

Discrete time-based chua chaotic generator

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Abstract

Chaos theory describes the behavior of certain nonlinear deterministic systems. These systems with evolving state time-dependence variation, may exhibit dynamics that is highly sensitive to initial conditions. Generally, the chaotic generators are classified into two main classes; continuous and discrete time generators. Continuous time such as systems based on Chua's circuit^[1] or the Lorenz^[2] and Rossler systems^[3], while discrete time systems such as Henon map^[4]. Many methods that use analog circuits have been proposed in the field of chaotic communication systems^[5-10]. Because recovery characteristics are sensitive to parameter mismatch between the receiver and the transmitter, one deficiency of these systems is that it must have both the transmitter and the receiver with very high component accuracy, to ensure correct information recovery. However, in practical situations, it is difficult to build both the transmitter and the receiver with very high component accuracy, since the component values are affected by aging, temperature...etc. Therefore, the analog implementation seems very difficult. Software implementations provide powerful computing tools in which complex numerical simulations of nonlinear phenomena are possible. They offer several advantages such as precision and ease of use to change the parameter values. Despite these privileges, applications such as spread spectrum and cryptography systems need high level of security; this is achieved by using hardware implementations instead of software implementations. In this paper, discrete time generators representing a modification of analog generators will be described. The goal here is to overcome the problems which face the analog circuits.

I. INTRODUCTION

Dynamical systems are represented by differential equations which relate the future states to the past states. The current state contains all the needed information

to completely describe the system in the future. These differential equations can be modeled using MATLAB; the nonlinearity can be simulated using many nonlinear functions like tanh function.

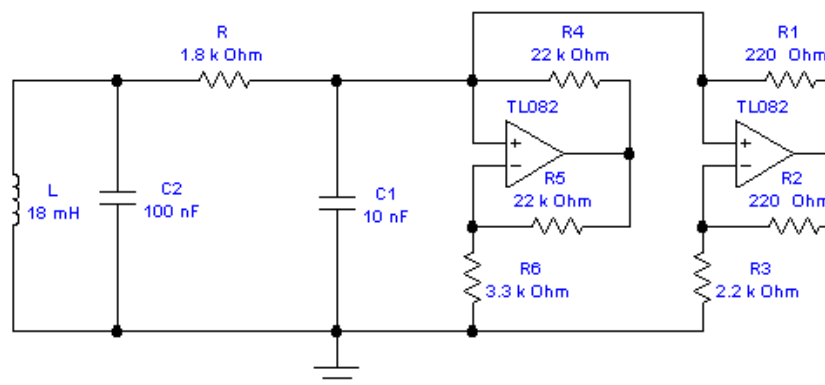


Fig. 1: Chua's physical circuit.

The main effort here is to find a mathematical expression with linear approximation for the nonlinear function so that the analog generator can be represented by a discrete model.

The paper is organized as follows. Section 2 describes Chua's circuit and the problems encountered. Section 3 discusses the modified discrete time generator of continuous time generators. Section 4 presents a simulation model to verify the chaotic behavior of the modified Chua generator. Conclusions and suggestions for further work are presented in section 5.

II. CHUA CHAOTIC GENERATOR CIRCUIT

Chua's circuit is one of the simplest circuits that can exhibit bifurcation and chaos. The Chua generator is like that discussed in^[1, 11] and is shown schematically in figure 1. The state equations of Chua's circuit are given by:

$$\begin{aligned} \dot{v}_{c_1} &= \frac{G}{c_1}(v_{c_2} - v_{c_1}) - \frac{1}{c_1}g_{c_1} \\ \dot{v}_{c_2} &= \frac{G}{c_2}(v_{c_1} - v_{c_2}) - \frac{1}{c_2}i_L \\ \dot{i}_L &= -\frac{1}{L}v_{c_2} \end{aligned} \quad (1)$$

where the nonlinear Chua function g_{c_1} is shown in figure 2, and is described by:

$$g_{c_1} = m_0 v_{c_1} + 0.5(m_0 + m_1)(|v_{c_1} + B_p| - |v_{c_1} - B_p|) \quad (2)$$

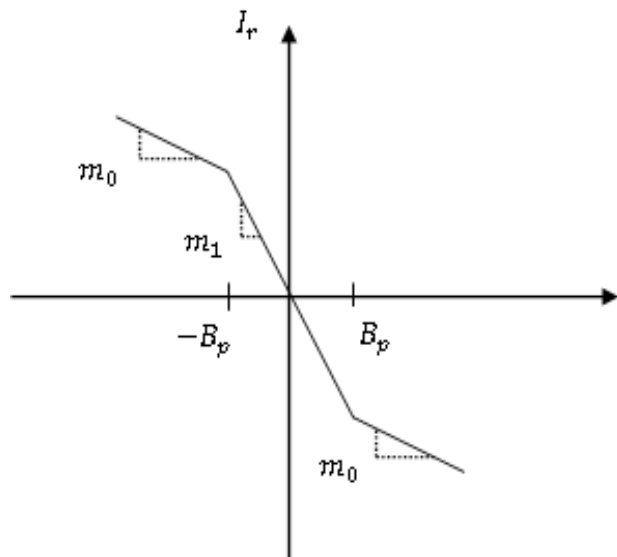


Fig. 2: Nonlinear characteristic of Chua diode.

where; are constants computed as follow:

$$\begin{aligned} m_0 &= -\frac{R_2}{R_1 R_3} + \frac{1}{R_4}, m_1 = -\frac{R_2}{R_1 R_3} - \frac{R_5}{R_4 R_6}, \\ B_p &= -\frac{R_3}{R_2 + R_3} \end{aligned} \quad (3)$$

Chua attractor is a fingerprint for this chaotic generator. The attractor is obtained by plotting the states of the system against each other. The attractor of a chaotic system is called a strange attractor where the chaotic signal is characterized by stretching and folding properties. Chua attractor is shown in figure 3.

III. PROPOSED DISCRETE TIME CHUA GENERATOR

The very beginnings of differential calculus^[12] shows that the gradient of a chord cutting a curve in two points (t_1, x_1) and (t_2, x_2) is given by:

$$\text{Gradient of chord} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \quad (4)$$

for which the gradient of the tangent at (t_1, x_1) is given, as $t_2 - t_1 = \delta t \rightarrow 0$, by:

$$\text{Gradient of tangent } \dot{x} = \frac{dx}{dt} \approx \frac{x_2 - x_1}{t_2 - t_1} \approx \frac{x_2 - x_1}{\delta t} \quad (5)$$

$$\text{Then: } x_2 = x_1 + \dot{x} * \delta t \quad (6)$$

Translating to the discrete system:

$$\dot{x} = \frac{dx}{dt} \approx \frac{x[(k+1)] - x[k]}{T} \quad (7)$$

The above equation called forward difference approximation. The difference equations for numerical solutions:

$$X_n = \dot{x}\Delta t + X_{n-1} \quad (8)$$

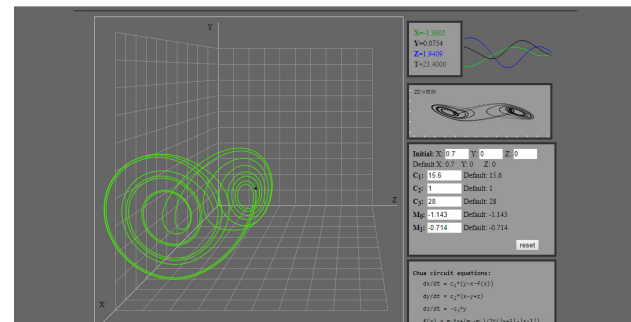


Fig. 3: Chua attractor.

Substituting the difference equation (8) in the state equations of Chua generator (1) yields:

$$\begin{aligned}
 V_{c_{1n}} &= \Delta t \left[\frac{G}{c_1} (V_{c_{2n-1}} - V_{c_{1n-1}}) - \frac{1}{c_1} g(V_{c_1}) \right] + V_{c_{1n-1}} \\
 V_{c_{2n}} &= \Delta t \left[\frac{G}{c_2} (V_{c_{1n-1}} - V_{c_{2n-1}}) + \frac{1}{c_2} I_{L_{n-1}} \right] + V_{c_{2n-1}} \\
 I_{L_n} &= \Delta t \left[-\frac{1}{L} V_{c_{2n-1}} \right] + I_{L_{n-1}}
 \end{aligned}
 \tag{9}$$

Using piecewise linear function instead of nonlinear function to can be implemented in digital circuits. Dividing the curve into 3 regions, each region will be represented by straight line function as follows

$$g_{c_1} = \begin{cases} g_1 - 0.49445v_{c_1} + 0.23095 & \text{for } -1 > v_{c_1} \\ g_2 - 0.7354v_{c_1} & \text{for } -1 < v_{c_1} < 1 \\ g_1^* - 0.49445v_{c_1} - 0.23095 & \text{for } +1 < v_{c_1} \end{cases}
 \tag{10}$$

Now, Chua generator can be transformed into discrete time model and can be simulated using MATLAB to verify

the chaotic behavior of the modified discrete time Chua generator.

IV. RESULT ANALYSIS

Using SIMULINK for modeling the continuous and the proposed discrete time Chua generator are shown in figure 4(a) and 5(a). Here, there are 9 new parameters added to the discrete time Chua generator, its values affect the behavior of the generator, where the step size of the solver is presented as an actual component, and the delay unit which used in the model has two parameters; the delay time (D) in second, and the initial value (IV) which represents the initial state of the dynamical system. The bifurcation diagrams for these new parameters are shown in figure 6. Figures 4(b) and 5(b) show the attractor of continuous time generator and the attractor of discrete time generator respectively, where tanh function is used as a nonlinear function for the continuous time model. Comparing the two results proves that the proposed model produces chaotic signals like that of analog model.

A discrete model of the well-known continuous time chaotic generators of Rossler attractor^[13] and Lorenz attractor^[14] can be achieved simply by substituting difference equation^[9] in their generators state equations.

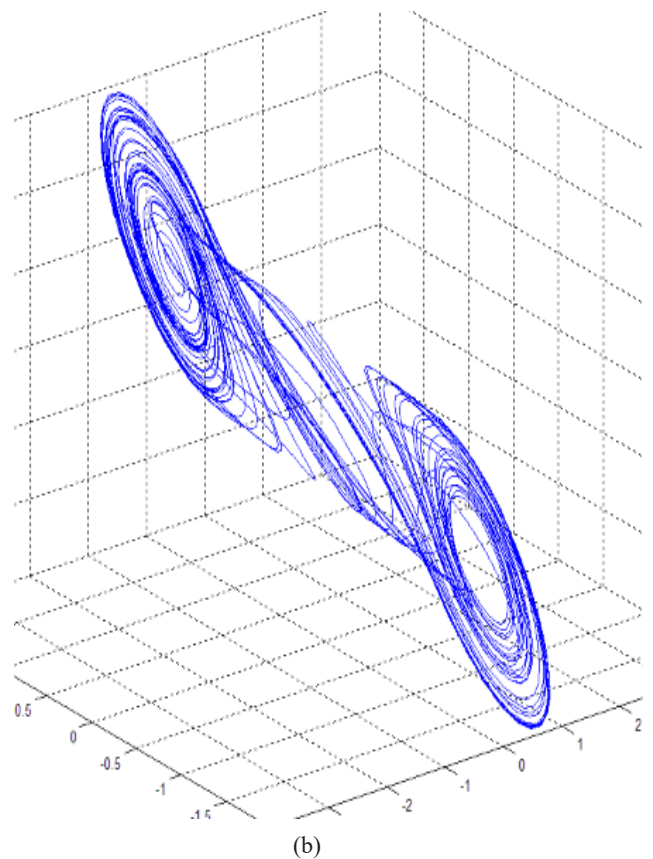
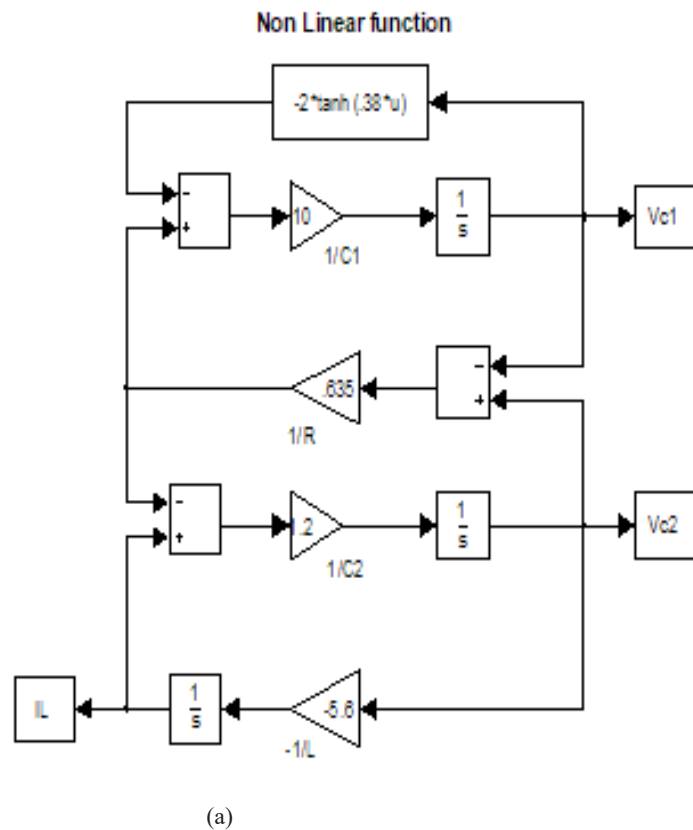


Fig. 4: Continuous time Chua generator; (a) Continuous time Chua model, (b) Continuous time Chua Attractor.

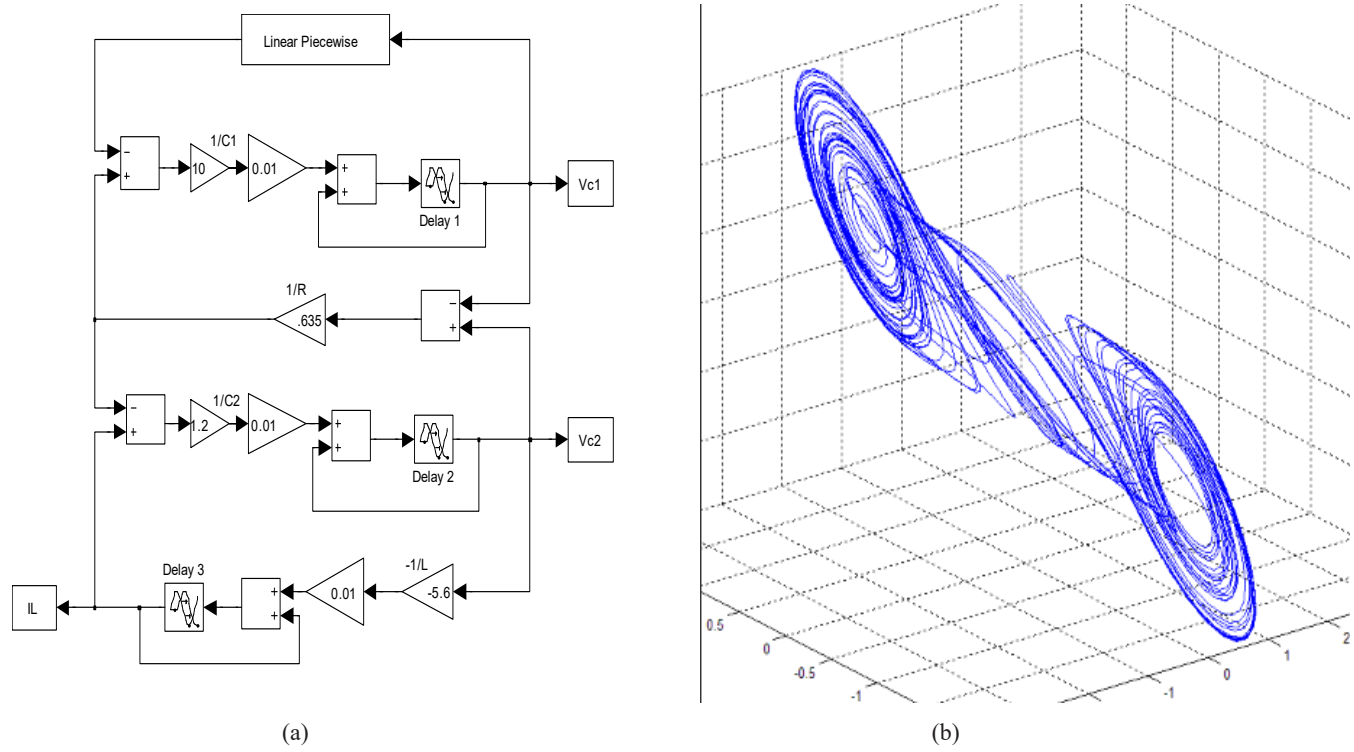
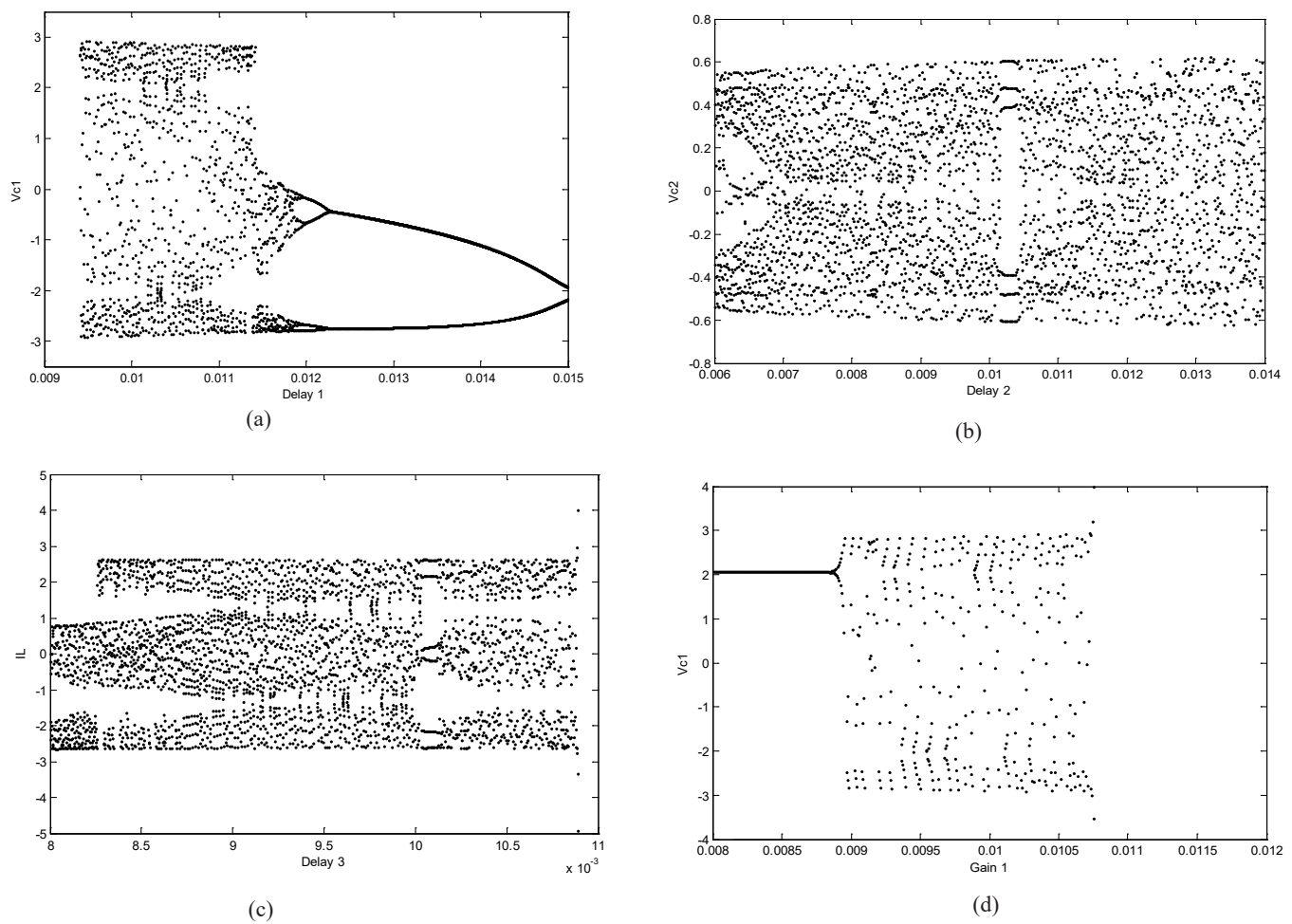
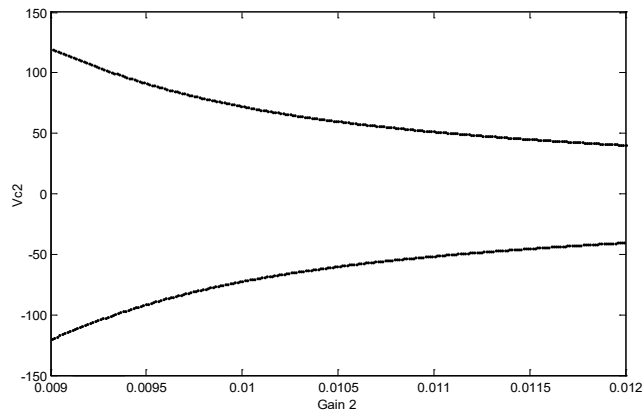
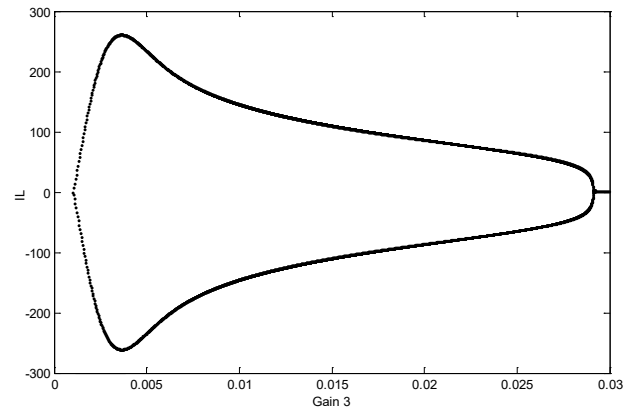


Fig. 5: Proposed discrete time Chua generator; (a) Discrete time Chua model, (b) Chua Attractor.

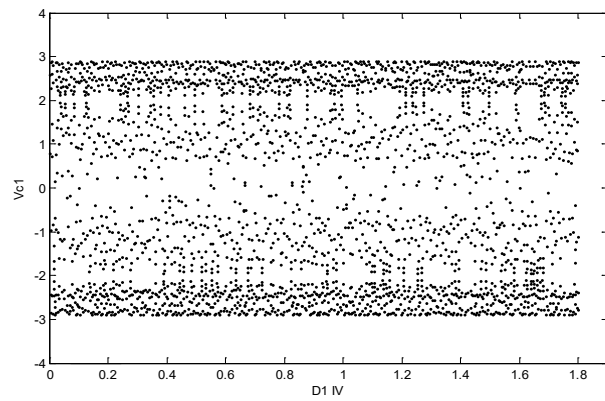




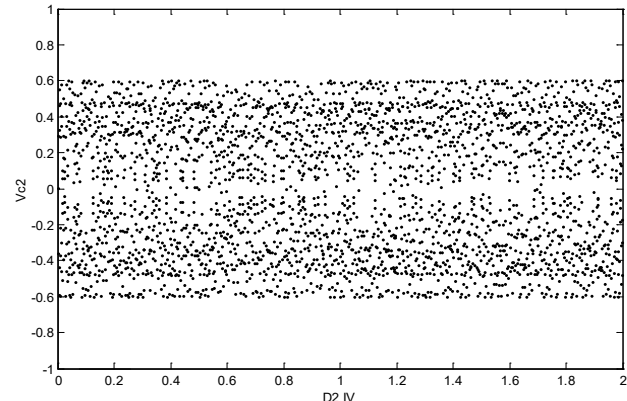
(c)



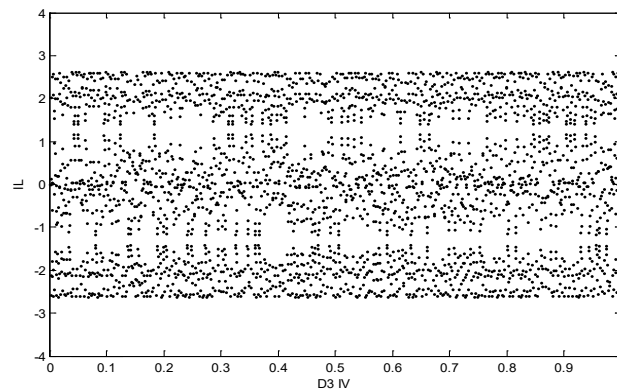
(f)



(g)



(h)



(i)

Fig. 6: Bifurcation diagram of proposed discrete time Chua generator. (a) State variable $Vc1$ versus delay1, (b) State variable $Vc2$ versus delay2, (c) State variable IL versus delay3, (d) State variable $Vc1$ versus gain1, (e) State variable $Vc2$ versus gain2, (f) State variable IL versus gain3, (g) State variable $Vc1$ versus $D1-IV$, (h) State variable $Vc2$ versus $D2-IV$, (i) State variable IL versus $D3-IV$. Where; delay1,2 and 3 are the values of different time delays, gain1,2 and 3 are the values of increments (Δt), $D1,2$ and $3-IV$ are the values of initial values assigned for each delay block.

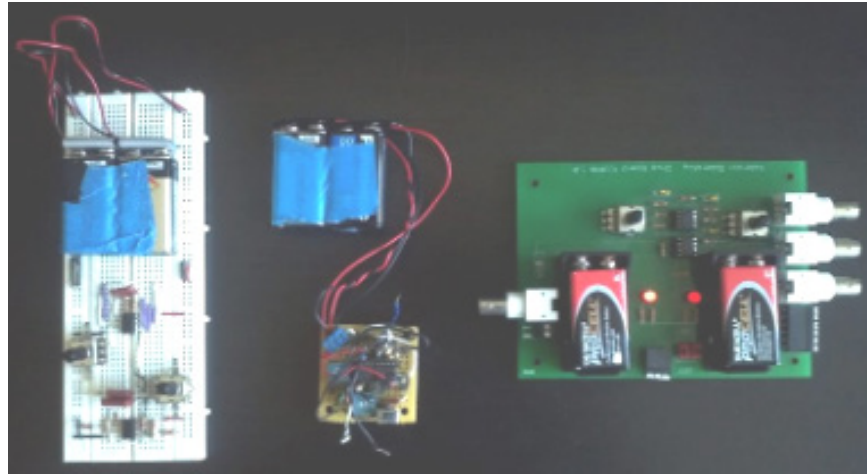


Fig. 7: Implementation of proposed Chua generator.

The proposed discrete time Rossler state equations will be:

$$\begin{aligned} X_n &= \Delta t(-Y_{n-1} - Z_{n-1}) + X_{n-1} \\ Y_n &= \Delta t(X_{n-1} + AY_{n-1}) + Y_{n-1} \\ Z_n &= \Delta t(B + Z_{n-1}(X_{n-1} - C)) + Z_{n-1} \end{aligned} \quad (11)$$

And, the proposed discrete time Lorenz state equations will be:

$$\begin{aligned} X_n &= \Delta t(A(Y_{n-1} - X_{n-1})) + X_{n-1} \\ Y_n &= \Delta t(BX_{n-1} - Y_{n-1} - 20X_{n-1}Z_{n-1}) + Y_{n-1} \\ Z_n &= \Delta t(5X_{n-1}Y_{n-1} - CZ_{n-1}) + Z_{n-1} \end{aligned} \quad (12)$$

The bifurcation diagrams for each proposed discrete time generator show the new parameters values added to these generators, controlling the trajectory of the chaotic signals.

V. CONCLUSION

SIMULINK models, with additional hardware implementation shown in figure 7, were used to evaluate the analog and discrete chaotic generators preserving the chaotic behaviour of these generators. This provides the discrete time Chua chaotic generator a precession over the ordinary continuous time Chua generator in two main points:

1) It overcomes the problem of accuracy, which faces the ordinary Chua circuit physical component used in ciphers, communication systems where the component values are

function of aging, temperature...etc.

2) The modified discrete Chua strengthens the cryptography systems where the added components have an important role in encrypting and decrypting the information.

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