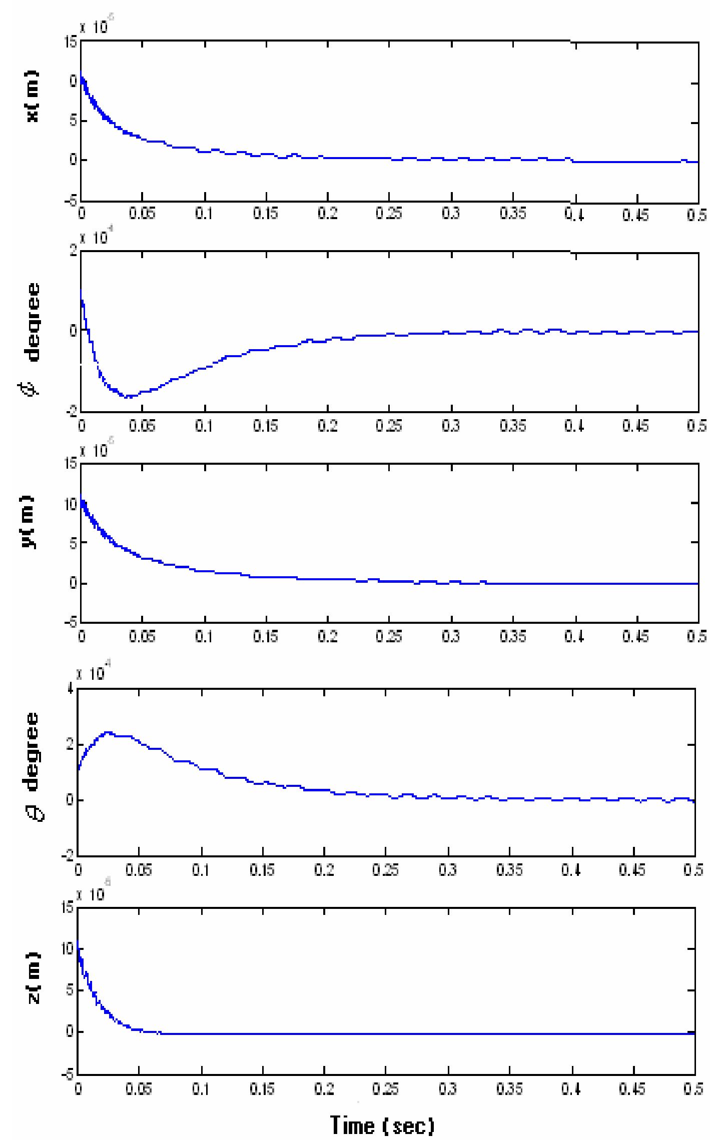


Fig.6 Response due to initial condition for $\omega = 2\pi 60 rad/\sec$

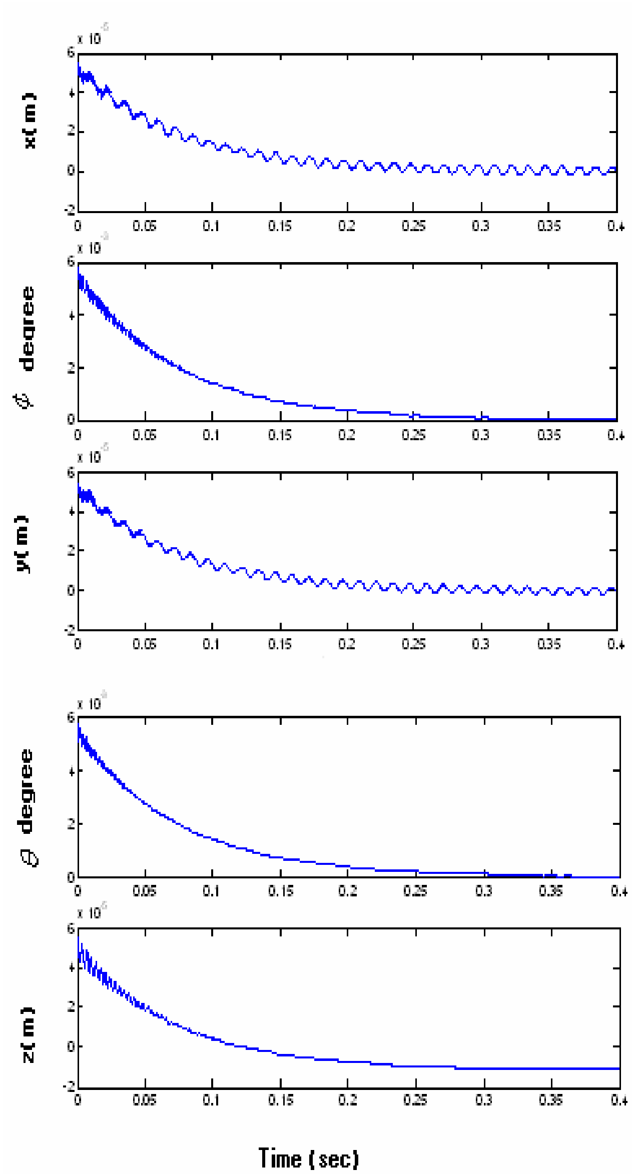


Fig.7 Response due to initial condition for $\omega = 2\pi 120 rad/\sec$

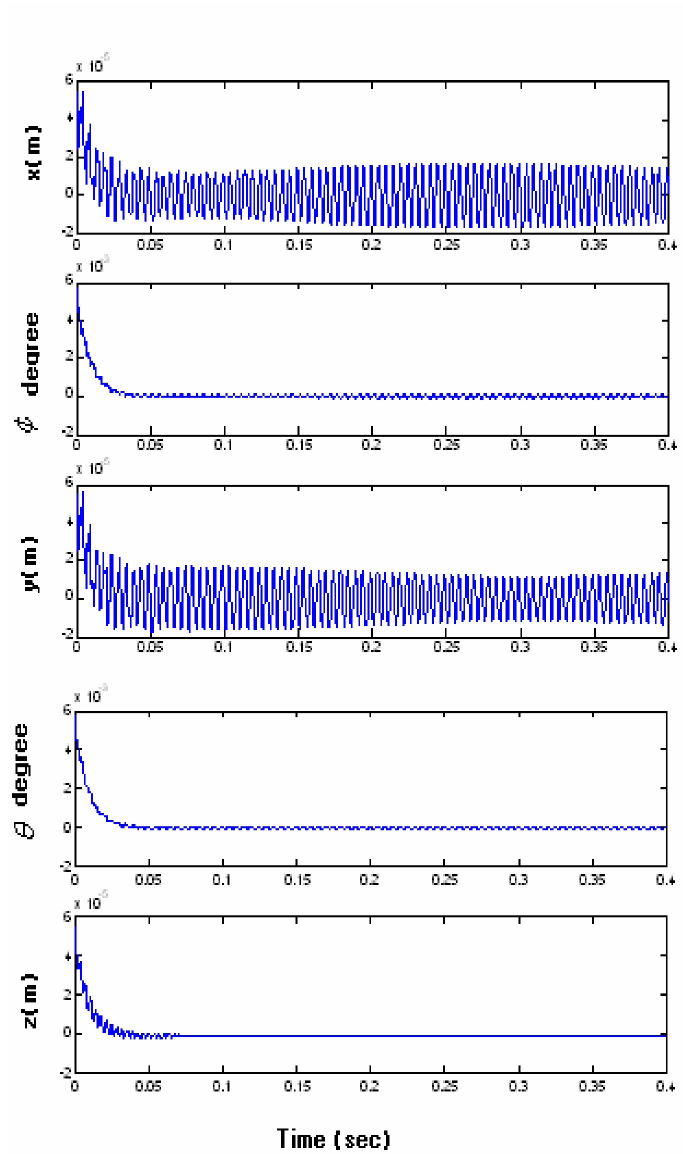


Fig.8 Response due to initial condition for $\omega = 2\pi 200 rad/\sec$