MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

SELF-EXCITED VIBRATION OF MULTI-DISC ROTOR SUPPORTED ON SEVERAL FLEXIBLE BEARINGS

E.M. BADAWY*, H.M. METWALLY**

F.K. SALMAN***

ABSTRACT

A calculation method incorporating the transfer matrix method and the characteristic-vector locus method has been developed for stability analysis of the self excited vibration of a rotating shaft system with many bearings and discs. The analysis is made for an elastic bearing mass, relatively large damping forces due to types of rotors fit, anisotropic foundation and rotors gyroscopic effect. A two-rotors-model is presented to show the influence of rotor and its gyroscopic action, support stiffness characteristics, internal and external damping on stability. A computer solution of the transfer matrix method shows the rotor stability is improved by damped support. The shrinkage fit type rotors are more stable than force fit, welding type and adhesion loctite or araldite type. The gyroscopic moment effect of the overhanging disc decreased the stability region of the rotor. A computer solution of the governing equations of motion is presented showing the shaft stability region for various speed ranges and the effect of the bearing stiffness ratio on the stability region. The effect of external damping on the stability region of a rotor supported on anisotropic bearings is obtained.

INTRODUCTION

Several incidents, in practice, were encountered where a series of failures in machines designed to operate above the first critical speed. These machines were subjected to occasional fits of more or less violent vibrations of unknown origin. It was observed that at the speeds above the first critical speed, the rotor would enter into a violent whirling in which the rotor centerline precessed at a rate equal to the first critical speed. If the machine rotational speed was increased above its initial whirl speed, the whirl amplitude would increase leading to eventual rotor failure, Newkirk [1].

Kimball [2], suggested that internal shaft friction could be responsible for the shaft whirling. He postulated that below the rotor critical speed the internal friction would damp out the whirl motion while above the critical speed the internal friction would sustain the whirl. Because of the small order of magnitude of the friction forces observed by Kimball, Newkirk concluded that the internal friction created by shrink fits of the impellers and spacers was the predominant cause of the observed whirl instability. Kimball at Newkirk's suggestion, constructed a special test rotor with rings on:

* Professor, Dept. of Mechanical Engineering.** Associate Professor, Dept. of Mechanical Engineering, Faculty of Engg. *** Doctor student, Dept. of Mathematic, Faculty of Engg., Alexandria Univ., Alexandria, Egypt.

hubs shrunk on the shaft Kimball [3], he did indeed confirm Newkirk's conclusion that the frictional effect of shrink fits is a more active cause of shaft whirling than the internal friction within the shaft itself.

The stability of the single mass rotor with internal friction on damped anisotropic supports was evaluated using the general Routh criterion, Gunter [4]. This analysis includes the effects of bearing mass and is not restricted to small values of damping as is the nalysis of Gunter [5]. Gunter [4] showed that a symmetric flexible foundation will reduce the rotor critical speed and also the whirl threshold in the absence of external damping. If external damping is added, the stability threshold can be greatly improved. It was also found that foundation asymmetry alone, without foundation damping, can create a large increase in the whirl threshold speed. This analysis showed also that there is a limiting value of external damping that should be used.

The stability due to shrink fit of a single-disc shaft on damped isotropic and anisotropic supports was evaluated by Metwally [6]. He showed that the rotor stability is improved by damped supports with isotropic stiffness properties and increasing the shrink fit effect will decrease the rotor stability.

Now, how " the stability is improved by the rotating shaft on flexible supports" has been explained theoretically by Gunter [5] or Tondl [7]. However, the former made numerical calculation by Hurwitz method and did not obtain analytical values of the stability threshold. The latter obtained analytically the stability threshold for the case without gyroscopic effects using an approximate solving method of the stability problem of linear simultaneous differential equations. But the derivation was rather involved.

Nonami and Miyashita [8] studied experimentally the structural damping caused by the method of fitting the rotor into the rotating shaft, the estimated values of the internal damping and stabilities of various fits for a central and an overhanging rotor. From these experimental results, it has been made clear that the fitting methods of the rotor and the shaft are very closely related with the structural damping, the internal damping and the stability of the whirling motion.

Lund [9] derived the characteristic equation of a rotating shaft system with many bearings and discs using the Myklestad-Prohl method. He used the QR method, Wilkinson [10] for the stability analysis of his system. Since the natural frequency of a shaft system is obtained at the same time, his method has several merits in analysis of the vibration of a rotating shaft system. On the other hand, it has demerits such as the needs for excessive memory in the computer, large computing time, and good-approximation initial values in convergence calculation.

A calculation method incorporating the transfer matrix method, Pestel and Leckie [11] is used to obtain the characteristic equation of the rotating shaft system with many bearings and discs on flexible damped supports. An analysis method only for the stability of the self-excited vibration of this shaft system is developed not involving essentially the convergence calculation. The stability Criterion used is the characteristic-vector locus method (Mikhailov stability Criterion), Netushi [12].

This paper deals with the analysis of an elastic bearing mass, relatively large damping forces due to types of rotors fits, isotropic and anisotropic foundation and rotors gyroscopic effect. A two-rotors-model is presented to show the influence of rotor and its gyroscopic action, support stiffness characteristics, internal and external damping on stability. A computer solution of the governing equations of motion is presented showing the shaft stability region for varions speed ranges.

CHARACTERISTIC EQUATION

In the derivation of the characteristic equation necessary in stability analysis of a rotating shaft system with many bearings and discs such as in Fig. 1, the transfer matrix is used. The distributed system as shown in Fig. 1 is replaced with an equivalent lumped parameter system by dividing the system into several sections, Fig. 2. The i-th section as a result is shown in Fig. 3 and is called a standard

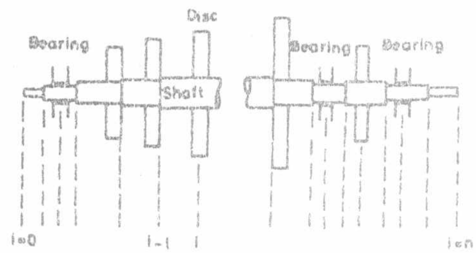

Fig. 1. Rotating shaft system.

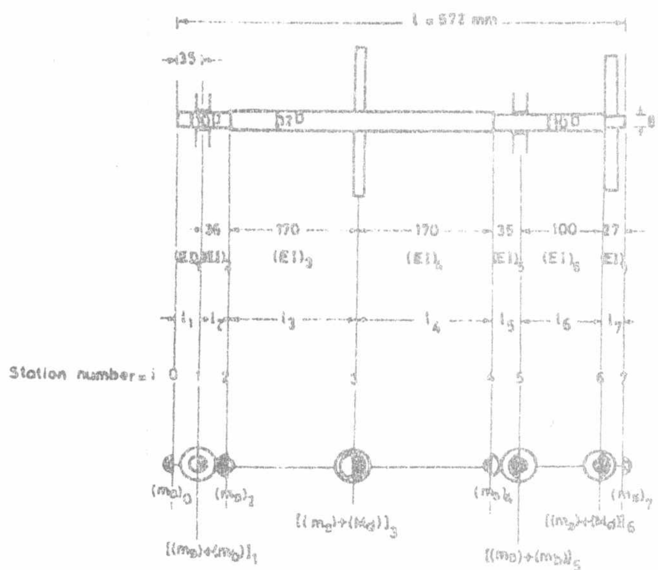

Fig. 2. Rotor-shaft system with two bearings and two discs.

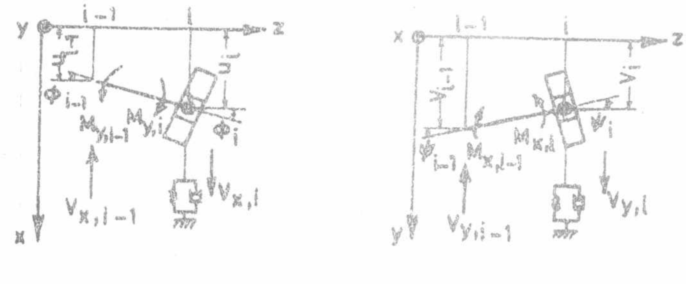

Fig. 3. Standard element.

element of the rotating shaft-system. It consists of the following; a flexible shaft, an equivalent lumped mass of shaft, a disc, a spring and damper. They are a direct spring and damper which have the action of forces in proportion to displacement and velocity, respectively.

For the free vibration of a rotating shaft system with rotational angular velocity w, the solution, taking u for example, is given as,

$$u = Re\left[Ue^{\lambda t}\right] = e^{\lambda_r t}(u_r \cos\lambda_i t - u_i \sin\lambda_i t) \qquad (1)$$

where

$$U = u_r + j\,u_i \;,\; \lambda = \lambda_r + j\lambda_i \qquad (2)$$

Since the bending free vibration is now considered, the state vector will be composed of eight state quantities; that is, deflections u and v, inclinations ϕ and ψ, bending moments M_y and M_x, and shearing forces V_x and V_y. Then the transfer matrix for the standard element as shown in Fig.3 and including the internal friction due to the fitting method of the disc on the shaft is obtained as

$$\begin{bmatrix} u \\ \phi \\ M_y \\ -V_x \\ -v \\ \psi \\ M_x \\ V_y \end{bmatrix}_i = \begin{bmatrix} 1 & l & \alpha & \beta & 0 & 0 & 0 & 0 \\ 0 & 1 & \gamma & \alpha & 0 & 0 & 0 & 0 \\ 0 & C_2 & C_3 & C_4 & 0 & C_6 & C_7 & C_8 \\ d_1 & (lS_{xx}+d_2) & (\alpha S_{xx}+d_3) & (\beta S_{xx}+d_4) & 0 & d_6 & d_7 & d_8 \\ 0 & 0 & 0 & 0 & 1 & l & \alpha & \beta \\ 0 & 0 & 0 & 0 & 0 & 1 & \gamma & \alpha \\ 0 & -C_6 & -C_7 & -C_8 & 0 & C_2 & C_3 & C_4 \\ 0 & -d_6 & -d_7 & -d_8 & d_1 & (lS_{yy}+d_2) & (\alpha S_{yy}+d_3) & (\beta S_{yy}+d_4) \end{bmatrix}_i \begin{bmatrix} u \\ \phi \\ M_y \\ -V_x \\ -v \\ \psi \\ M_x \\ V_y \end{bmatrix}_{i-1} \qquad (3)$$

or in the matrix form :

$$Z_i = T_i \cdot Z_{i-1} \qquad (4)$$

where,

$$\alpha = \frac{l^2}{2EI},\ \beta = \frac{l^3}{6EI},\ \gamma = \frac{l}{EI},$$

$$S_{xx} = -\left[k_{xx} + C_{xx}\lambda\right],\quad S_{yy} = -\left[k_{yy} + C_{yy}\lambda\right],$$

$$C_2 = \left[\left(\frac{3}{2}f_{22} - lf_{21}\right)\lambda + I_d\lambda^2\right],\ C_3 = \left[1 + (\gamma f_{22} - \alpha f_{21})\lambda + \gamma I_d\lambda^2\right] \qquad (5)$$

$$C_4 = \left[1 + (\alpha f_{22} - \beta f_{21})\lambda + \alpha I_d\lambda^2\right],\ C_6 = \left[\left(lf_{21} - \frac{3}{2}f_{22}\right)w + I_p w\lambda\right],$$

$$C_7 = \left[(\alpha f_{21} - \gamma f_{22})w + \gamma I_p w\lambda\right],\ C_8 = \left[(\beta f_{21} - \alpha f_{22})w + \alpha I_p w\lambda\right],$$

$$d_1 = -\left[(m_s + M)\lambda^2\right],\ d_2 = \left[(2f_{12} - lf_{11})\lambda - l(m_s+M)\lambda^2\right],$$

$$d_3 = \left[(\gamma f_{12} - \alpha f_{11})\lambda - \alpha(m_s+M)\lambda^2\right],\ d_4 = \left[1 + (\alpha f_{12} - \beta f_{11})\lambda - \beta(m_s+M)\lambda^2\right]$$

6 $$d_6 = \left[(1f_{11} - 2f_{12})\, w \right], \quad d_7 = \left[(\alpha f_{11} - \gamma f_{12})\, w \right],$$

$$d_8 = \left[(\beta f_{11} - \alpha f_{12})\, w \right].$$

As seen in Eqs. (3) and (5), the transfer matrix T_i is expressed as a function of the unknown λ. When the transfer matrix T_i for the i-th section is thus determined, from Eq. (4) the Eq. (6) results for the overall shaft system with n divided sections.

$$Z_n = T_n \cdot T_{n-1} \dots T_2\, T_1\, Z_o = TZ_o \tag{6}$$

In actual form, it is expressed as

$$\begin{bmatrix} u \\ \phi \\ M_y \\ -V_x \\ -v \\ \psi \\ M_x \\ V_y \end{bmatrix}_n = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{18} \\ t_{21} & t_{22} & \dots & t_{28} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ t_{81} & t_{82} & & t_{88} \end{bmatrix} \cdot \begin{bmatrix} u \\ \phi \\ M_y \\ -V_x \\ -v \\ \psi \\ M_x \\ V_y \end{bmatrix}_o \tag{7}$$

Assuming free boundary conditions at both ends of the shaft system,

$$\begin{aligned} M_{x,n} &= M_{y,n} = M_{x,o} = M_{y,o} = 0 \ ; \\ V_{x,n} &= V_{y,n} = V_{x,o} = V_{y,o} = 0 \end{aligned} \tag{8}$$

Applying Eq. (8) to Eq. (7) , therefore,

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} t_{31} & t_{32} & t_{35} & t_{36} \\ t_{41} & t_{42} & t_{45} & t_{46} \\ t_{71} & t_{72} & t_{75} & t_{76} \\ t_{81} & t_{82} & t_{85} & t_{86} \end{bmatrix} \cdot \begin{bmatrix} u_o \\ \phi_o \\ -v_o \\ \psi_o \end{bmatrix} \tag{9}$$

Eq. (10) must hold in order for Eq. (9) to have a solution with non-zero values of u_o, ϕ_o, v_o and ψ_o.

$$\Delta \equiv \begin{vmatrix} t_{31} & t_{32} & t_{35} & t_{36} \\ t_{41} & t_{42} & t_{45} & t_{46} \\ t_{71} & t_{72} & t_{75} & t_{76} \\ t_{81} & t_{82} & t_{85} & t_{86} \end{vmatrix} = 0 \qquad (10)$$

Since t_{ij} is a function of the unknown λ, Δ is also a function of λ. Therefore, Eq. (10) is the characteristic equation of the given shaft system. Eq. (10) is a λ polynomial of the form:

$$\Delta(\lambda) = a_o\lambda^m + a_1\lambda^{m-1} + \dots + a_{m-1}\lambda + a_m = 0 \qquad (11)$$

In a shaft system divided into n sections, m=8 n

STABILITY CRITERION

The characteristic equation for a shaft system with n sections is after all a polynomial of order 8 n with respect to λ. In actual calculation using a computer, however, it's not obtained as a polynomial of degree m in λ (Eq. (11)) with λ being as unknown; Δ is derived only numerically. Such a stability criterion as Routh-Hurwitz is thus not applicable for this case. Therefore, considering that the characteristic equation in form is a λ polynomial of higher order such as Eq. (11), in the present analysis method a stability criterion by the characteristic-vector locus method is utilized.

With the formula (12) in Eq. (11),

$$\lambda = j\Omega \qquad (12)$$

the characteristic vector as Eq. (13) is obtained:

$$\Delta(j\Omega) = \Delta_r(\Omega) + j\Delta_i(\Omega) \qquad (13)$$

Then as shown in Fig. 4, the principle of stability criterion depends on whether the characteristic-vector locus starts with $\Omega = 0$ at the positive real axis (Δ_r) on Δ plane and then proceeds through an m number of quadrants turning around the origin of Δ plane in a positive direction (that is, anticlockwise) towards $\Omega \to \infty$. The system will be found stable if the condition described above is satisfied.

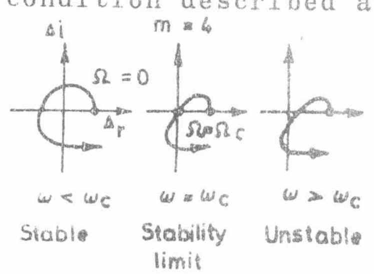

Fig. 4. Stability criterion by the characteristic-vector locus

As seen, the present method makes a stability criterion only through value of the characteristic vector, so it is applicable also in a numerical characteristic equation such as in the transfer matrix method. And further, it does not involve any convergence calculation in the stability criterion; difficulty in numerical calculation is thus eliminated. This is both advantageous and convenient.

6

NUMERICAL CALCULATION METHOD

In a rotating shaft system, the internal damping forces and the gyroscopic moment of the discs depend on the rotational angular velocity w, so the characteristic vector, Eq. (13), differs with the rotational angular velocity w. It is therfore necessary to make stability analysis at each rotational speed.

(I) Stability Analysis at a Rotational Angular Velocity w

In the characteristic-vector locus method, Ω is varied from zero to infinity. And, stability criterion can be made by:

(1) Whether or not the characteristic-vector locus turns anticlockwise successively through respective quadrands of the Δ plane;
(2) Whether or not the argument of the characteristic-vector increases monotonously; and
(3) Whether or not zero points of the real and the imaginary part in the characteristic-vector appear alternately.

However, the second technique is employed. The variation of frequency Ω is set automatically in the computer program.

(II) Threshold Rotational Speed of the Self-Excited Vibration (Stability Limit) w_c :

The threshold rotational speed is obtainable by performing the stability Criterion of (I) at each rotational speed w finely given. Then, to obtain automatically the w_c value by a sort of the bisection method, the region between stability rotational speed w_1, and instability rotational speed w_2 is successively divided, so w_c with a desired accuracy is derived.

(III) Whirl Frequency Ω_c at the Threshold Rotational Speed of the Self-Excited Vibration

In occurrence of the self-excited vibration, as indicated in Fig.4 the characteristic-vector locus passes through the origin of Δ plane. The Ω value when the vector locus is at the origin is thus the whirl frequency in the self-excited vibration. Therefore, in numerical calculation the Ω value closest to the origin in locus with $w=w_c$ can give a value of Ω_c as accurate as the w_c value.

RESULTS AND DISCUSSION FOR TWO DISC SYSTEM

These results are obtained for the model shaft system shown in Fig. 2 with the following data:

$(m_s)_o = 0.01078$ Kg., $(m_s)_1=0.02156$ kg., $(m_s)_2=0.0861784$ Kg.,
$(m_s)_3 = 0.1507968$Kg., $(m_s)_4=0.0861784$Kg., $(m_s)_5=0.04158$ Kg.,
$(m_s)_6 = 0.03612224$Kg., $(m_s)_7=0.00532224$, $(m_b)_1=(m_b)_5=19.2$ kg.
$(M_d)_3 = 1.57$kg., $(M_d)_6=1.1$ kg., $(I_p)_3=2\ (I_d)_3=0.0157\ m^4$
$(I_p)_6 =2(I_d)_6=0.00891\ m^4$, $(k_{xx})_1=(K_{yy})_1=0.331683 \times 10^5$ N/m,
$(k_{xx})_5=(k_{yy})_5=0.331683 \times 10^5$ N/m, $(C_{xx})_1=(C_{yy})_1=C_e$ NS/m,
$(C_{xx})_5 = (C_{yy})_5=C_e$ Ns/m.

The computer results for the model shaft system with two discs mounted on the shaft by different types of fit is shown in Fig. 5. This figure gives the relation between the external damping coefficient

Curve	A	B	C	D	E	F
$(C_i)_6 \times 10^{-4}$	10.904	8.832	3.574	2.703	1.468	1.468
$(C_i)_3 \times 10^{-4}$	3.494	3.494	1.288	1.033	1.220	0.543
Type of fit	Adhesion	Adhesion	Force Fit	Welding	Shrink Fit	Shrink Fit

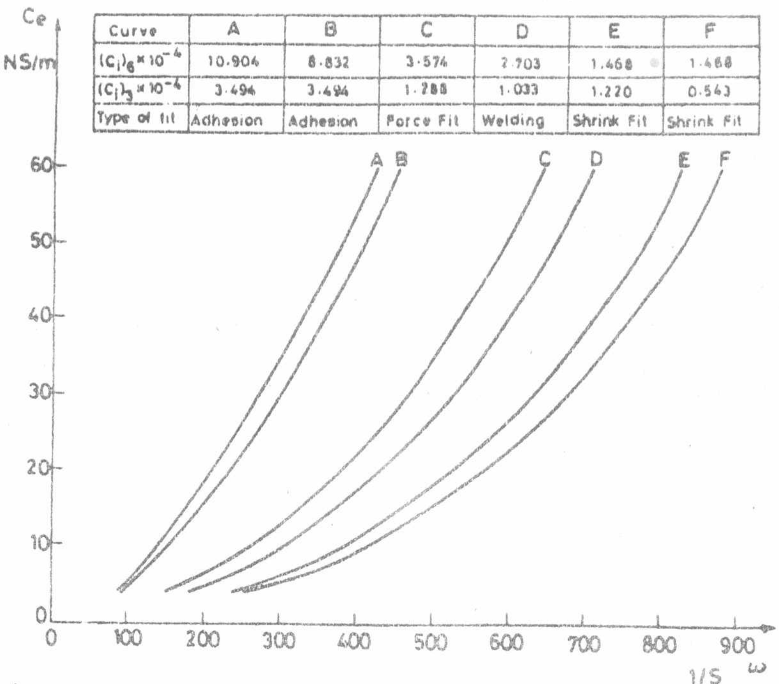

Fig. 5 . The external damping coefficient and the stability region for different kinds of fit .

and the stability region. The left hand side of each curve represents the stable region whereas the right hand side of it represents the unstable region. The internal damping coefficient of the central rotor $(C_i)_3$ and the overhanging rotor $(C_i)_6$.

The gyroscopic effect of the central rotor and the overhanging rotor for the best shrink fit type is shown in Fig. 6. The stability region is on the left hand side of each curve.

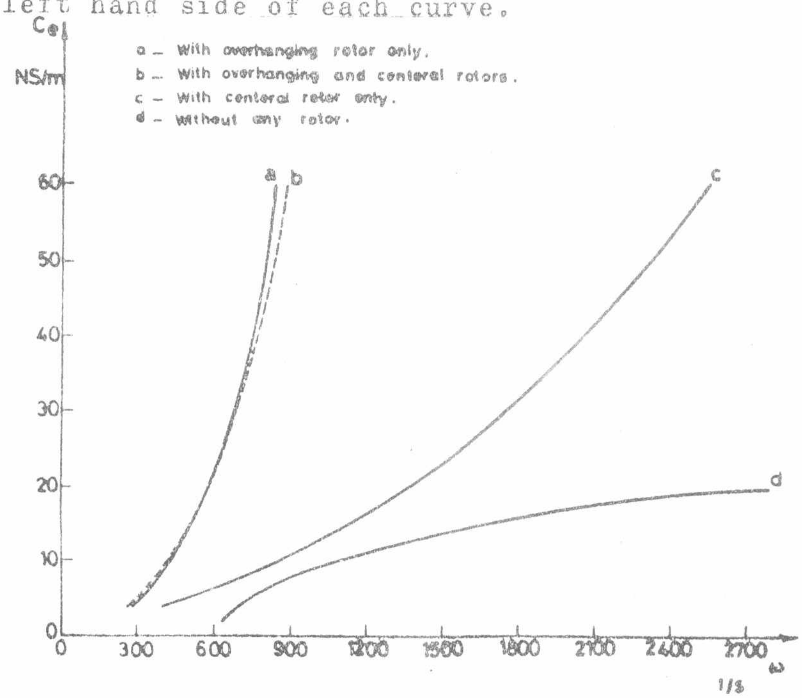

Fig. 6. Effect of gyroscopic action on stability .

The effect of bearing stiffness on stability is shown in Fig. 7. The left hand side of each curve is the stable region.

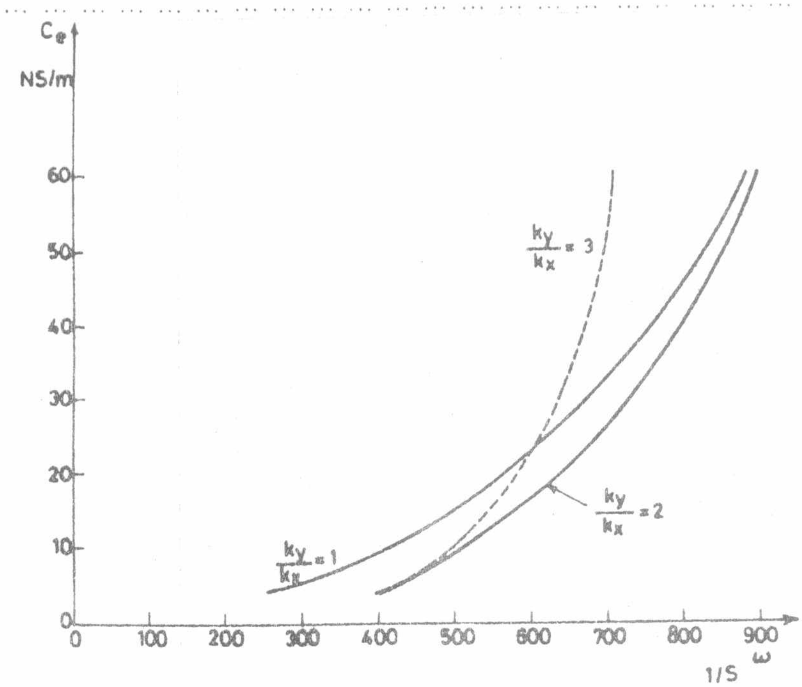

Fig. 7. Effect of bearing stiffness on stability.

The optimum values of the external damping for the given model shaft with a specified bearing stiffness and running at a certain speed can be estimated from Fig. 8.

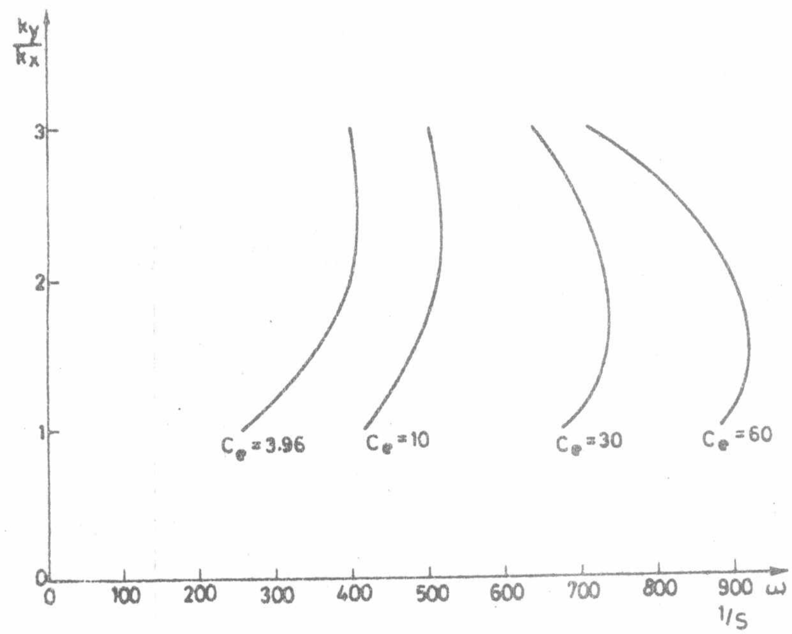

Fig. 8. The optimum external damping for different bearing stiffness.

CONCLUSION

1. The estimated values of the internal damping due to the type of fit by which the two-disc-rotors are mounted on the shaft have a large influence on the stability in certain shaft system. The type of fit of the overhanging rotor will affect greatly the stability region. The more the coefficient of internal damping for the over-hanging rotor, the less will be the stability region. The shrink fit type will have greater stability region than taht of the welding type, the force fit type, the adhesion by loctite and araldite.

Therefore, with a rise of the value of the coefficient of internal damping for the overhanging rotor, the rotational speed initiating the self-excited vibration decreases and the stability region will be decreased.

2. The gyroscopic moment plays a big role in the stability region, it decreases the region of stability by a large amount for the case of overhanging rotor rather than the case without gyroscopic action as shown in Fig. 6.

3. Changing of the bearing stiffness from isotropic to anisotropic will vary the stability region as shown in Fig. 7.

4. We can estimate the optimum values of external damping for any shaft system having a specified bearing stiffness and running at a certain speed as shown in Fig. 8.

5. For the improvement of the stability, it is important to minimize the internal damping of fitted parts. In particular, for rotor systems having fitted parts, the shrinkage fitting is desirable.

6. Generally, the stability region increases with the increase of the external damping for both isotropic and anisotropic bearings for a certain value of bearing stiffness.

REFERENCES

1. Newkirk, B.L., "Shaft Whipping", General Electric Review, Vol. 27, 169 , (1924).
2. Kimball, A.L., "Internal Friction Theory of Shaft Whirling", General Electric Review, Vol. 27, No.4, 244, (1924).
3. Kimball, A.L., "Measurement of Internal Friction in a Revolving Deflected Shaft", General Electric Review, Vol. 28, 554, (1925).
4. Gunter, E.J., JR., and Trumpler, P.R., "The Influence of Internal Friction on the Stability of High Speed Rotors With Anisotropic Supports", J. of Eng. for Ind., Vol. 91,1105-1113, Nov. (1969).
5. Gunter, E.J., JR., "The Influence of Internal Friction on the Stability of High Speed Rotors", J.of Eng. for Ind., Trans. ASME, Vol. 89, 683-688, Nov. (1967).
6. Metwally, H.M., "Effect of Shrink Fits of a Disc on a Rotating Shaft", The Bulletin of the Faculty of Engineering, Alexandria University, Vol. XIX, No.1, 1-18, (1980).
7. Tondl., A., "Some Problems of Rotor Dynamics", Chapmann and Hall, London,(1965).
8. Nonami, K. and Miyashita, M., "Internal Damping and Stability of Rotor-Shaft Systems", Bulletin of the JSME, Vol. 24, No. 187, January (1981).
9. Lund, J.W., Trans. ASME, Ser. B, Vol. 96, No.2, 509, (1974-5).
10. Wilkinson, J.H., "The Algebraic Eigenvalue Problem ", Oxford University Press, London, (1965).
11. Pestel, E.C, and Leckie, F.A., " Matrix Methods In Elastomechanics", McGraw-Hill Book Company, Inc., (1963).
12. Netushi, A., " Theory of Automatic Control", Mir Publishers Moscow English translation, 310-313, (1973).

NOMENCLATURE

C_{xx}, C_{yy}	damping coefficients at support in the x and y directions, respectively. ($C_{xx}=C_{yy}=C_e$)
C_i	coefficient of internal damping.
C_{11}, C_{12}, C_{22}	stiffnesses of the shaft.
E	Young's modulus.
e	base of natural logarithm.
f_{11}, f_{12}	coefficient of forces of internal friction.
f_{21}, f_{22}	coefficient of moments of internal friction.
I	second moment of shaft area.
I_p, I_d	moments of inertia of disc (polar, diametral, respectively)
j	imaginary unit ($j = \sqrt{-1}$).
k_{xx}, k_{yy}	support stiffness in the x and y directions, respectively.
l	length of shaft.
M	mass of shaft.
M_y, M_x	bending moments.
m_s	effective mass of shaft.
n	number of sections.
T	transfer matrix.
t	time.
t_{ij}	element of transfer matrix.
u, v	deflections in directions x,y, respectively.
U etc.	complex variables.
V_x, V_y	shear forces.
Δ	characteristic equation.
Δ_r	real part of $\Delta(\Omega)$.
Δ_i	imaginary part of $\Delta(\Omega)$.
λ	complex eigenvalue ($\lambda = \lambda_r + j\lambda_i$).
Ω	angular velocity.
Ω_c	whirl frequency.
w	rotational angular velocity.
w_c	threshold rotational speed.