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# Analytical Solution Developed for Optimizing Double-Pipe Flow Systems

*Mohamed M. A. Somaida[1] ( Faculty of Engineering, Port Said University)*


## Abstract *:*

In the present study an equation has been developed to estimate the optimal design diameters of double-pipe flow systems . The equation is based on hydraulic and economic considerations and is applicable in annular flow turbulent regions. The equation is tried on data obtained from an experimental work , conducted on a specified annular flow system . The equation gives good results as illustrated from computations and has revealed that, considerable reductions have been achieved in pipe, pumping and pump costs by the selection of the optimal diameters of the double-flow pipe system


**Keywords** *:* Annular flow, Pipe cost, Pump and pumping costs, Annulus friction factor, equivalent diameter, Optimization, Optimal design-diameter ratio

## 1 . Introduction

The double-pipe flow systems , are known for a **wide range of applications in practice; flushing** purposes well-drilling operations, double-pipe heat exchangers, jacketed pipes and similar equipments. In designing such double-pipe systems, the pipe costs, pump and pumping costs should be taken into account.

### 2. Background

Recent studies about double-pipe flow systems are listed here. Luke, /1/, presented an experimental and a numerical investigation on flow in a short annulus. Polderman, /2/, studied analytically and experimentally the wall friction and velocity profiles of lubrication flow in an annular space with a moving core. Also, Yowakin, /3/, investigated experimentally the swirl flows in a cylindrical annulus. Somaida et al, /4/, studied the frictional data , pressure variations , and friction head losses for inner and annular flow formed between commercial galvanizes steel pipes in laboratory. Also, Somaida, /5, studied the minimum-cost design diameters of a double-pipe flow system, under smooth turbulent flow conditions using a hypothetical annuli. However, the optimization problem of the double-flow pipe system, can be solved using the derivative method to reach the most economical design. In the present study, an analytical solution has been reached and applied. This is being based on the data obtained from the experimental work given in, /4/. The optimum diameter pipe ratios reached, should satisfy minimizing, pipe, pump, pumping and global costs in the double-pipe flow system.

### 3. Cost Functions

In double-flow pipe systems, Fig(1), the most important cost elements are; pipe costs, pumping costs and pump cost, /6/ :

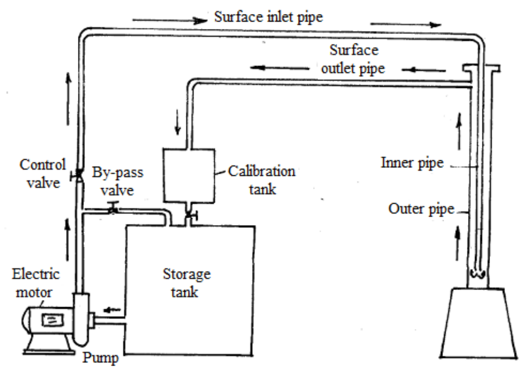


Figure 1: Diagrammatic illustration for apparatus

$$C_{pipes} = A\, L\, ((D^x) + (kd)^x \qquad (1)$$

Where D = inner diameter of outer pipe, d = outer diameter of inner pipe, K = constant <1, expresses the inner diameter in terms of outer diameter of pipe, A = pipe cost coefficient, L = length of each pipe and x = pipe cost exponent

$$C_{pumping} = A_1\, Q\, (H_1 + H_2) \qquad (2)$$

Where Q = pumping rate, $H_1$ = heads lost by friction in inner pipe, $H_2$ = head lost by friction in annulus and $A_1$ = Unit cost of pumping which is given by :

$$A_1 = \frac{w\ p\ N\ Y}{1000\ e} \qquad (3)$$

Where w = specific weight of water, p = power cost/kwh, N = average hours pumping /annum, Y = life period of the scheme and e = efficiency of pump

$$C_{pump} = A_2\, Q\, (H_1 + H_2) \qquad (4)$$

Where A2 = unit pump cost which is given by :

$$A_2 = \frac{w\ cp}{1000\ e} \qquad (5)$$

Where cp = cost of pump/kw. Adding equations (2) and (4), then :

$$C_{pumping} + C_{pump} = (A_1 + A_2 A2)\, Q\, (H_1 + H_2)(6)$$

The term $H_1$ and $H_2$ in equation(6) can be written in terms of flow rate Q and diameters using Darcy's Weisbach equation as follows :

For inner flow, $H_1 = B\,F\,\frac{Q^2}{(K\,d)^5}$

For annulus flow, $H_2 = B\,Fe\,\frac{Q^2}{De^5}$

Where B = constant equal to $(\frac{8\,L}{g\,\pi^2})$ , De = hydraulically equivalent diameter (D-d), /7/, F = coefficient of friction for inner pipe flow and Fe = coefficient of friction for annulus flow. Hence, equation(6) can be written as :

$$C_{pumping} + C_{pump} = (A_1 + A_2)\ B\,Q^3\ (\frac{F}{(K\,d)^5} + \frac{Fe}{(D-d)^5}) \quad (7)$$

Finally, the total cost will be given by :

$$C_{T=}\,A_1\ (D^x + (K\,d)^x) + (A_1 + A_2)\ B\,Q^3(\frac{F}{(k\,d)^5} + \frac{Fe}{(D-d)^5}) \quad (8)$$

## 4. Analytical Formulation of Optimum Diameter Ratio

For minimum cost design of double-pipe system, Differentiate $C_T$ in equation (7) with respect to d and equating to zero :

$$\text{ALxK}\ (Kd)^{x-1} + (\text{A1 + A2})\text{B}\ Q^3((\frac{-5\,K\,F}{(K\,d)^6} + \frac{1}{(Kd)^5}\frac{\partial F}{\partial d}) + (\frac{5\,Fe}{(Dn-d)^6} + \frac{1}{(D-d)^5}\frac{\partial Fe}{\partial d})) = 0 \quad (9)$$

By the same method, differentiate $C_T$ in equation (7) with respect to D and equating to zero:

$$\text{ALx}\ D^{x-1} + (A_1 + A_2)\ \text{B}\ Q^3((\frac{-5\,Fe}{(D-d)^6} + \frac{1}{(D-d)^5}\frac{\partial Fe}{\partial d})) = 0 \quad (10)$$

Combining equations (9) and (10) by subtraction gives :

$$\text{Alx}(\text{K}(\ Kd)^{x-1} - D^{x-1})) + (A_1+A_2)\ \text{B}\ Q^3(\frac{-5\,K\,F}{(Kd)^6} + \frac{1}{(Kd)^6} + \frac{1}{(Kd)^5}\frac{\partial F}{\partial d} + \frac{10\,Fe}{(D-d)^6} + \frac{1}{(D-d)^5}\frac{\partial Fe}{\partial d} - \frac{1}{(D-d)^5} - \frac{1}{(D-d)^5}\frac{\partial Fe}{\partial D} = 0 \quad (11)$$

For inner flow, the pipe friction factor F is dependent upon Reynold's number R and the relative roughness $(\frac{k}{d})$ of pipe. For turbulent flow, The pipe friction factor according to Swamee and Jain, /8/, is given by :

$$\text{F} = \frac{1.325}{(\ln(\frac{1.325}{3.7\,Kd}) + \frac{5.74}{R^{0.9}}\ ))^2} \quad (12)$$

Differentiating equation(12) with respect to d and simplifying, it is reached that, (9), :

$$\frac{\partial F}{\partial d} = -\frac{1.74}{d}\ F^{1.5}\ (\ 1- \frac{5.74\ e^{\frac{1.15}{\sqrt{F}}}}{R^{0.9}}\ ) \quad (13)$$

For annular flow, it is noticed from experiments that Fe can be related to Reynold's number Re in a power law of the following form, (4,7) :

$$\text{Fe} = \frac{a}{(\ Re\ )^b} \quad (14)$$

Where a and b are constants. Differentiating equation(14) with respect to d, yields :

$$\frac{\partial Fe}{\partial d} = -\text{ a b } (Re)^{-b-1}\,\frac{\partial Re}{\partial d} \quad (15)$$

The Reynold's number Re for annular flow is given by :

$$\text{Re} = \frac{4\,Q}{\vartheta\ \pi\ (D+d)} \quad (16)$$

Where $\vartheta$ is the kinematic viscosity of the fluid . Differentiation of equation(16) with respect to d leads to :

$$\frac{\partial Re}{\partial d} = --\frac{4\,Q}{\vartheta\pi\,(D+d)^2} = -\ \frac{Re}{(D+d)} \quad (17)$$

Substitution in equation(15) yields :

$$\frac{\partial Fe}{\partial d} = \frac{a\ b}{(D+d)(Re)^b} \quad (18)$$

To determine $(\frac{\partial Re}{\partial d})$, the same previous mathematical procedure is used and the result is :

$$\frac{\partial Fe}{\partial d} = \frac{a\ b}{(D+d)(Re)^b} \quad (19)$$

Substituting by the partials $(\frac{\partial F}{\partial d})$, $(\frac{\partial Fe}{\partial d})$, and $(\frac{\partial Fe}{\partial D})$ of equations (13), (18) and (19) in equation(11) and simplifying, then :

$$(A_1 + A_2)\ \text{B}\,Q^3 + \frac{10\,Fe}{(D-d)^6} = \text{ALx}\ (D^{x-1} - \text{K}(Kd)^{x-1} + (A_1+A_2)\ \text{B}\ Q^3(\frac{5\,F}{K^5 d^6} + \frac{1.74\,F^{1.5}}{K^5 d^6}\ (\ 1 - \frac{0.574\ e^{\frac{1.15}{\sqrt{F}}}}{R^{0.9}})) \quad (20)$$

Equation(20) can be written in the following form :

$$\frac{10\,Fe}{(D-d)^6} = \frac{ALx\ (D^{x-1} - (Kd)^{x-1}}{(A_1+A_2)BQ^3} + \frac{1}{K^5 d^6}(\ 5\ \text{F} + 1.74\ F^{1.5}\ (1 - \frac{0.574\ e^{\frac{1.15}{\sqrt{F}}}}{R^{0.9}})) \quad (21)$$

More simplification of equation(21) leads to the following form :

$$(\frac{D-d}{d})^6 = (\frac{Alx\ (D^{x-1} - K(Kd)^{x-1}}{10\ Fe\ (A_1+A_2)BQ^3}\ d^6 + \frac{1}{10\ Fe\ K^5}\ (5\ \text{F} + 1.74\ F^{1.5}\ (1- \frac{0.574\ e^{\frac{1.15}{\sqrt{F}}}}{R^{0.9}})\ ))^{-1} \quad (22)$$

The final form of equation(22) will be :

$$(\frac{D}{d}) = 1 + (\frac{ALx(D^{x-1} - K(Kd)^{x-1}}{10\ Fe(A_1+A_2)BQ^3}\ d^6 + \frac{1}{10\ FeK^5}\ (\ 5\ \text{F} + 1.74F^{1.5}\ (1- \frac{0.574\ e^{\frac{1.15}{\sqrt{F}}}}{R^{0.9}})\ ))^{\frac{-1}{6}} \quad (23)$$

In which $(\frac{D}{d})$ is the optimum diameter ratio of the double-pipe flow system. It is noted that, all the parameters of the developed equation are dimensionless and for a single pipe flow, the equation reduces to $(\frac{D}{d}) = 1$, i.e., one-pipe diameter system. Equation(23), can be further simplified and solved as will be shown within the scope here .

## 5. Experimental Work

The frictional data on inner and annular flows collected from experiments, Fig(1),/4/, are employed under various conditions for applying the analytical solution developed. The experimental procedure consists of three tests conducted at diameter ratios ( $\frac{D}{d}$ ) of 1.82, 2.77 and 3.7. Each test consists of 4 runs at flow rates of 1.026, 1.364, 2.222 and 3.077 lit/sec. The utilized pipes are made of commercial galvanized steel.

### 5.1 Inner Flow

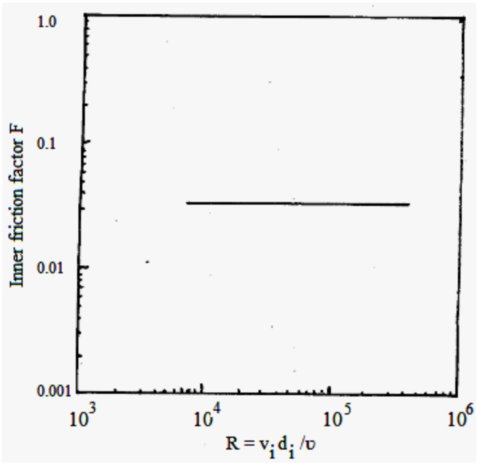

Figure 2: Reynold's number R versus friction factor F for inner flow, /4/

In all the runs one inner pipe is used of 25.4 and 27.94 mm diameters. For this pipe size, the relative roughness $\frac{k}{d}$ = 0.006, (10), the computed Water velocity ($v_i$) varies from 2.026 and 6.075 m/sec. The inner Reynold's number R = ($\frac{vi\ di}{\vartheta}$) ranges from 51435 to 154255 respectively. Fig(2), shows the plot of F calculated according to Swamee versus R. The F-R plot is found to be nearly horizontal and F has a constant value of 0.034 regardless of the flow rate. This means that, F is independent of R, where the inner flow is in the rough turbulent zone.

## 5.2 Annular Flow

In annular flow, the inside diameter of the outer pipe is variable by changing the outer pipe size during the experimental procedure. The annulus Reynold's numbers Re = ($\frac{va\ De}{\vartheta}$) are calculated based on the hydraulically equivalent annular diameter (De = D-d) utilized, which are; 22.86, 48.26 and 37.66 mm. Water velocities in annulus ($v_a$), are estimated and found to range from 0.137 to 2.18 m/sec respectively. The corresponding Re are computed and range from 9600 to 47000, indicating annulus turbulent-flow conditions. Fig(3) shows the plot of Re-Q plot .

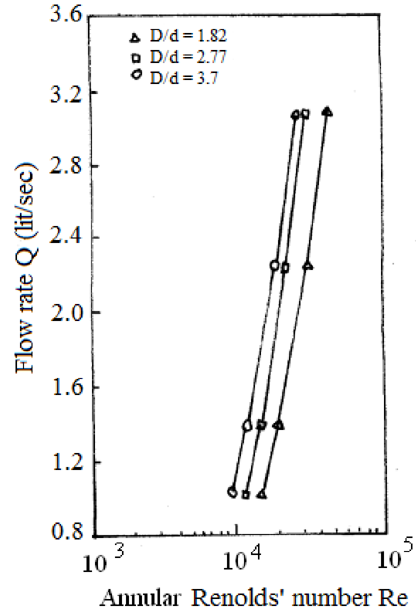

Figure 3: Rate of flow Q versus Reynold's number Re for annular flow, /4/

The plot shows the dependence of Re on rate of flow and the utilized diameter ratio.

**Annulus Friction Factor Fe**

From the experimental data,/4/, and use of the following equation Fe can be estimated :

$$Fe = \frac{hf}{\Delta z}\left(\frac{g\ De}{2\ (va)^2}\right) \qquad (24)$$

where $\frac{hf}{\Delta z}$ = annulus hydraulic gradient ( annulus pressure head drop per unit length of the annulus) and g = gravitational acceleration

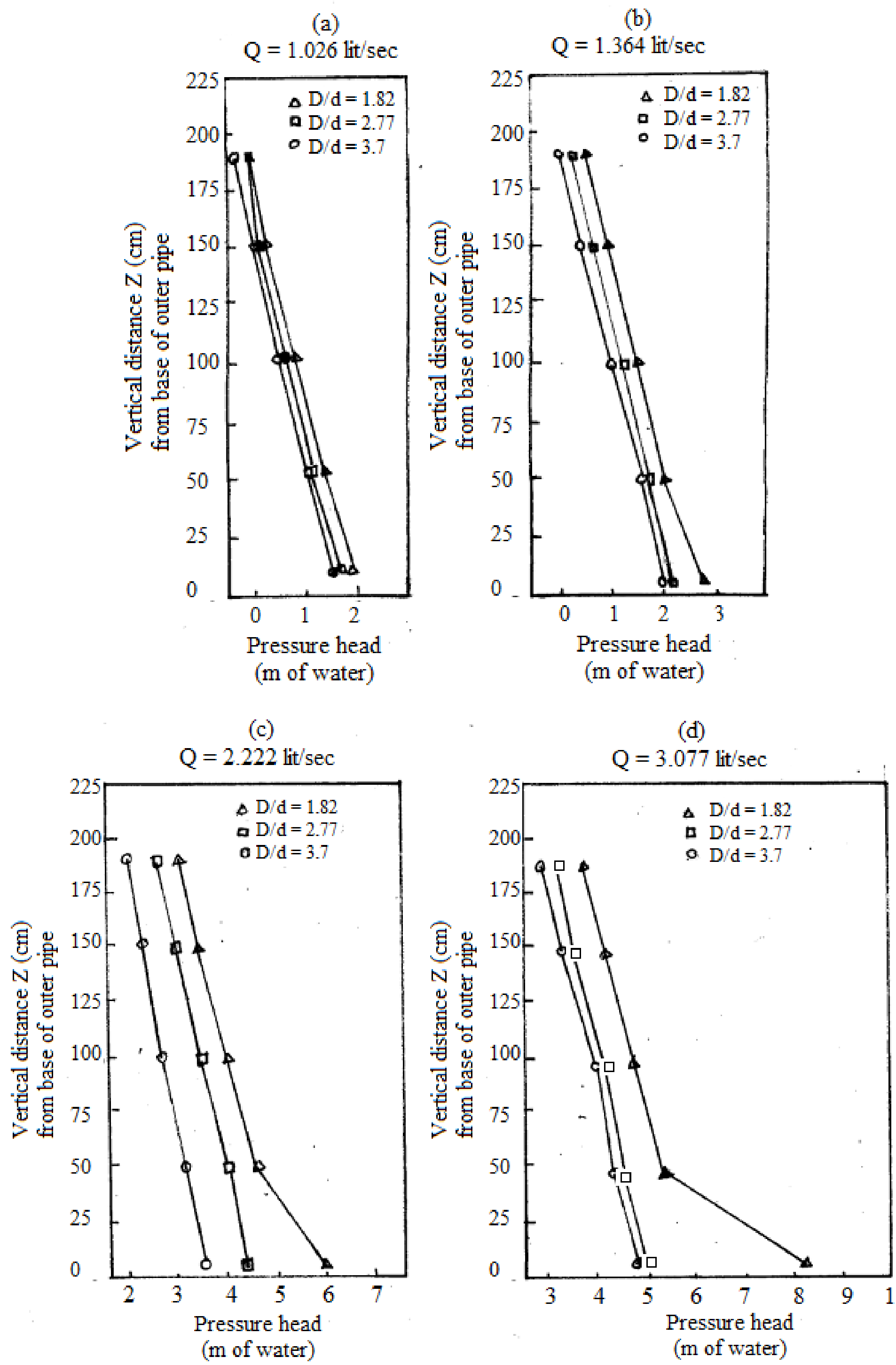


Figure 4: Annular pressure head versus vertical distance from the base of the outer pipe at different flow rates, /4/

At different annular velocities and the use of the annulus experimental pressure profile of the experiment, Fig(4), (values of Fe are determined by the application of equation(24).

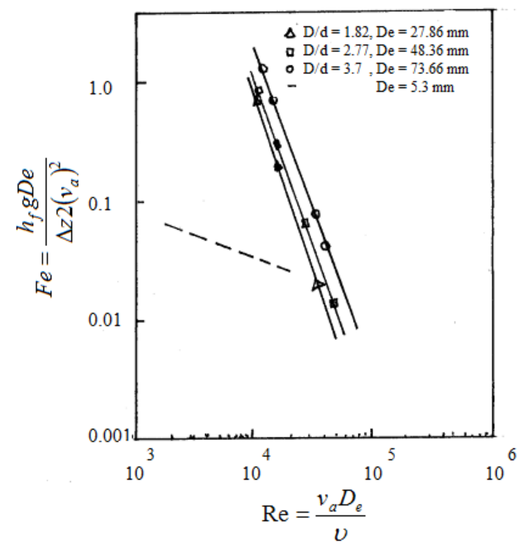


Figure 5: Reynold's number Re versus Friction factor Fe calculated for annular flow at different diameter ratios.

Fig(5) is plotted to show the log-log relation of Re and Fe at different De. This figure indicates the following :

(1)The dependence of Fe on Re and De, since each De has its own plot and Fe may be expressed a logarithmically linear function of Re

(2) At the same value of Re, as De increases, Fe increases, i.e., the resistance to flow increases and the fluid experiences a considerable frictional resistance in annulus by the larger surface areas of pipes

(3) At the same value of De, Fe decreases with increase of Re in an inverse proportionality. This agrees with the Fe-Re plot presented by Carpenter et al, (7), for annular flow at De = 5.3 mm, Fig(4).

(4) The rate of change of Fe with Re in the recent plots is higher than that in the earlier one. This is attributed to the steeper hydraulic gradient, the larger De, the smaller $(v_a)$and the high annular frictional resistance.

## 6. Computational Steps

### 6.1 Evaluation of Parameters

For application of equation(23) for optimal design of double-pipe flow systems, the different parameters in the equation are developed as follows :

(A)The pipe cost coefficient A is taken 370 and the pipe cost exponent x = 1.1, /6/,and K = 0.91.The length of the annular pipe system = 1 m

(B) The unit-cost of pumping $A_1$ is determined using equation(3) for the following data : w = 9810 N/$m^3$, p = 0.09 $ /kwh, N = 365x 8 = 4380 hours/year, Y = 10 years and e = 0.7. However, $A_1$ is found = 36829.5 $/($m^3$/sec)* m

(C)The unit cost of pump $A_2$ is determined using equation(5) for cp =200 $/kw, and is equal to 2802.26 $/($m^3$/sec) * m

(D) The term B = $\frac{8\,L}{\pi 2\, g}$ , is evaluated at L = 1 m and found = 0.0827 $sec^2$, hence $(A_1$+$A_2)$ B = 3277.55

(E) For inner flow, R = 31435 in the first run and 154255 in the last run and F = 0.034 in all runs

(F) For annular flow, Fe is variable and is interpreted by the use of Fig(5). For the first run, $\frac{D}{d}$ = 3.7, d =0.02794 m, D = 0.0508 m, Q = 0.001026 $m^3$/sec, Re = 16602 and Fe = 0.2049. While, in the last run, $\frac{D}{d}$ = 3.7, d = 0.02794 m, D = 0.10338 m, Q = 0.003077 $m^3$/sec and Fe = 0.02.

### 6.2 Simplification of The Developed Analytical Solution

To simplify equation(23), it can be put in the following form :

$\frac{D}{d}$ = 1 +( $\frac{M}{10\, Fe\, K^5}$ + N )$^{\frac{-1}{6}}$ (25) where,

M = (5 F + 1.74 $F^{1.5}$ ( 1- $\frac{0.574\, e^{\frac{1.15}{\sqrt{F}}}}{R^{0.9}}$ )) (26),

and N = $\frac{ALx\,(D^{x-1} - K(Kd)^{x-1} d^6}{10\, Fe\, (A_1+A_2) B\, Q^3}$ (27)

All the parameters in equations(26) and (27) are evaluated previously and are employed to evaluate the quantities M and N, for the first and last runs of the experimental procedure

**Quantity M**

For the first run, F = 0.034 and R = 51435. For the last run, F = 0.034 and R = 154255, then equation(26), gives M = 0.180814 for the first run and = 0.180814 for the last run( nearly equal values). On the average, M may be taken 0.18087. Also with K = 0.91, substitution in equation(25), yields :

$$\frac{D}{d} = 1 + \left(\frac{0.029}{Fe} + N\right)^{\frac{-1}{6}} \quad (28)$$

**Quantity N**

For the first run, Fe = 0.2049, N = 0.003 from equation(27). Application of equation(28) gives $\frac{D}{d}$ = 2.3804. By omitting N, then optimum $\frac{D}{d}$ = 2.3852. The error is - 0.0020124 (negligible).

In the last run, Fe = 0.1468, N = 0.00023 from equation(27). Application of equation(28) gives optimum $\frac{D}{d}$ = 2.3101. By omitting N, then optimum $\frac{D}{d}$ =2.31035. The error is – 0.00011 (negligible). This enhances the omission of N from equation(28), accordingly :

$$\frac{D}{d} = ( 1 + 1.8\, Fe^{\frac{1}{6}} ) \qquad (29)$$

It is evident that :

(A) Equation(29) is the simplest form of the developed analytical solution and can be applied without the need for computer programming

(B) The optimal diameter ratio $\frac{D}{d}$ , is a function only of Fe only regardless of the orientation of the double-pipe flow system

(C) Errors in computation of optimum $\frac{D}{d}$ are negligible

**6. 3 Computational Procedure**

The recommended procedure for calculating the optimal $\frac{D}{d}$ , is as follows :

(A)For each test run, determine Fe by the use of Fig(5) and substitute in equation(29) to determine the optimal value of $\frac{D}{d}$

(B) Compute the total cost $C_T$ for the tested $\frac{D}{d}$ and the optimal $\frac{D}{d}$ by the use of equation(8). For correctness of the solution, $C_T$ for optimal $\frac{D}{d}$ , must be less than that optimized as will be shown in the following example

**6.4 Illustrative Example**

In the first run of the experiment, the tested $\frac{D}{d}$ =1.82, Q = 1.0206 lit/sec, d = 27.94 mm, D = 1.82 x 27.94 = 50.8 mm, De = 22.86 , L = 1 m and the kinematic viscosity of water $\vartheta$ = 0.000006 $m^2$/sec. The results of calculations are listed in Table(1)

Table 1. Calculated parameters of the illustrative example

| Parameter | Test | Optimized |
|---|---|---|
| Va m/sec | 0.72626 | 0.36303 |
| Re | 16602 | 13977 |
| Fe, Fig(5) | 0.2048 | 0.52 |
| $\frac{D}{d}$ | 1.82 | 2.3 |
| Pipes costs $ | 21.015 | 25.9871 |
| ( Pumping +pump ) costs $ | 125.745 | 32.1226 |
| Total cost $C_T$ $ | 146.76 | 58.11 |

The previous results indicate the following :

(A) The total cost estimate according to the optimal diameter ratio is less than the corresponding diameter ratio used in the experiment

(B) The optimized diameter ratio leads to a reduction of 60.4 percent in the total cost

(C) Regardless of the increase of the pipe costs by 4.972 $, achieved by the increase of the optimized diameter ratio, but the total cost is reduced by 88.65 $

## 7. Evaluation of The Optimal Design Equation

### 7.1 Diameter-Ratio Evaluation

**Fe - Optimum ( $\frac{D}{d}$ ) Plots**

The plots of optimum $\frac{D}{d}$ versus annulus friction factor Fe for the different tests are shown in Fig(6). It is noted that, as $\frac{D}{d}$ increases, Fe increases, i.e., the resistance to flow increases because the fluid experiences a considerable frictional resistance by the larger surface area of annulus.

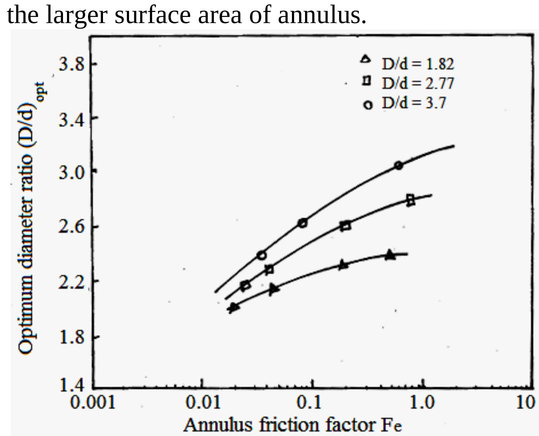


Figure 6: optimal diameter ratio $(D/d)_{opt}$ versus Annular friction factor F

**Re – ($\frac{D}{d}$) Plots**

The plots of optimum $\frac{D}{d}$ versus annulus Reynold' s number Re for the different tests are shown in Fig (7). In all the plots, the Re-optimum $\frac{D}{d}$ relations are represented by segments sloping downward in the direction of increasing Re, indicating that the optimum $\frac{D}{d}$ decreases with increase of Re as the developed analytical solution implies. It is noted that all the plots exhibit nearly similar trends

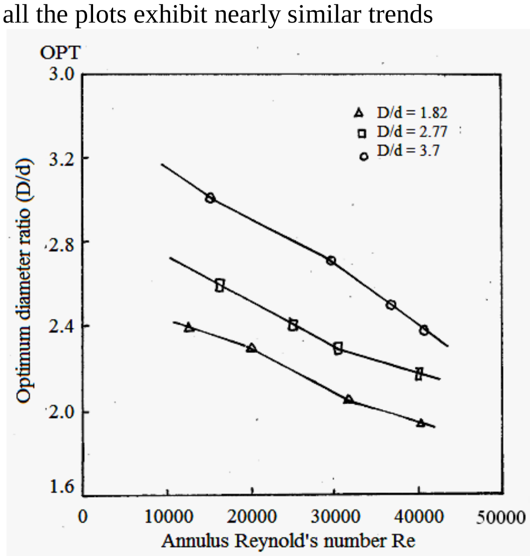


Figure 7: optimal diameter ratio $(D/d)_{opt}$ versus Annular Reynold's number Re

**Q – Optimum ($\frac{D}{d}$) Plots**

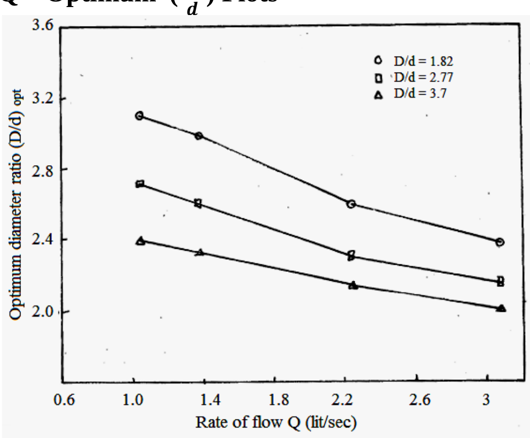


Figure 8: optimal diameter ratio $(D/d)_{opt}$ versus Annular Rate of flow Q.

Fig (8) illustrates the relationship between the optimum $\frac{D}{d}$ and the flow rate Q for the three tests. All the plots show, a decrease in optimum $\frac{D}{d}$ with increase of Q, which are similar to the trends of Re versus optimal $\frac{D}{d}$ in Fig (7), since Re and Q are in accordance. These trends are logic, because in the derivation of the analytical solution , both parameters Q and $\frac{D}{d}$ , are inversely proportional

### 7.2 Cost Evaluation

For cost evaluation , the pipe, pumping , pump and global costs are estimated using the optimal design developed equation at different flow rates for each test at the experimental diameter ratio in the twelve runs of the experimental work. The cost plots in Figs( 9-a,b,c,) show the variations of the different costs with flow rates, noting that the double-pipe system is of one meter length.

**Plot (Fig(9) – a )**

In this plot the experimental $\frac{D}{d}$ = 1.82. It is evident that the Optimal pipe cost is slightly higher than the actual one.. The pumping and pump costs according to optimal $\frac{D}{d}$ are much reduced compared with those according to the utilized $\frac{D}{d}$. In both cases, these costs increase with increase of flow rate, i.e., being lower at lower flow rates and vice versa. Similar trends are found in the total-cost plots regardless of the increase in pipe costs, since, the pipe –cost difference is very slight.

**Plot (Fig(9) – b)**

In this plot the utilized $\frac{D}{d}$ = 2.77. It is indicated that, the optimal and actual pipe costs are of nearer magnitudes at lower flow rates and then get smaller with increase of flow rates. This is attributed to the reduction in the estimated optimum diameter ratios $\frac{D}{d}$. Also, the optimal pumping and pump costs are lower than those at the experimental $\frac{D}{d}$ . It is noted that, the pumping and pump costs are increasing rapidly with increase of flow rate due to the smaller equivalent diameters which lead to considerable annular friction losses regardless of the lower pipe costs. However, the optimal global cost is lower than the actual global cost due to the proper estimation of $\frac{D}{d}$.

**Plot (Fig(9) – c)**

In this plot the experimental $\frac{D}{d}$ = 3.7. It is noted that, the actual pipe cost is higher than the optimal pipe cost but, these are being small compared with the other costs. However, the reduction of the pipe costs has a marked effect on the total costs, since, it compensates the increase of pump and pumping costs over the actual particularly at higher flow rates, Fig(9-c).

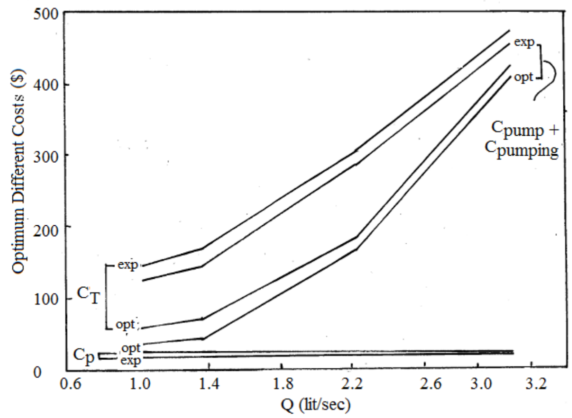


Figure 9-a: optimized different costs versus rate of flow, test 1 at used D/d = 1.82 and L = 1.0m

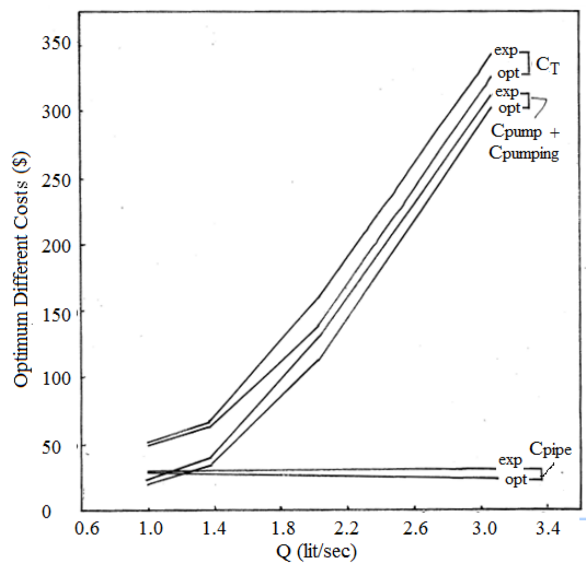


Figure 9-b: optimized different costs versus rate of flow, test 2 at used D/d = 2.77 and L = 1.0m

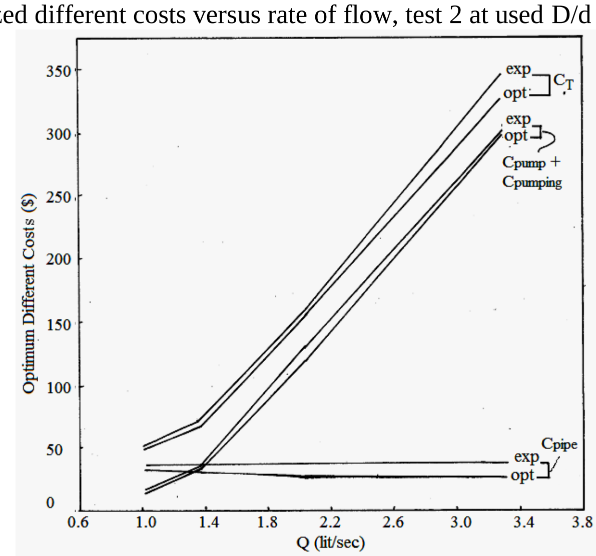


Figure 9-c: optimized different costs versus rate of flow, test 3 at used D/d = 3.7 and L = 1.0m

## 8. Conclusions

According to the previous discussions, the following conclusions are reached :

(A)The analytical solution developed for estimating the optimal design diameter ratio in double-pipe flow systems has a simplest form, since, the optimal diameter ratio depends on the annular fanning factor only. However, it can be easily applied without the need for computer programming.

(B) The developed equation is applicable for turbulent-flow regimes which cover a range of 50000 to 155000 for inner Reynold's number and from 10000 to 50000 for annular Reynold's number.

(C) The developed analytical solution gives a unique value for optimal pipe ratio that minimizes either the pipe costs and or the pumping and pump costs with a marked saving in the global cost. It is evident that, the use of the optimal design diameter ratio reduces the impacts on the fixed costs

(D) It is recommended to conduct further experimental work using diameter ratios less than 1.82 and more than 3.7 for more evaluation of the developed analytical solution.

## 9. Nomenclature

A = pipe-cost coefficient
L = length of pipe
D = inside diameter of outer pipe
d = outer diameter of inner pipe
x = pipe-cost exponent
K = inner diameter/outer diameter, for inner pipe
Q = rate of flow
$H_1$ = head lost by friction in inner pipe
$H_2$ = head lost by friction in annulus
$A_1$ = unit cost of pumping
p = power cost/ kwh
N = average hours pumping/annum
Y = life period of scheme
e = pump efficiency
$A_2$ = unit cost of pump
cp = cost of pump/kw
g = gravitational acceleration
B = coefficient equal to $(\frac{8\,L}{g\,\pi^2})$
F = coefficient of friction for inner pipe
Fe = annulus friction factor
De = equivalent diameter of annulus (D-d)
$C_T$ = Global cost
k = pipe roughness height
$\frac{k}{Kd}$ = relative roughness of inner pipe
R = Reynold's number for inner flow
Re = Reynold's number for annular flow
$\frac{D}{d}$ = diameter ratio in double-flow system
a,b = constants for a specific diameter ratio
$v_i$ = velocity of flow in inner pipe
$v_a$ = velocity of flow in annulus
Z = vertical distance from the base of the outer pipe
$\vartheta$ = kinematic viscosity of water.

## 10. References


/1/ *Luke, m., Mihelcic, M., Wingerath, K., and Pfister, G., 1984 : Flow in a small annulus between concentric cylinders. J. Fluid Mech., Vol. 140, 43-53*

*/2/ Polderman, H.G., Velereeds, G., and Knol., 1986 : Turbulent lubrication in an annular channel. Trans. ASME., J. Fluid Engineers, Vol. 108, No. 2, 185-192*

*/3/ Youwakin, F.M., and Kind, R.J., 1988 : Mean flow and turbulent measurements of annular swirling flows. Trans. J. Fluid Engineers, Vol. 110, No.3, 257-26*

*/4/ Somaida, M.M., and Elotla F., 1991 : A study of water flow through vertical pipe annulus. J. of Engineering and Applied science, Faculty of Eng., Cairo universityVol.38 , No. 4, 691-706*

*/5/ Somaida, M.M., 1991 : Developed equation for minimum cost deign diameters of double-pipe flow systems. J., of Engineering and Applied Science, Faculty of Eng., Cairo University, Vol.38, No. 6, 1081-1094*

*/6/ Somaida, M.M., 1994: Economical design of water-distribution networks supplied from a groundwater source. Engineering Bull., Faculty of Eng. , Suez-Canal University, Vol. 6, No. 1, 16-28*

*/7/ Carpenter, F.G., Colburn, A.P., and Schoenborn, E.M., 1946 : Heat transfer and friction of water in annular space. Trans. Amer. Inst. Chem. Engineers, Vol. 42, 165-178,*

*/8/ Swamee, P.K., and Jain, A.K., 1976 : Explicit equations for pipe-flow problems. ASCE, Vol. 102, No. HY5, 657-664*

*/9/ Somaida, M.M., 1994 : Optimal design of pipe-nozzle line discharging free jets. Engineering Bull., Faculty of Eng., Suez-Canal University, Vol. 6, No. 1, 1-15*

*/10/ Daughtery, R.L., and Franzini, J.B., 1977 : Fluid mechanics with engineering applications. Mc-Graw Hill Inc.*