ICCAE

Military Technical College
Kobry Elkobbah,
Cairo, Egypt

7th International Conference
On Civil & Architecture
Engineering

COMPARISON AMONG DIFFERENT TECHNIQUES OF PREDICTING GEOID UNDULATION

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1. ABSTRACT

The height measurements by Global Positioning System ,GPS, which measures the heights with respect to the reference ellipsoid make confusion with the height of Mean Sea Level (MSL) which determined with respect to geoid. The datum that defines the MSL (also called the geoid) is a complex surface that requires dense and accurate data to define its shape.

In this study, ellipsoidal heights of data points were determined from GPS measurements, with respect to WGS84 reference ellipsoid, and we know accurately the undulation of these points at specific locations inside the cairo city. The objective of this research is to predict the undulation of unknown points by using different prediction techniques.

Four different prediction methods were used to determine geoid undulation. These methods are: inverse distance weighting method, triangulation method, radius search method, and nearest neighbor method.

Comparison among different techniques were applied. Distance as a common variable for all different techniques is used to compare the accuracy for the different resulted values of Geoid undulation. Most results show that the triangulation
prediction method is the best for prediction of Geoid undulation.

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2. INTRODUCTION

The fact that the topographic surface of the earth is highly irregular makes it difficult for the geodetic calculations, for example the determination of the user's location. To overcome this problem, geodesists adopted reference surfaces, to approximate the irregular shape of the earth [1,2].

These reference surfaces are: topographic surface which represents the physical surface of the earth, geoid which defined as a level surface of gravity field with best fit to mean sea level and ellipsoid that represents a mathematical surface approximating the physical reality while simplifying the geometry [3].

The height varies corresponding to variation of reference surface (datum). The Geoid is valuable for oceanographers and maritime industries. Scientist use the Geoid for research purposes such as the study of ' isostatic' Or`Post-glacial rebound'[4]. and it has many practical applications such as:

1.the geoid is the fundamental reference surface for classical height system i.e. the orthometric height of any point is defined relative to the geoid as the distance measured along the plumb line between the point and the geoid [4].

2. the geoid is an intermediate surface for the reduction of the geodetic data from the terrain to the reference ellipsoid.

3. the knowledge of the geoid in any country is essential to the establishment of a geodetic datum, i.e. a certain best fitting ellipsoid for geodetic computations which has a minimum deviation from the geoid.

4. It may be connected with the analysis of dynamics of artificial earth satellite orbit determination.[5].

In general, the basic function of both the geoid and the ellipsoid can be defined as: geoid refers to the orthometric height and ellipsoid refers to the geometric (ellipsoidal) height [6].

GPS as a very useful tool in various scientific life applications has many advantages especially in surveying : Intervisibility between stations is not necessary, the system is independent of weather conditions, GPS provides three-dimensional information, GPS can be used 24 hours a day, position

accuracy is largely a function of interstation distance, and not of network "shape" or "geometry",….,etc. Also GPS has a few disadvantages, the more important one is The GPS height is reduced to an ellipsoid reference, but we have to reduce the heights to mean see level (more precisely to the geoid).

Geoid undulation plays a major role in achieving the best possible accuracy when evaluating orthometric heights on the basis of GPS-derived ellipsoidal heights.

The main task of this work was to compare between the different methods of predicting the geoid undulations of group (2), Unknown points, by using points of group (1), 28 known geoid undulation points. Four different prediction techniques were used and comparison between the results and the actual geoid undulations were done.

3. DESCRIBTION OF USED DATA:

38 different data points were available for this research. These points were established and observed by the surveying consulting unit, faculty of engineering, Ain shams university. Such points are representing the main frame of the control network, established for the purpose of the surveying of new Cairo city. So, the (3-D) curvilinear coordinates of these points are computed using GPS static survey missions. Also, the orthometric heights of these points are observed using spirit leveling technique. Consequently, the geoid undulations at the considered 38 points are now available. See figure (1): shows the location and distribution of these points.

To evaluate the different techniques of geoid undulation prediction, the available 38 points are divided into two groups. The first group, which consists of 28 points, is used as data points, whereas the second group, which consists of the remaining 10 points, is used as check points. Figures (2) and (3) show the distribution of the two groups.

The main idea of this research is that the points of first group is used to predict the geoid undulation at the points of the second group using different prediction techniques. Then, prediction errors at the 10 check points are computed by comparing the actual geoid undulation with the corresponding predicted

undulation, so judgment for the accuracy of the considered prediction technique can be assessed by analyzing such prediction errors.

4. PREDICTION METHODS

There are many prediction methods, four prediction methods were applied, these methods are:

I- Inverse distance weighting method.

The inverse distance weighting prediction technique depends on calculation the undulation as weighted mean of the most nearest surrounding observed (computed) undulations Here ,the weights are taken inversely proportional to the distance between the prediction point and the data points.

$$N = \frac{\sum_{i=1}^{n} \frac{Ni}{d_i^n}}{\sum_{i=1}^{n} \frac{1}{d_i^n}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (1)$$

N :geoid undulation d : distance between predicted point and data point
n : power of weight [7]. Predicted geoid undulations by using inverse distance weighting method were calculated and tabulated in table (1).

II- Nearest neighbor method

In this method we usually adopt the value of undulation of the nearest data point. (value of prediction = undulation of the nearest data point).See table(2).

III-Triangulation method

As a special case from the inverse distance weighting prediction technique, which is called the prediction using triangulation , is applied for the most nearest three known undulation points , with the condition that the prediction point is lying inside the triangle formed by the three data points. The resulted undulation were tabulated in tables: 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

IV- Radius search method

In general, this method is usually applied using the data point undulations that lying inside a specific radius around the prediction points as shown in tables 13,14, 15, 16 [7].

Analysis of results:

The main task of this work is to predict the geoid undulations based on known geoid undulation points, Different prediction techniques were used and comparison between the results and the actual geoid undulations was carried out.

In inverse distance weighting see table (1), all points in group (1) were used to predict each unknown point. The relative position and point distribution effect strongly the accuracy of predicted Geoid undulation.

In nearest neighbor prediction method was adopted the nearest value of Geoid undulation, the variation of Geoid undulation in surrounding area plays strong rule for accuracy of prediction see table (2).

The nearest three vertices of triangle which surround the predicted point were chosen in triangulation prediction method. The position of the predicted point inside the triangle play important rule for required accuracy see tables (3, 4, 5, 6, 7, 8, 9, 10, 11, 12).

For radius search prediction, different radius was applied to predict the Geoid undulation see tables (13, 14, 15, 16.). It is noted that the error of predicting decreases with the increasing the power of the weight as illustrated in tables (13,14,15,16) and it increases with the increasing of distances.

To compare the accuracy for the different resulted values of Geoid undulation, the biggest distance between the predict point and the vertices of triangle (which was chosen for triangulation method) was applied as a value of radius to predict the Geoid undulation for each predict point, and compare the results with other techniques see tables (17, 18, 19, 20, 21, 22, 23, 24, 25, 26), and see figures (4, 5, 6, 7, 8, 9, 10, 11, 12, 13.)

Most results show that the triangulation prediction method is the best for prediction of Geoid undulation.

6. CONCLUSION

Geoid undulation prediction is based on surrounding known Geoid undulation points. The distribution of these known points is not necessarily uniform. To determine the Geoid undulation, by using mathematical prediction models, the accuracy depends on many factors:

- Distribution of the undulation data and relative position of the predicted point with respect to the known undulation points.
- The distances between predicted point and known undulation points.
- Adopted mathematical prediction technique.

The power of the weight does not effect the accuracy of predicted undulation by using one data point undulation.

For large values of radius search, the accuracy increases with increasing power.

In general triangulation method is the best technique for predicting geoid undulation.

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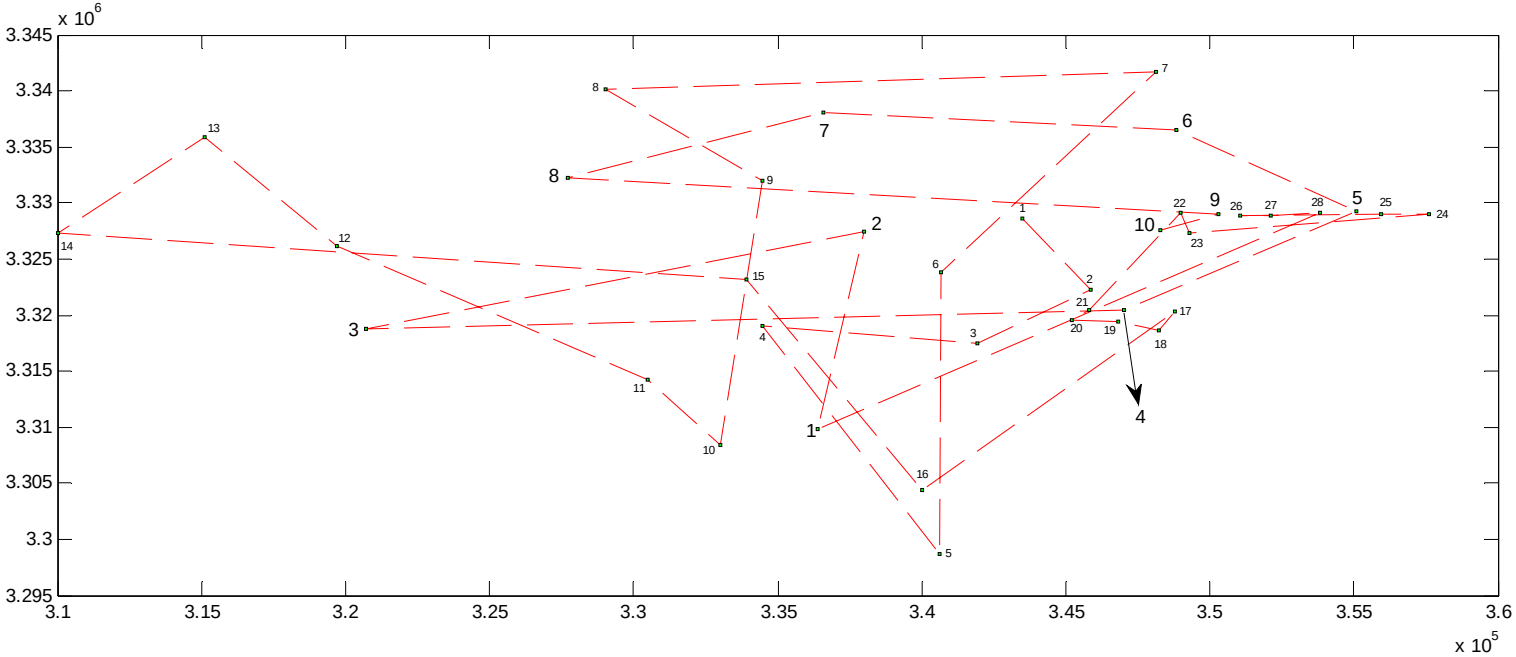

Figure (1): distribution of 38 points.

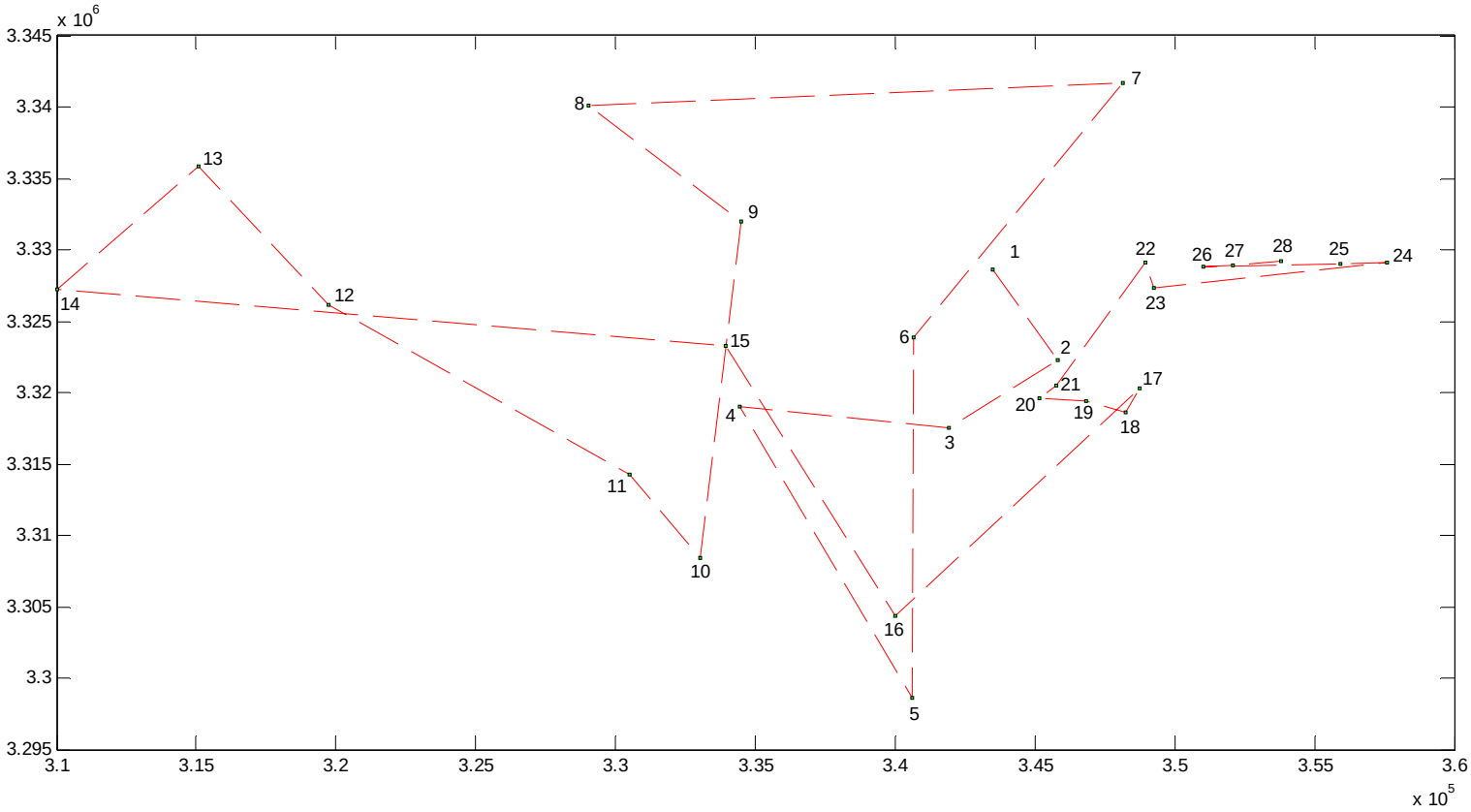

Figure (2): Distribution of data points.

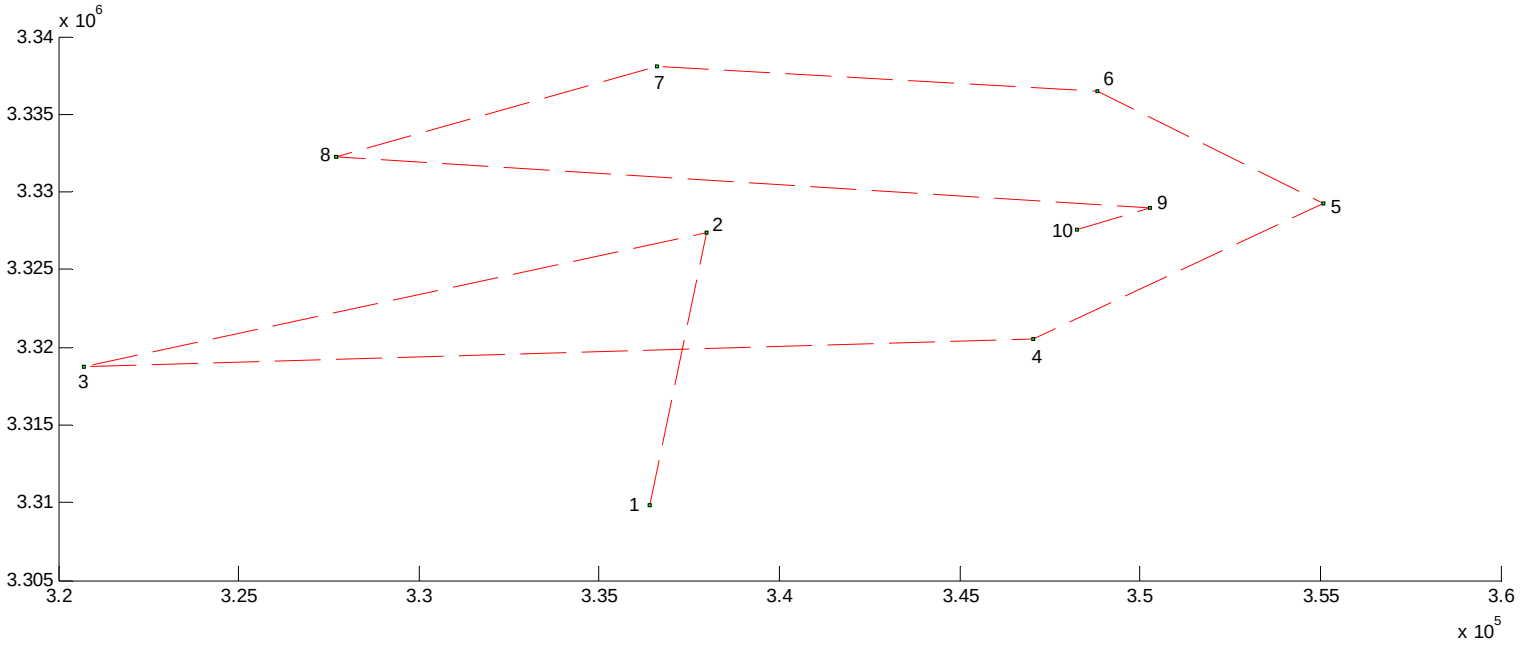

Figure (3): Distribution of check points.

IDW-01 TABLE (1): INVERSE DISTANCE WEIGHTING FOR PREDICTING UNDULATION

NO	TN	RS-P	TN1	TN2	TN3	TN4	TN8	ERROR1	ERROR2	ERROR3	ERROR4	ERROR8
1	37.886	28	37.890	37.829	37.763	37.722	37.687	0.004	0.057	0.123	0.164	0.199
2	37.814	28	37.918	37.903	37.878	37.857	37.830	0.104	0.089	0.064	0.043	0.016
3	37.502	28	37.838	37.754	37.676	37.619	37.543	0.337	0.252	0.174	0.117	0.041
4	38.108	28	38.039	38.084	38.094	38.098	38.107	0.069	0.024	0.014	0.010	0.001
5	38.162	28	38.072	38.131	38.139	38.140	38.142	0.090	0.031	0.023	0.022	0.020
6	37.908	28	37.974	38.007	38.013	38.007	37.971	0.066	0.099	0.105	0.099	0.063
7	37.668	28	37.885	37.831	37.763	37.709	37.671	0.217	0.163	0.095	0.041	0.003
8	37.433	28	37.842	37.764	37.700	37.664	37.657	0.409	0.331	0.267	0.231	0.224
9	38.450	28	38.034	38.064	38.068	38.071	38.075	0.416	0.386	0.382	0.379	0.375
10	38.036	28	38.014	38.047	38.053	38.056	38.061	0.022	0.011	0.017	0.020	0.025

Where: TN: Actual geoid undulation of prediction point.

RS-P: Number of points which are entered in calculation of prediction.

TN1, TN2, TN3, TN4, TN8 : Are values of prediction corresponding to values of power(1, 2, 3, 4, 8) consequently.

ERR1, ERR2, ERR3, ERR4, ERR8: The errors of prediction corresponding to values of power(1, 2, 3, 4, 8) consequently.

TABLE (2): NEAREST NEIGHBOR PREDICTION

NO	TN	TN1	ERROR1	P
1	37.886	37.685	0.201	10
2	37.814	37.823	0.009	6
3	37.502	37.531	0.029	12
4	38.108	38.123	0.015	19
5	38.162	38.143	0.019	25
6	37.908	37.958	0.050	7
7	37.668	37.709	0.041	9
8	37.433	37.709	0.276	9
9	38.450	38.076	0.374	26
10	38.036	38.062	0.025	23

P: Nomer the point which adopted as NN.

TRI-011 TABLE (3): TRIANGULATION PREDICTION METHOD FOR CHECK POINT (1).

NO	TN	TRI-P	D	N	M.N	ERR-M.N	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8
		4	9451.325	37.884												
1	37.886	10	3632.743	37.685	37.828	0.058	37.790	37.754	37.726	37.708	37.687	0.096	0.132	0.160	0.178	0.199
		16	6552.069	37.915												

TRI-012 TABLE (4): TRIANGULATION PREDICTION METHOD FOR CHECK POINT (2).

NO	TN	TRI-P	D	N	M.N	ERR-M.N	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8
		1	5661.244	37.910												
2	37.814	9	5791.230	37.709	37.840	0.026	37.840	37.841	37.841	37.841	37.843	0.026	0.027	0.027	0.027	0.029
		15	5826.779	37.900												

Where: TN: Actual geoid undulation of prediction point.
TRI-P: Nomers of points which are chosen as triangle.
D: The distance between predict point and vertices of triangle.
N: Actual geoid undulation of vertices of triangle.
TN1, TN2, TN3, TN4, TN8 : Are values of prediction corresponding to values of power(1, 2, 3, 4, 8) consequently.
ERR1, ERR2, ERR3, ERR4, ERR8: The errors of prediction corresponding to values of power (1, 2, 3, 4, 8) consequently.

TRI-013 TABLE (5): TRIANGULATION PREDICTION METHOD FOR CHECK POINT (3).

NO	TN	TRI-P	D	N	M.N	ERR-M.N	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8
		10	16081.323	37.685												
3	37.502	12	7468.421	37.531	37.556	0.054	37.545	37.537	37.533	37.531	37.531	0.043	0.035	0.031	0.029	0.029
		14	13658.863	37.451												

TRI-014 TABLE (6): TRIANGULATION PREDICTION METHOD FOR CHECK POINT (4).

NO	TN	TRI-P	D	N	M.N	ERR-M.N	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8
		2	2146.711	37.994												
4	38.108	17	1745.117	38.141	38.088	0.02	38.098	38.107	38.114	38.118	38.123	0.010	0.001	0.006	0.010	0.015
		19	1080.380	38.128												

Where: TN: Actual geoid undulation of prediction point.
TRI-P: Nomers of points which are chosen as triangle.
D: The distance between predict point and vertices of triangle.
N: Actual geoid undulation of vertices of triangle.
TN1, TN2, TN3, TN4, TN8 : Are values of prediction corresponding to values of power(1, 2, 3, 4, 8) consequently.
ERR1, ERR2, ERR3, ERR4, ERR8: The errors of prediction corresponding to values of power (1, 2, 3, 4, 8) consequently.

TRI-015 TABLE (7): TRIANGULATION PREDICTION METHOD FOR CHECK POINT (5)

NO	TN	TRI-P	D	N	M.N	ERR-M.N	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8
		23	6087.425	38.062												
5	38.162	25	877.676	38.143	38.111	0.051	38.131	38.137	38.139	38.140	38.142	0.031	0.025	0.023	0.022	0.019
		28	1242.739	38.128												

TRI-016 TABLE (8): TRIANGULATION PREDICTION METHOD FOR CHECK POINT (6)

NO	TN	TRI-P	D	N	M.N	ERR-M.N	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8
		7	5192.185	37.958												
6	37.908	22	7416.137	38.015	38.016	0.108	38.008	37.999	37.990	37.983	37.965	0.100	0.091	0.082	0.075	0.057
		26	7970.870	38.076												

Where: TN: Actual geoid undulation of prediction point.
TRI-P: Nomers of points which are chosen as triangle.
D: The distance between predict point and vertices of triangle.
N: Actual geoid undulation of vertices of triangle.
TN1, TN2, TN3, TN4, TN8 : Are values of prediction corresponding to values of power(1, 2, 3, 4, 8) consequently.
ERR1, ERR2, ERR3, ERR4, ERR8: The errors of prediction corresponding to values of power (1, 2, 3, 4, 8) consequently.

TRI-017 TABLE (9): TRIANGULATION PREDICTION METHOD FOR CHECK POINT (7)

NO	TN	TRI-P	D	N	M.N	ERR-M.N	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8
		7	12069.345	37.958												
7	37.668	8	7821.872	37.462	37.710	0.042	37.679	37.660	37.651	37.649	37.668	0.011	0.008	0.017	0.019	0.000
		9	6428.701	37.709												

TRI-018 TABLE (10): TRIANGULATION PREDICTION METHOD FOR CHECK POINT (8)

NO	TN	TRI-P	D	N	M.N	ERR-M.N	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8
		8	7985.026	37.462												
8	37.433	9	6780.513	37.709	37.567	0.134	37.578	37.590	37.602	37.613	37.652	0.145	0.157	0.169	0.180	0.219
		12	10057.295	37.531												

Where: TN: Actual geoid undulation of prediction point.
TRI-P: Nomers of points which are chosen as triangle.
D: The distance between predict point and vertices of triangle.
N: Actual geoid undulation of vertices of triangle.
TN1, TN2, TN3, TN4, TN8 : Are values of prediction corresponding to values of power(1, 2, 3, 4, 8) consequently.
ERR1, ERR2, ERR3, ERR4, ERR8: The errors of prediction corresponding to values of power (1, 2, 3, 4, 8) consequently.

TRI-019 TABLE (11): TRIANGULATION PREDICTION METHOD FOR CHECK POINT (9)

NO	TN	TRI-P	D	N	M.N	ERR-M.N	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8
9	38.450	22 23 28	1322.525 1868.533 3552.852	38.015 38.062 38.128	38.068	0.382	38.051	38.039	38.031	38.026	38.018	0.399	0.411	0.419	0.424	0.432

TRI-0110 TABLE (12): TRIANGULATION PREDICTION METHOD FOR CHECK POINT (10)

NO	TN	TRI-P	D	N	M.N	ERR-M.N	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8
10	38.036	2 22 23	5819.543 16089.016 1034.604	37.994 38.015 38.062	38.024	0.012	38.039	38.048	38.053	38.056	38.061	0.003	0.012	0.017	0.020	0.025

Where: TN: Actual geoid undulation of prediction point.
TRI-P: Nomers of points which are chosen as triangle.
D: The distance between predict point and vertices of triangle.
N: Actual geoid undulation of vertices of triangle.
TN1, TN2, TN3, TN4, TN8 : Are values of prediction corresponding to values of power(1, 2, 3, 4, 8) consequently.
ERR1, ERR2, ERR3, ERR4, ERR8: The errors of prediction corresponding to values of power (1, 2, 3, 4, 8) consequently.

RS1-1000 TABLE (13): RADIUS SEARCH PREDICTION METHOD Radius for value = 1000 m

NO	TN	RS1.P	TN1	TN2	TN3	TN4	TN8	ERROR1	ERROR2	ERROR3	ERROR4	ERROR8
1	37.886	0	0	0	0	0	0	0	0	0	0	0
2	37.814	0	0	0	0	0	0	0	0	0	0	0
3	37.502	0	0	0	0	0	0	0	0	0	0	0
4	38.108	0	0	0	0	0	0	0	0	0	0	0
5	38.162	1	38.143	38.143	38.143	38.143	38.143	0.019	0.019	0.019	0.019	0.019
6	37.908	0	0	0	0	0	0	0	0	0	0	0
7	37.668	0	0	0	0	0	0	0	0	0	0	0
8	37.433	0	0	0	0	0	0	0	0	0	0	0
9	38.450	1	38.076	38.076	38.076	38.076	38.076	0.374	0.374	0.374	0.374	0.374
10	38.036	0	0	0	0	0	0	0	0	0	0	0

RS1-2500 TABLE (14): RADIUS SEARCH PREDICTION METHOD Radius for value = 2500 m

NO	TN	RS1-P	TN1	TN2	TN3	TN4	TN8	ERR0R1	ERROR2	ERROR3	ERROR4	ERROR8
1	37.886	0	0	0	0	0	0	0	0	0	0	0
2	37.814	0	0	0	0	0	0	0	0	0	0	0
3	37.502	0	0	0	0	0	0	0	0	0	0	0
4	38.108	6	38.088	38.092	38.095	38.098	38.107	0.020	0.016	0.013	0.010	0.001
5	38.162	2	38.137	38.138	38.139	38.140	38.142	0.025	0.024	0.023	0.022	0.020
6	37.908	0	0	0	0	0	0	0	0	0	0	0
7	37.668	0	0	0	0	0	0	0	0	0	0	0
8	37.433	0	0	0	0	0	0	0	0	0	0	0
9	38.450	4	38.062	38.065	38.068	38.071	38.075	0.388	0.385	0.382	0.379	0.375
10	38.036	2	38.044	38.049	38.053	38.056	38.061	0.008	0.013	0.017	0.020	0.025

RS1-10000 TABLE (15): RADIUS SEARCH PREDICTION METHOD Radius for value = 10000 m

NO	TN	RS-P	TN1	TN2	TN3	TN4	TN8	ERROR1	ERROR2	ERROR3	ERROR4	ERROR8
1	37.886	5	37.800	37.764	37.733	37.712	37.687	0.086	0.122	0.153	0.174	0.199
2	37.814	6	37.858	37.849	37.842	37.838	37.829	0.044	0.035	0.028	0.024	0.015
3	37.502	1	37.531	37.531	37.531	37.531	37.531	0.029	0.029	0.029	0.029	0.029
4	38.108	13	38.070	38.087	38.094	38.098	38.107	0.038	0.021	0.014	0.010	0.001
5	38.162	7	38.124	38.136	38.139	38.140	38.142	0.038	0.026	0.023	0.022	0.020
6	37.908	7	38.030	38.022	38.013	38.002	37.970	0.122	0.114	0.105	0.094	0.062
7	37.668	2	37.598	37.609	37.621	37.632	37.666	0.070	0.059	0.047	0.036	0.002
8	37.433	2	37.596	37.606	37.615	37.625	37.656	0.163	0.173	0.182	0.192	0.223
9	38.450	11	38.069	38.067	38.068	38.071	38.075	0.381	0.383	0.382	0.379	0.375
10	38.036	15	38.055	38.052	38.054	38.056	38.061	0.019	0.016	0.018	0.020	0.025

RS1-40000 TABLE (16): RADIUS SEARCH PREDICTION METHOD Radius for value = 40000 m

NO	TN	RS-P	TN1	TN2	TN3	TN4	TN8	ERROR1	ERROR2	ERROR3	ERROR4	ERROR8
1	37.886	28	37.890	37.829	37.763	37.722	37.687	0.004	0.057	0.123	0.164	0.199
2	37.814	28	37.918	37.903	37.878	37.857	37.830	0.104	0.089	0.064	0.043	0.016
3	37.502	28	37.838	37.754	37.676	37.619	37.543	0.336	0.252	0.174	0.117	0.041
4	38.108	28	38.039	38.084	38.094	38.098	38.107	0.069	0.024	0.014	0.010	0.001
5	38.162	26	38.078	38.131	38.139	38.140	38.142	0.084	0.031	0.023	0.022	0.020
6	37.908	28	37.974	38.007	38.013	38.007	37.971	0.066	0.099	0.105	0.099	0.063
7	37.668	28	37.885	37.831	37.763	37.709	37.671	0.217	0.163	0.095	0.041	0.003
8	37.433	28	37.842	37.764	37.700	37.664	37.657	0.409	0.331	0.267	0.231	0.224
9	38.450	27	38.036	38.064	38.068	38.071	38.075	0.414	0.386	0.382	0.379	0.375
10	38.036	28	38.014	38.047	38.053	38.056	38.061	0.022	0.011	0.017	0.020	0.025

TABLE (17): ***Comparison among different techniques*** for point (1).

NO	TN	MODEL	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8	POINTS
1	37.886	IDW	37.890	37.829	37.763	37.722	37.687	0.004	0.057	0.123	0.164	0.199	28points
1		TRI	37.790	37.754	37.726	37.708	37.687	0.096	0.132	0.160	0.178	0.199	4,10,16
1	R=9451.33	RS	37.768	37.743	37.722	37.707	37.687	0.118	0.143	0.164	0.179	0.199	4,10,11,16
1		NN	37.685					0.201					

TABLE (18): ***Comparison among different techniques*** for point (2).

NO	TN	MODEL	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8	POINTS
2	37.814	IDW	37.918	37.903	37.878	37.857	37.830	0.104	0.089	0.064	0.043	0.016	28points
2		TRI	37.840	37.841	37.841	37.841	37.843	0.026	0.027	0.027	0.027	0.029	1,9,15
2	R=5827	RS	37.835	37.834	37.833	37.833	37.829	0.021	0.020	0.019	0.019	0.015	1,6,9,15
2		NN	37.823					0.009					

IDW: Inverse distance weighting.
TRI: Triangulation method.
RS: Radius search method.
NN: Nearest neighbor method.
R: value of radius (The biggest distance between predicted point and vertices of triangle which was used in triangulation method).
POINTS: Points which are entered in prediction calculation.

TABLE (19): ***Comparison among different techniques*** for point (3).

NO	TN	MODEL	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8	POINTS
3	37.502	IDW	37.838	37.754	37.676	37.619	37.543	0.337	0.252	0.174	0.117	0.041	28points
3		TRI	37.545	37.537	37.533	37.531	37.531	0.043	0.035	0.031	0.029	0.029	10,12,14
3	R=16082	RS	37.656	37.638	37.616	37.593	37.543	0.154	0.136	0.114	0.091	0.041	10,12,14,4,11,15
3		NN	37.531					0.029					

TABLE (20): ***Comparison among different techniques*** for point (4).

NO	TN	MODEL	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8	POINTS
4	38.108	IDW	38.039	38.084	38.094	38.098	38.107	0.069	0.024	0.014	0.010	0.001	28points
4		TRI	38.098	38.107	38.114	38.118	38.123	0.010	0.001	0.006	0.010	0.015	2,17,19
4	R=2147	RS	38.079	38.086	38.092	38.096	38.107	0.029	0.022	0.016	0.012	0.001	2,17,19,20,21
4		NN	38.123					0.015					

IDW: Inverse distance weighting.
TRI: Triangulation method.
RS: Radius search method.
NN: Nearest neighbor method
R: value of radius (The biggest distance between predicted point and vertices of triangle which was used in triangulation method).
POINTS: Points which are entered in prediction calculation.

TABLE (21): ***Comparison among different techniques*** for point (5).

NO	TN	MODEL	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8	POINTS
5	38.162	IDW	38.072	38.131	38.139	38.140	38.142	0.090	0.031	0.023	0.022	0.020	28points
5		TRI	38.131	38.137	38.139	38.140	38.142	0.031	0.025	0.023	0.022	0.019	23,25,28
5	R=6088	RS	38.130	38.137	38.139	38.140	38.142	0.032	0.025	0.023	0.022	0.020	23,24,25,26,27,28
5		NN	38.143					0.019					

TABLE (22): ***Comparison among different techniques*** for point (6).

NO	TN	MODEL	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8	POINTS
6	37.908	IDW	37.974	38.007	38.013	38.007	37.971	0.066	0.099	0.105	0.099	0.063	28points
6		TRI	38.008	37.999	37.990	37.983	37.965	0.100	0.091	0.082	0.075	0.057	7,22,26
6	R=7971	RS	38.008	37.999	37.990	37.983	37.965	0.100	0.091	0.082	0.075	0.057	7,22,26
6		NN	37.958					0.050					

IDW: Inverse distance weighting.
TRI: Triangulation method.
RS: Radius search method.
NN: Nearest neighbor method
R: value of radius (The biggest distance between predicted point and vertices of triangle which was used in triangulation method).
POINTS: Points which are entered in prediction calculation.

TABLE (23): ***Comparison among different techniques*** for point(7).

NO	TN	MODEL	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8	POINTS
7	37.668	IDW	37.885	37.831	37.763	37.709	37.671	0.217	0.163	0.095	0.041	0.003	28points
7		TRI	37.679	37.660	37.651	37.649	37.668	0.011	0.008	0.017	0.019	0.000	7,8,9
7	R=12069.35	RS	37.723	37.693	37.674	37.663	37.670	0.055	0.025	0.006	0.005	0.002	1,7,8,9
7		NN	37.709					0.041					

TABLE (24): ***Comparison among different techniques*** for point (8).

NO	TN	MODEL	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8	POINTS
8	37.433	IDW	37.842	37.764	37.700	37.664	37.657	0.409	0.331	0.267	0.231	0.224	28points
8		TRI	37.578	37.590	37.602	37.613	37.652	0.145	0.157	0.169	0.180	0.219	8,9,12
8	R=10058	RS	37.578	37.590	37.602	37.613	37.652	0.145	0.157	0.169	0.180	0.219	8,9,12
8		NN	37.709					0.276					

IDW: Inverse distance weighting.
TRI: Triangulation method.
RS: Radius search method.
NN: Nearest neighbor method
R: value of radius (The biggest distance between predicted point and vertices of triangle which was used in triangulation method).
POINTS: Points which are entered in prediction calculation.

TABLE (25): ***Comparison among different techniques*** for point (9).

NO	TN	MODEL	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8	POINTS
9	38.450	IDW	38.034	38.064	38.068	38.071	38.075	0.416	0.386	0.382	0.379	0.375	28points
9		TRI	38.051	38.039	38.031	38.026	38.018	0.399	0.411	0.419	0.424	0.432	22,23,28
9	R=3553	RS	38.068	38.066	38.068	38.071	38.075	0.382	0.384	0.382	0.379	0.375	22,23,28,26,27
9		NN	38.076					0.374					

TABLE (26): ***Comparison among different techniques*** for point (10).

NO	TN	MODEL	TN1	TN2	TN3	TN4	TN8	ERR1	ERR2	ERR3	ERR4	ERR8	POINTS
10	38.036	IDW	38.014	38.047	38.053	38.056	38.061	0.022	0.011	0.017	0.020	0.025	28points
10		TRI	38.039	38.048	38.053	38.056	38.061	0.003	0.012	0.017	0.020	0.025	2,22,23
10	R=16089	RS	38.043	38.050	38.054	38.056	38.061	0.007	0.014	0.018	0.020	0.025	19points
10		NN	38.062					0.025					

19 Points: 1,2,3,6,7,9,15,17,18,19,20,21,22,23,24,25,26,27,28.

IDW: Inverse distance weighting.
TRI: Triangulation method.
RS: Radius search method.
NN: Nearest neighbor method
R: value of radius (The biggest distance between predicted point and vertices of triangle which was used in triangulation method).
POINTS: Points which are entered in prediction calculation.