MIXED MODE CUMULATIVE FATIGUE DAMAGE

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ABSTRACT

The present work attempts to understand early fatigue damage accumulation in a mechanical sense. The effect of changing the mode of load application from torsion cycling to push-pull cycling and vise-versa, on the same specimen, during the course of its life was investigated. Torsion prior to push-pull and push-pull prior to torsion mixed mode cumulative damage tests in both low and high cycle fatigue regimes were conducted. The cumulative damage tests were carried out between two strain levels selected such that when applied individually and separately, in each mode, would give the same endurance. It was found that when torsion cycling had been applied first, a decrease in the residual life in push-pull was observed indicating a positive damage due to the prior torsion. On the otherhand, prior push-pull cycling caused an increase in the residual torsional fatige life, which reached a max. of 170% after a push-pull damage of 70%, in the high cycle fatigue regime.

INTRODUCTION

Damage accumulation through the fatigue life has been emphasized in the literature at both the micro and macro scales [1]. The aspects of the latter have been intensively studied quantitatively and qualitatively [2-4]. However, damage at the microscopic level is known to play an important role in the whole damage process, especially at long endurances. The significance of this role was recognized in situations where the state of strain is kept unchanged throughout the fatigue life.

Two cases of strain states, that lead to two different cracking systems, designated A and B [5], have been created due to the application of reversed torsion and uniaxial push-pull modes respectively, as shown in Fig. 1.

Pure reversed torsion can be represented by Case A, Fig.1, where $\varepsilon_1 = -\varepsilon_3$, and $\varepsilon_2 = 0$. The cracks which are nucleated initially on the surface will propagate primarily along planes of intense shear deformation. Two planes of maximum shear arise at the surface: one is parallel to the specimen's

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Fracture mechanics equations quantify the growth of small cracks that are created by both torsion and push-pull loading [10-11] . The present work attempts to understand early fatigue damage accumulation in a mechanical sense during mixed mode application of cyclic loading.

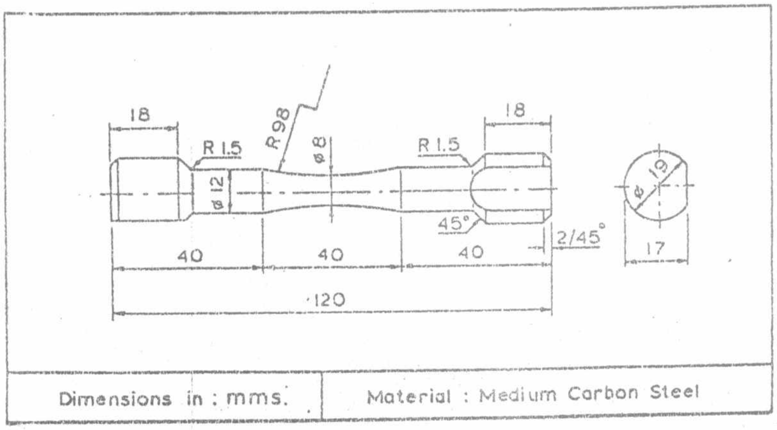

Fig. 4. Specimen Geometry

SPECIMEN GEOMETRY AND MATERIAL PROPERTIES

A specimen having an hour-glass profile with large curvature, Fig.4, was made from medium carbon steel, having the following chemical composition (in percent) : 0.40C, 0.13 Si, 0.007 S, 1.03 Mn, and remainder Fe. The average grain size measured on the transverse plane to the bar axis was ≈ 56 μm. Before testing, the specimens were carefully polished and then inspected under a microscope to insure that no serious scratches remain.

The fatigue properties were obtained seperately in push-pull and torsion modes of loading and the results are summerized in Table 1. Details of testing procedure and machinery can be found in Ref. [12] . Fig. 5 shows the endurance curves in push-pull and torsion as plots of the equivalent "Tresca" shear stress range, $\Delta\tau$, versus number of cycles to failure, N_f.

Table 1. Cyclic Properties of 0.4 % C Steel

	Push-Pull	Torsion
Cyclic stress-strain behaviour	$\Delta\sigma = 2.48\text{x}10^3(\Delta\varepsilon_p)^{0.218}$	$\Delta\tau = 1.18\text{x}10^3(\Delta\gamma_p)^{0.225}$
Fatigue endurance curves	$\Delta\varepsilon_p . N_f^{0.488} = 0.194$	$\Delta\gamma_p . N_f^{0.42} = 0.85$
Fatigue limit	$\Delta\sigma_e = 474$ MPa.	$\Delta\tau_e = 300$ MPa.

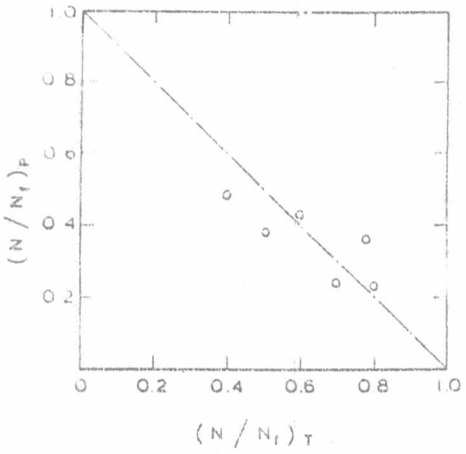

Fig.6. Fraction of life spent in Torsion $(N/N_f)_T$ versus Fraction of Residual Life Measured in Push-Pull $(N/N_f)_P$, at $N_f=10^3$ cycles.

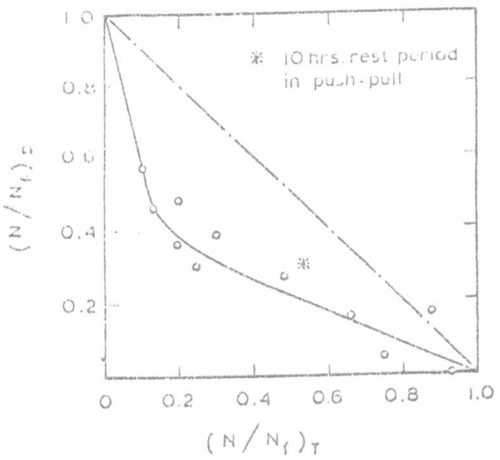

Fig.7. Fraction of life spent in Torsion $(N/N_f)_T$ versus Fraction of Residual Life Measured in Push-Pull $(N/N_f)_P$, at $N_f=4 \times 10^5$ cycles.

N.B.

N_f = Number of cycles to failure.

$(N/N_f)_P$ = Fraction of life spent in push-pull cycling.

$(N/N_f)_T$ = Fraction of life spent in torsion cycling.

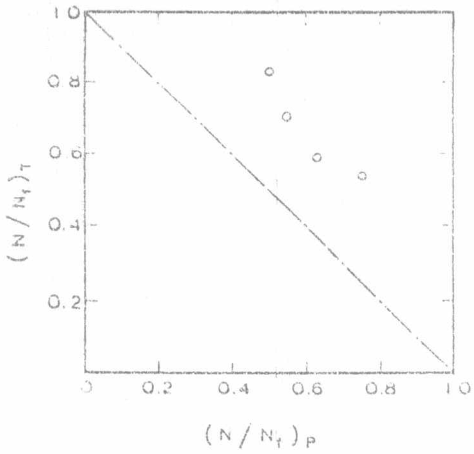

Fig.8. Fraction of Life Spent in Push-Pull $(N/N_f)_P$ versus Fraction of Residual Life Measured in Torsion $(N/N_f)_T$, at $N_f= 10^3$ cycles.

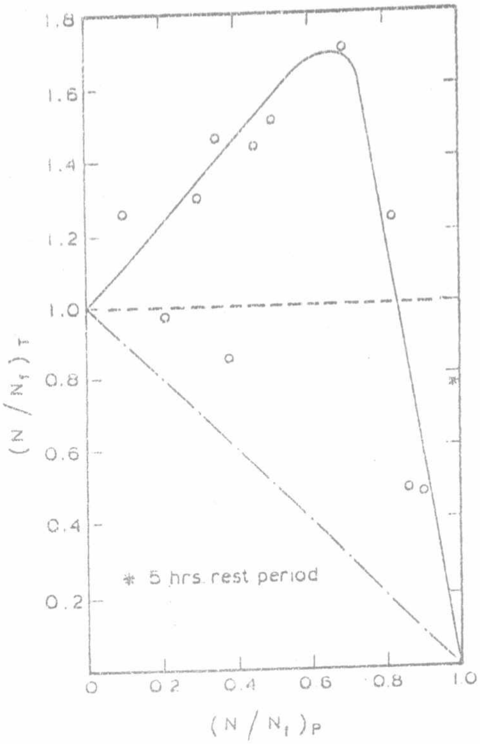

Fig.9. Fraction of Life Spent in Push-Pull $(N/N_f)_P$ versus Fraction of Residual Life Measured in Torsion $(N/N_f)_T$, at $N_f= 4 \times 10^5$ cycles.

The test features that are involved in this effect are given elsewhere [12], and they are summerized as :

i- The initial total shear strain range $\Delta\gamma_t$ in torsion is 0.82% which produces a cyclic softening. Consequently, the enhanced plasticity will accelerate or may even eliminate the crack initiation phase in the subsequent push-pull mode.

ii- The equivalent (Tresca) plastic shear strain range $\Delta\bar{\gamma}_p$ in push-pull is $\sim$ 0.15% compared with that in torsion of 0.37%. This implies that the sequence of loading can be considered as a high-to-low (H-L) type cumulative damage. It has been established, previously [15], that such a sequence results in a summation of life fractions less than unity, even when accounting for residual stresses.

iii- The maximum decrease in push-pull life occured after 0.23 of the fatigue life was spent in torsion, although the initiation phase in torsion is thought to be completed only after a fraction of 0.74 of the total life [18] . The minimum summation, $\sum N/N_f$, equals to 0.6. Such reduction in the push-pull life may be attributed to the orientation of the planes of maximum shear with regard to the state of strain. One of the two planes of maximum shear resulting from the reversed torsion lies in a plane normal to the maximum principal stress in push-pull.

This implies that when the initial torsional loading is within the initiation phase, the normal stress resulting from the new stress state will promote quicker development of Stage II cracks.

iv- When the initial damage in torsion exceeds the initiation phase and some Stage I cracking has occured, the change to push-pull will allow several Mode I cracks to develop. If torsion alone has had continued, only one of the crack systems would operate, but the application of push-pull will open out other cracks and permit cross linking.

(c) Push-Pull Prior to Torsion in LCF Regime

The mixed mode cumulative test obtained at the LCF regime, as represented by Fig.8 , shows that the total summation of damage terms has a maximum value of $\sim$ 1.34. The deviation from linearity is attributed mainly to the importance of the crack front geometry in the damage process. In the initial push-pull test, cracks are inclined at 45° to the specimen surface and axis, and then will propagate on a plane perpendicular to the specimen axis, see Fig. 2. However, the crack area will be roughly semi-circular and be only a few grains deep. On introducing the torsion mode, these cracks will now have to propagate around the surface, as they would have done without the initial push-pull cycles, in a Mode II sense before any substantial propagation into the interior by Mode III. Note that in the context of this work damage is equivalent to crack growth that reduces the cross-sectional area.

Due to shortage of specimens in this test series a curve representing the MMCD effect could not be drawn, a point which will be considered in future study.

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