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VIBRATION ANALYSIS OF FULLY AND PARTIALLY DAMPED AND RIGIDAMPED SANDWICH BEAMS WITH VISCOELASTIC CORE

S.H.FARGHALY*,M.M.EL-Maddah** and M.K.Ismail***

ABSTRACT

The reduction of vibratory energy transmission has been the subject of investigation within past years. The present paper is concerned with the increase of the internal damping in mechanical systems as a method for their vibration control. The inclusion of viscoelastic materials within the structural configurations of the mechanical system is proposed to increase its internal damping.

Analytical solutions are presented for the fully damped sandwich beam. In addition, laboratory experiments have been performed to verify the analytical predictions. Moreover, the optimal damping distribution for partially damped and the case of rigidamped sandwich beams are considered experimentally as a practical design proposition.

INTRODUCTION

Vibration control has an important role in designing structures and mechanical systems to function within a dynamic environment. The reduction of the transmission of vibratory energy has been the object of investigation in many years. The high vibration and noise levels associated with modern high energy power sources exsist over a broad frequency range. The amplification of power source vibration by structural members creates large dynamic stresses and results in excessive noise,structure fatigue and component failure.

The quality of some products depends on the vibration and noise levels which may cause discomfort to occupants. The increase of the internal damping in the mechanical system usually reduces its dynamic response. This reduction is caused by energy dissipation properties of the material and the system. Material damping, Lazan [1], is a name for the phenomenon by which energy is dissipated in a vibrating mechanical system consisting of a volume of macro continuous (solid) matter. However the system damping, involves configurations of distinguishable parts.

In the presented paper we shall be concerned with the increase of the internal damping in the system as a method for its vibration control.

* Assistant Prof. Dept. of Mech. Design. Faculty of Eng. and Tech. (Mataria). University of Helwan , Cairo, Egypt.
** Lt.Col.Dr. Dept. of Mechanics and Elasticity;
*** Maj. Gen. Dr., Head of Basic Sciences Dept.,Military Technical College, Cairo, Egypt.

When layers of viscoelastic materials are applied to the surface of a conventional structural element, the bending of this element will produce predominantly extensional strains in these attached layers, Ungar [2] . This type of damping has been termed extensional, and the viscoelastic layer have been designated as free. In contrast, constrained viscoelastic layer is bonded with two adjacent elastic in such way that flexural motion of the sandwich produces primarily shear strains and hence shear damping in the viscoelastic materials. Figs. 1-2, illustrates a comparison of damping capabilities for various materials and design configurations.

The analytical treatment of the sandwich beam is reported by many authors [4-10] . The optimal design problems are presented in the cases of fully and partially covered damped sandwich beams. In many cases, particularly when the weight saving is important. One may do well to cover a part of structural layer with a damped sandwich element. The location of this element may be chosen for damping some particular modes of vibration (see Fig.3).

The excitations and loading may damage the bonding between layers and may cause an additional resonant frequencies. For these reasons the rigidamping treatment is suitable to be used from the design standpoint (see Fig.4).

Laboratory experiments have been performed to verify the analytical analysis for the fully damped sandwich beam cases. The partially damped and rigidamped sandwich beam configurations are presented as practical design consideration problems.

THEORY

The fully damped sandwich beam considered in this paper consists of a pair of uniform structural and constraining layers separated by and bonded to viscoelastic core (see Fig.5). All the layers have the same width. The differential equation of flexural vibration of sandwich beams is a complex equation of six order. This results from the following essential assumptions:

a- The shear strain in the core is uniform across the depth of the core.
b- The longitudinal direct stresses and transverse direct strains in the core are negligible.
c- The shear strains in the face plates are very small.
d- The rotary inertia of the whole layers is not taken into consideration.
e- The internal damping of the face plates are negligible compared to that of core layer.

The non-dimensional form of the differential equation of motion can be written as [4].

$$\bar{v}^{VI} - X^{*}(1+Y)\bar{v}^{IV} + (\ddot{\bar{v}}^{II} - X\ddot{\bar{v}}) = 0 \qquad (1)$$

where

$$X^{*} = \frac{G_2 L^2}{2H_2}\left(\frac{\bar{K}}{K_1}\right) \quad \text{(the complex shear parameter)} \qquad (2)$$

$$Y = \left(\frac{C^2}{B}\right)\left(\frac{K_1}{\bar{K}}\right) \quad \text{(the geometric parameter)} \qquad (3)$$

and

$$G_2^* = G_2 \ (1 + j\ \eta_2) \qquad \text{(the complex shear modulus)}$$

$$\bar{K} = (K_1 + K_3)/\ K_3$$

$$K_1 = E_1\ A_1$$

$$K_3 = E_3\ A_3$$

$$C = H_1 + 2H_2 + H_3$$

$$B = E_1 I_1 + E_3 I_3 \qquad \text{(overall bending stiffness)}$$

$$X^* = X(1 + j\ \eta_2)$$

$$\bar{v} = v(\bar{x}, \bar{t})$$

$$\bar{x} = x\ /L$$

$$\bar{t} = t/to$$

$$= t\ / \sqrt{\frac{mL^4}{B}}$$

Assuming transverse motion to be of the form

$$v\ (x,t) = V(x)\ \text{Exp. } j(\Omega t) \tag{4}$$

and introducing the dimensionless modal frequency parameter

$$a_{ns} = \omega_{no} \sqrt{\frac{mL^4}{B}} \tag{5}$$

where n is the mode number .

After substituting eq.(4) into eq.(1) we obtain

$$V^{VI} - X^* (1+Y)\ V^{IV} - a_{ns}^2\ (V^{II} - X^* V) = 0 \tag{6}$$

The values of the composite modal frequency Parameters a_{ns} and the composite loss factors for different boundary conditions are reported by, Rao [5].

The unknown function V(x) can be found in the form

$$V(x) = \sum_{i=1}^{6} A_i \ \text{exp. } jr_i x \tag{7}$$

Substituting one term of eq.(7) into eq.(6) we obtain the characteristic equation

$$Z_i^3 + X^* (1+Y)\ Z_i^2 - a_{ns}^2\ (Z_i + X^*) = 0 \tag{8}$$

where $Z_i = r_i^2$

The real p rt of this complex characteristics equation represents the case of undamped sandwich beam. The solution of that equation gives three real roots: one positive and two negative.

EXPERIMENTAL WORK

The response of the different test specimens is acquired by an accelerometer mounted at the measuring point. The vibration exciter (112 N) , which is driven by a sinusoidal exciter control (5Hz-10KHz) through a power amplifier of (120 VA) power. The measurements include the loss factors and resonant frequencies for each test specimen of fully and partially sandwich beams. Fig.6 indicates the outline of the experimental instrumentations.

RESULTS AND DISCUSSION

The use of rivets may increase the rigidity in the case of fully damped sandwich beam. This will be denoted as rigidamped design configuration.

Table 1, indicates a typical comparison for the resonant frequencies and the composite loss factors, the theoretical results are in good agreement with the experimental ones for the fully damped sandwich beam.

Table 1. The effect of the rigidamping on the composite loss factor and resonant frequencies.

Mode	Parameter	Fully Damped		Rigidamped
		Analytical	Experimental	Experimental
I	f_o, Hz	89.32	110	125
	η	0.2	0.318	.229
II	f_o, Hz	502.18	500	600
	η	0.0513	0.082	.073

For the case of partial coverage sandwich beam, the main structural specimen is divided into equal elements. Each of these elements has the length of the damping element in sandwich form. This damping element is displaced successively along the vibrating main structural specimen. The displacement has an equal pitch which is equal to the smallest length of the damping element, (see Fig. 7). The values of loss factors are plotted in a single graph as shown in Figs.8-11. For desired modes of vibration using damping elements for maximizing damping capability, the location of the damping element is preferred to be near the node of vibration. Thus the shear strain energy takes its maximum values at the nodal points of the modal shape.

CONCLUSION

Experimental verification of the theory for the fully damped sandwich beams are presented. The correlation between the analytical results and the experimental ones are in a good agreement. The use of rigidamped beams proved to be a suitable design. This design is fulfilled by using rigid rivets to protect the bonding adhesives between layers.

Design graphs, practically considered for the partial covered beam with damped sandwich element, from which the composite loss factor and the composite resonant frequency can be read directly. One interesting conclusion which may be drawn from these experiments is that the higher damping occurs when the damping element location is near the nodal points.

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NOMENCLATURE

Latin letters

a_n	Mode frequency factor for homogeneous beam.	
B	Uncoupled flexural rigidity $(E_1I_1 + E_3I_3)$.	$N.m^2$
C	Central distance between structural and constraining layers.	m
E_i	Young's modulus of i th layer	$N.m^{-2}$
f_o	Resonant frequency	Hz
G_2	Real part of the complex shear modulus, G_2^*	$N.m^{-2}$
H_i	Half thickness of ith layer	m

I_i	Area moment of inertia of ith layer about its own midline.	m^4
K_1	Extensional stiffness of structural layer per unit width.	$N.m^{-1}$
K_3	Extensional stiffness of constraining layer per unit width	$N.m^{-1}$
$\bar{K}$	Stiffness ratio $(K_1+K_3)/K_3$	
L	Vibrating length of sandwich beam specimen	m
X	Shear parameter	
$\bar{x}$	Length ratio (x/L)	
Y	Geometrical parameter	

Greek Letters

η	Loss factor.	
ω_{on}	Resonant frequency at mode n	$rad.sec.^{-1}$
Ω	Excitation frequency.	

Superscripts

-	Designates the dimensionless parameters.
*	Designates the complex parameters.
.	Designates the derivation with respect to time.
I	Designates the derivation with respect to x

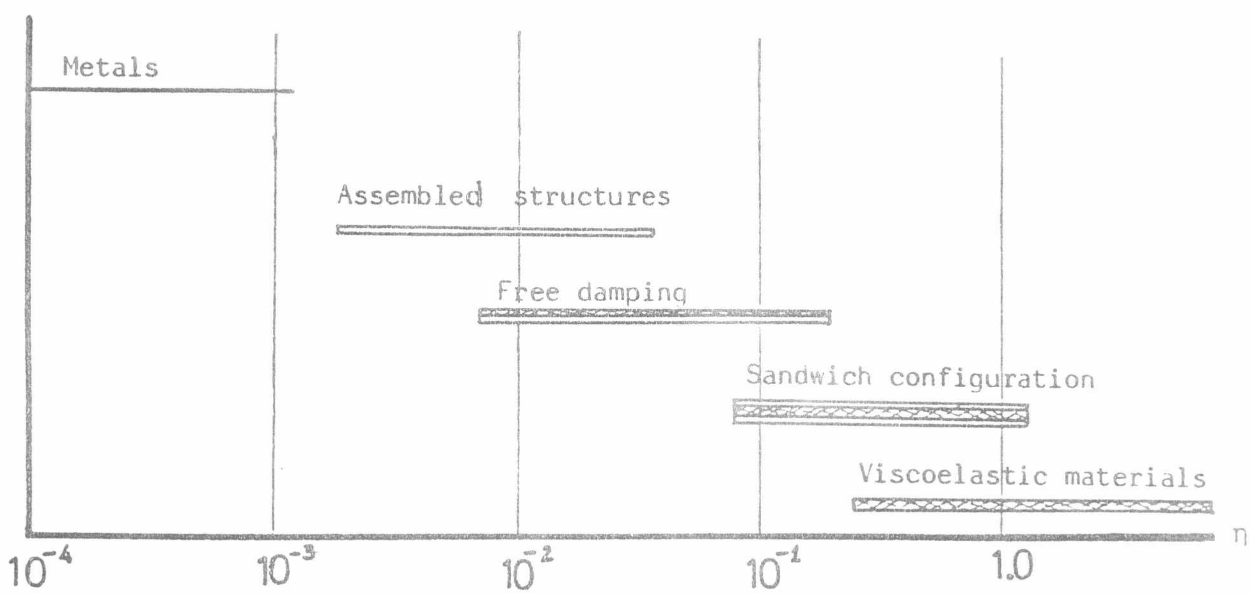

Fig. 1 : Damping merits for different materials and systems, (Santini [3])

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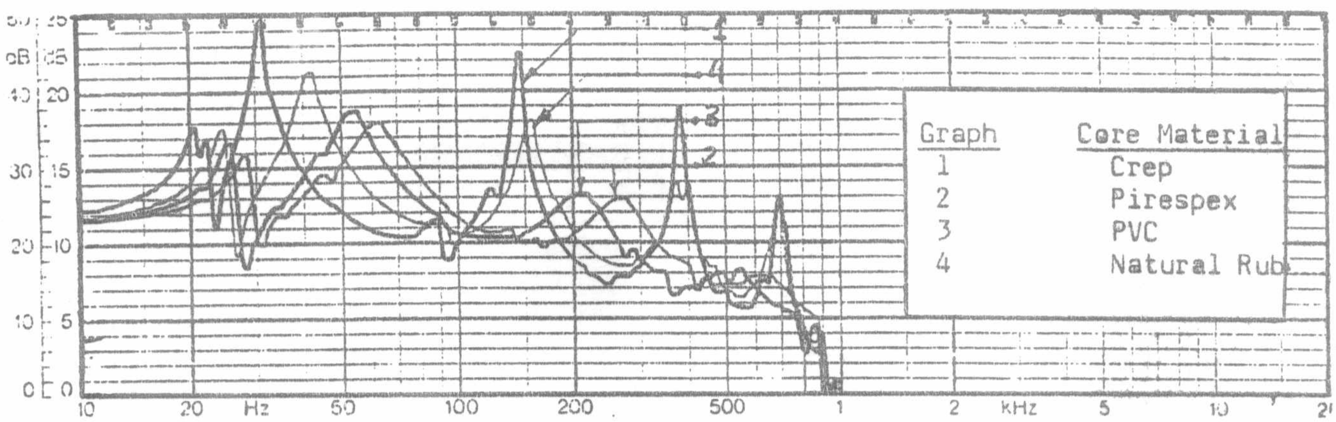


Fig. 2 : Comparative damping for typical viscoelastic materials (Farghaly [4])

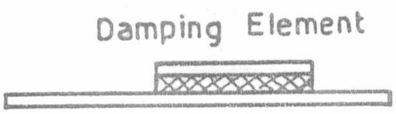


Fig. 3 :Partial damped beam configuration.

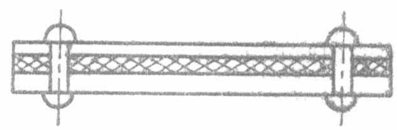


Fig. 4 : Rigidamped beam element.

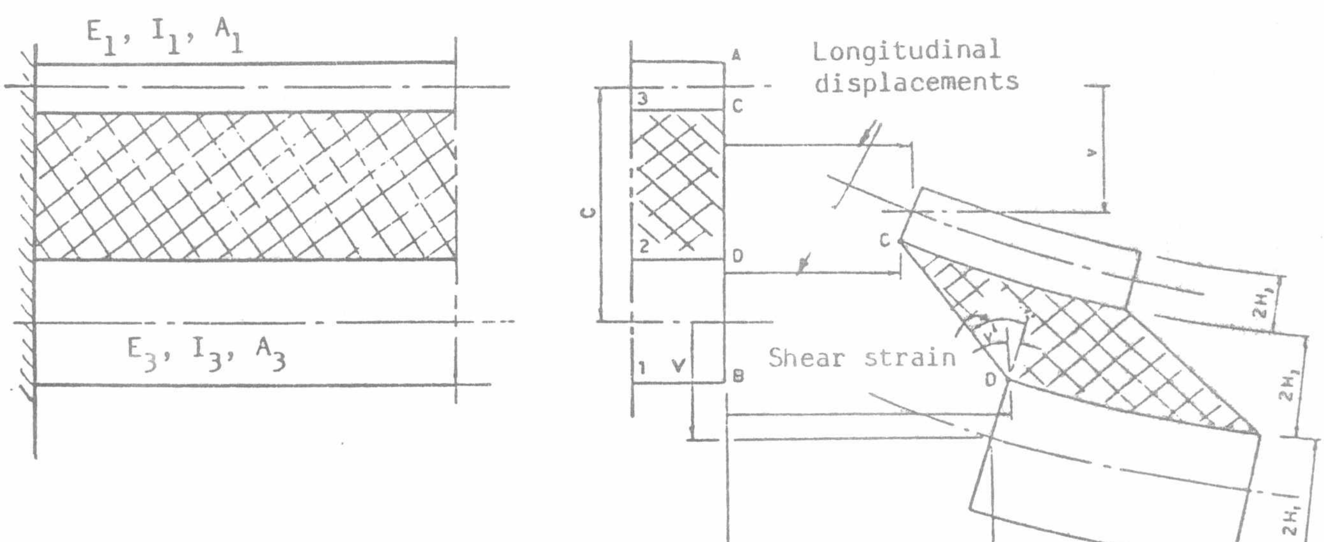


Fig. 5 : Undeflected and deflected beam element.

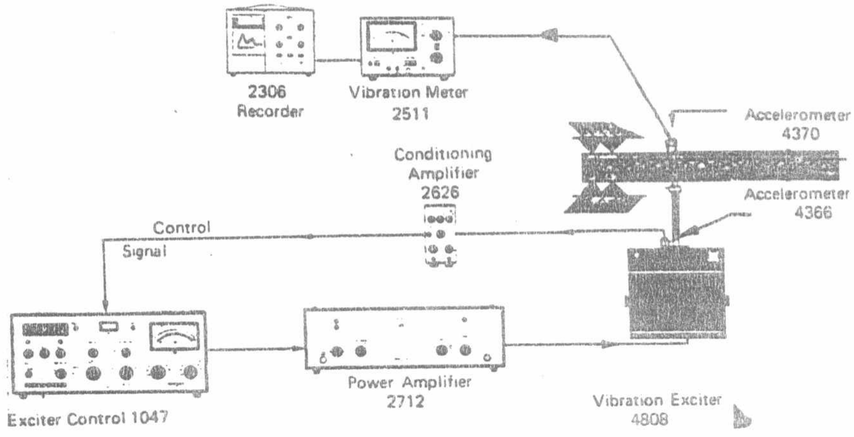
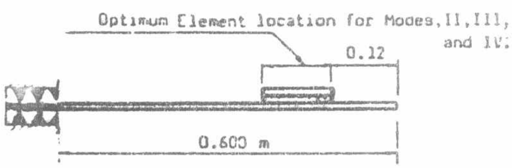


Fig. 6 : Instrumentation arrangement.

Fig. 7 : Size and locations of different damping elements.



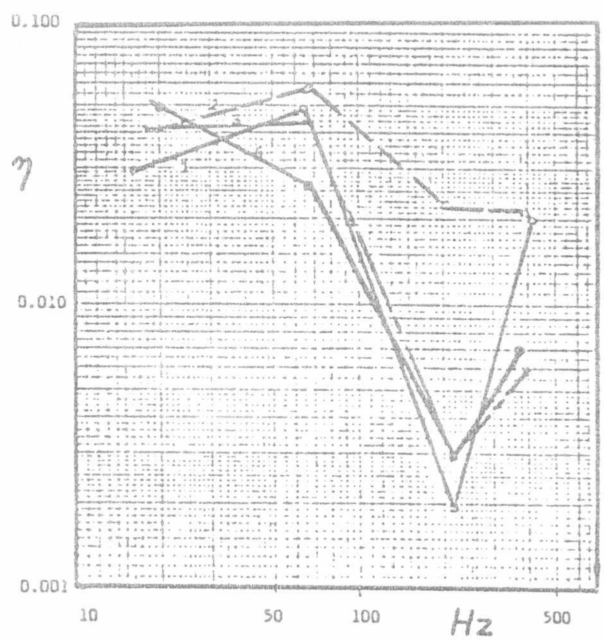


Fig. 8 ; Optimum element location for the smallest damping element.

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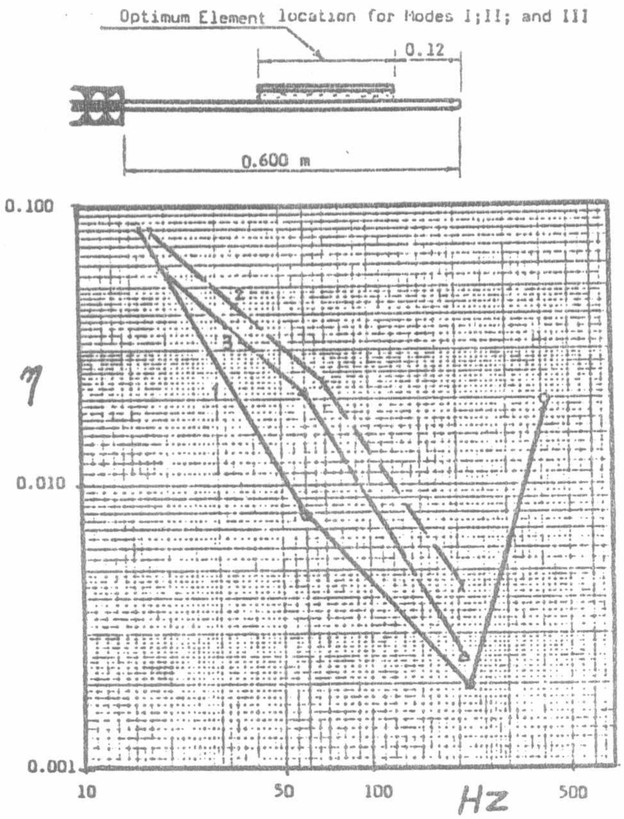

Fig. 9 : Optimum element location for medium damping element.

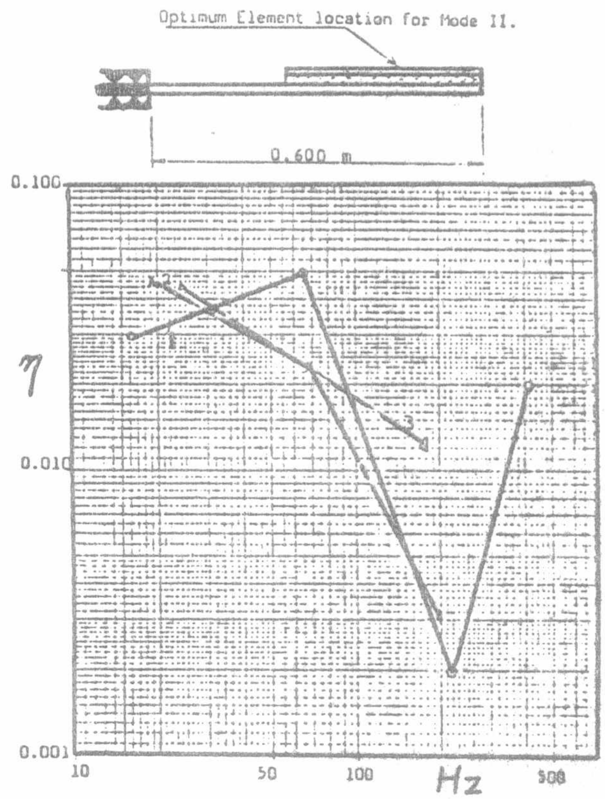

Fig. 10 : Optimum element location for large damping element.

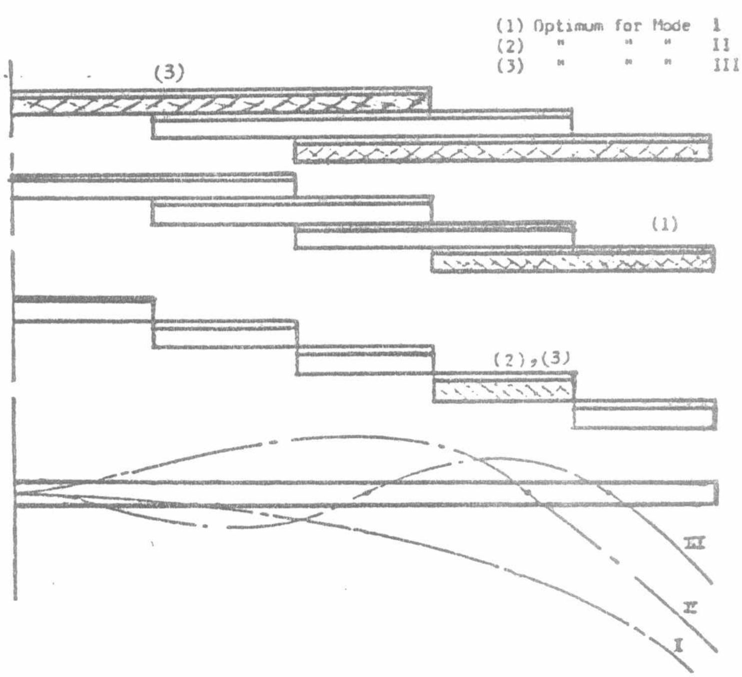

Fig. 11 : Optimum locations for damping elements.