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EMPIRICAL EQUATION FOR TREATING CUMULATIVE FATIGUE DAMAGE

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ABSTRACT

Cumulative damage in fatigue has received a great deal of attention in recent years and many methods have been developed for predicting fatigue life. However, it does not yield predictions that are in all cases in agreement with the experimental results. In the following a simple equation was developed to interpet the cumulative damage occured by the stress levels and cycles ratio. The derivation is based on analogy and on intuition rather than on well-established natural laws due to the complicated process of cumulative damage and the many factors encountered in it.

Comparison of the results obtained from the proposed empirical equation with the most commonly known methods of linear damage ruleLDR, double linear damage rule DLDR and the experimental results,available in the literature for SAE 4130 steel and maraging 300 CVM steel, proved its true validity. The proposed equation give better life prediction than DLDR and LDR. On average its estimate for life prediction approximately 8% higher or lower than experimental while the DLDR provide estimates 12% high and LDR provide estimates 37% high. However more study is needed to obtain the good perspective of its merits.

INTRODUCTION

Fatigue prediction in design has, in the past, been the subject of extensive experimental work and theoretical study because it plays a dominant role in almost every engineering endeavour. Many publications have appeared on this subject and several damage theories devoted to various aspects of fatigue have been developed [1 - 8] .

Fatigue tests are normally carried out with constant stress or strain amplitude because of the straight forward presentation

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of data. However this seldom represents the loading conditions to which engineering parts are subjected in service, where fluctuating stress or strain may vary widely in amplitude often in a random manner [5], [6] .

The earliest method for quantitatively evaluating the effect of a varying stress level on the accumulation of fatigue damage is the linear damage rule, Miners rule. Experiments have shown that this simple summation rule is inadequate for many situations. This method does not take into account the order of stress application nor the reduction in the endurance limit of a prestressed material. However it received widespread recognition and was extensively used because of its simplicity [7].

In an attempt to overcome some deficiencies of the linear damage rule, a large number of theories have been proposed . A generalized concept presented by many workers [3], [4], [8] , they hypothesis for the determination of the degree of damage of a material by a function interms of the cycle ratio and the exponent depends on the applied stress level. Others [7] to [12] , presented a physical mechanism of metal fatigue by considering that the life of a material is composed of two stages : crack initiation and crack propagation to final failure.

Manson and co-workers [12] followed this lead in explaining the order effect as well as developing an independent approach known as, the Double Linear Damage Rule (DLDR). They recognized that the mechanism of fatigue failure consists of two phases, and provided a formula for determining the " Knee point " where the transition between the two phases occured. Although a number of practical cases have been studied by DLDR, experience is still limited and descripances exists between experimental and mathematical expressions.

Other conventional methods involve constant cycling of stress or strain (or their equivalents) at different amplitudes were used to determine an S-N curve and the endurance limits. Such methods are expensive and time consuming as they require many specimens tested for long periods, especially at low stress levels. In a more recent approach, a method for accelerating fatigue testing of materials, based on monotonically increasing the stress amplitude with the number of cycles, until failure. The limited experimental results which were presented, show a fair agreement with the analytical predictions. Much more experiments work need to be done to verify the validity of this method [10 - 14] .

Concept damage in present theories was developed from ad hoc mathematical descriptions containing coefficients which must be determined from fit to experimental test results or from the physical mechanism of internal crack growth. But, tracking the growth of a single dominant crack developed involves many complicated process. In order to account for these processes, they make use of empirical information. However the developed

6

theories does not yield predictions that are in all cases in agreement with the experimental results. Hence in the present work the cumulative damage problem is again treated in a more rational way to develop a simple and useful emperical form.

PROPOSED RATIONAL APPROACH

Review is made for the different cumulative damage theories available in the literature. Many approaches are based on the physical interpretation and crack propagation. However the micro-aspects of failure cyclic loading are of extreme complexity. Therefore, quantitative prediction of fatigue failure on the basis of microstructural considerations appears to be a forbiddingly difficult problem, even for the much simple case of constant amplitude cycling. In order to account for all processes, an empirical information is used in the developed theories.

Other approaches treat the cumulative damage in phenomenological way, the approach consisting of attempts to assess the damage produced by cycling and its accumulation due to changing amplitude.

In the following the cumulative damage problem is again treated in reasonable simple approach based on analogy and on intuition rather than on well-established natural laws.

A simple basic concept in the rational approach is to hyposize the damage function which defines in some sense the " damage" produced in a specimen when subjected to stress amplitude. The first step to establish a model for damage accumulation. It is recognized that the major manifestation of damage is based on physical mechanism.

Tracking the growth of a single dominant crack developed involves many complicated processes such as dislocation agglomeration, subcell formation, multiple microscopic crack formation and the independent growth of these cracks until they link and form the dominant crack. Clearly, the fatigue damage increases with initiation of cracks and their growth, but strengthening of material occurs as a result of the development of dislocation structures or strain-aging. Therefore, the cumulative damage is not considered as a linear function of stress cycle, as assumed in the Miner rule. However a nonlinear power law, for the damage curve is the most suitable. Thus the general form in which damage occured depends on the number of cycles and stresses may take the form:

$$D = f(n,s) \qquad (1)$$

Using the simple power law, then damage occured will be

$$D = \left(\frac{n}{N}\right)^{\alpha} \qquad (2)$$

Where α is a function of stress S and life N, but specific functional forms for α interms of the life values and stresses involved have not proposed. In this analysis, based on analogy and intuition, rational or phenomenological functional form for the exponent α interms of cycle ratio and stresses was proposed for multi-stage loading, i, as:

$$\alpha_{i-1} = \left[\left(\frac{S_{i-1}}{S_i}\right) \cdot \left(\frac{N_{i-1}}{N_i}\right) \right]^m \qquad \text{Where } \alpha_i = 1$$

Where the second exponent m is a fraction depends on the material and various between (0.25 - 0.5), [8] , [13] .

For two load level S_1 and S_2 with lifes N_1 and N_2 respectively the damage equation take the simple form :

$$D = \left(\frac{n_1}{N_1}\right)^{\left[\frac{S_1}{S_2} \cdot \frac{N_1}{N_2}\right]^m} + \frac{n_2}{N_2} \qquad (3)$$

When failure occurs, D = 1, the remaining number of cycles n_2 / N_2 will be:

$$\frac{n_2}{N_2} = 1 - \left(\frac{n_1}{N_1}\right)^{\left[\frac{S_1}{S_2} \cdot \frac{N_1}{N_2}\right]^m}$$

In the more general case of multi-stage loading, when i loadings are applied, before failure occurs, the equation for the damage may be considered as the cumulative summing of the damage occured.Proceed in the same manner as before, based on analogy and intuition [13] , adding each stage with its power value, the general equation assume the form :

$$D = \left[\left\{\left(\left(\frac{n_1}{N_1}\right)^{\alpha_2} + \frac{n_2}{N_2}\right)^{\alpha_3} + \frac{n_3}{N_3}\right\}^{\alpha_4} + \dots\dots\dots \frac{n_{i-1}}{N_{i-1}}\right]^{\alpha_i} + \frac{n_i}{N_i} \qquad (4)$$

$$= \left[\left(\left(\frac{n_1}{N_1}\right)^{\left(\frac{S_1}{S_2} \cdot \frac{N_1}{N_2}\right)^m} + \frac{n_2}{N_2}\right)^{\left(\frac{S_2}{S_3} \cdot \frac{N_2}{N_2}\right)^m} + \dots\dots \frac{n_{i-1}}{N_{i-1}}\right]^{\left(\frac{S_{i-1}}{S_i} \cdot \frac{N_{i-1}}{N_i}\right)^m} + \frac{n_i}{N_i}$$

Because of the inherent complexity in calculation of damage from this form. A simpler general form for the damage equation may be reached by separating the damage occured at each load, life stage and neglecting the accumulation effect. Thus the proposed equation can simply be presented in the general form:

$$D = \sum_{2}^{i-1} \left(\frac{n_{i-1}}{N_{i-1}}\right)^{\alpha_{i-1}} + \frac{n_i}{N_i}$$

$$= \left(\frac{n_1}{N_1}\right)^{\alpha_1} + \left(\frac{n_2}{N_2}\right)^{\alpha_2} + \ldots\ldots\ldots + \left(\frac{n_{i-1}}{N_{i-1}}\right)^{\alpha_{i-1}} + \frac{n_i}{N_i}$$

$$= \left(\frac{n_1}{N_1}\right)^{\left(\frac{S_1}{S_2}\cdot\frac{N_1}{N_2}\right)^m} + \ldots\ldots\ldots + \left(\frac{n_{i-1}}{N_{i-1}}\right)^{\left(\frac{S_{i-1}}{S_i}\cdot\frac{N_{i-1}}{N_i}\right)^m} + \frac{n_i}{N_i} \qquad (5)$$

When failure occurs D = 1, the remaining cycles ratio to failure n_i/N_i can be determined from :

$$\frac{n_i}{N_i} = 1 - \left[\left(\frac{n_1}{N_1}\right)^{\left(\frac{S_1}{S_2}\cdot\frac{N_1}{N_2}\right)^m} + \ldots\ldots\ldots + \left(\frac{n_{i-1}}{N_{i-1}}\right)^{\left(\frac{S_{i-1}}{S_i}\cdot\frac{N_{i-1}}{N_i}\right)^m}\right]$$

Equation(5) is very symmetrical and its application is actually simple once the analyst becomes accustomed to its use.

DAMAGE CURVE ANALYSIS

Cumulative fatigue damage analysis for equations(3) and (5) through the use of damage curves establish its use for multi stage loading. Fig. 1 shows the curves plot for the accumulation of "damage" as a function of cycle ratio for various life and stress values. The LDR, Minors Rule would require that the curves be coincident for all life level, it is their separateness that produces the loading order effect. All curves start at the origin which represents the initial condition of the material where the damage state is zero, and terminate at failure F where D = 1.0 .

The damage curve concept is that, damage accumulation proceeds along the damage curves associated with the life level ratio and load level ratio at which a cycle ratio is applied. For example if a cycle $n_1/N_1 = n_A/N_1$ is first applied at the life level N_1 and load or stress S_1, the damage occured will pend upon the type of loading sequence i.e., if n_2 cycles are then applied at the life level N_2 and stress S_2, then the damage occured is located on damage curve D_1 at B (for the arbitrary choosen loading condition High-Low) or at B' on damage curve D_4 (for the arbitrary choosen loading condition Low-High), fig.1. Similarly, if n_3/N_3 is applied at the life N_3 and stress level S_3, we locate the point C at the same level as B on D_2 (the left or right side of B depends on the value of life cycles and stress ratios, i.e. $(S_2/S_3 \, . \, N_2/N_3)$ < or > $(S_1/S_2 \, . \, N_1/N_2)$, in this arbitrary condition it is 0.017 < .133). Then the damage will go from C to D by the cycle ratio n_2/N_2, and so on for any arbitrary sequence of loading. According to equation(5) the damage occures for the last stage of loading will follows the LDR, Miners line. Hence for two-stage loading we proceed from B to C then along Miners line to D' and for three-stage loading we proceed from D to D" then along Miners line to E. When point F is reached failure occurs, fig.1.

Developed form is used to calculate the remaining number of cycles for two stage loading, fig.2. The remaining number of cycles is greatly affected by the value of the exponent and type of loading. Also the damage occured is function of both the load and the cycle levels ratio. The extent of loading order effect is clearly shown in figure 2. When the first load is applied at the low level followed by a loading at a higher level, then the remaining cycle will be greater than that obtained from Miners and the sum of the cycle ratio will be greater than unity. However if the high load is applied first, followed by the low load level, then the summation of the cycles ratio is less than unity. This agree with early experimental results which failed to satisfy Miner's rule[1 - 7] .

Simple plots of equation(5) is shown in figure 2.for two stage loading. The meaning of the plots is as follows : if a specimen is first cycled for n_1 cycles at stress level S_1 with associated life $N_1(S_1)$, then for subsequent cycling at S_2 with associated life time $N_2(S_2)$ the remaining life time is n_2. It is seen that there is considerable deviation from Miner's rule. It is quite considerable deviation from Miner's rule, depend on the order of loading, the stress level and life time or number of cycles to failure.

PRACTICAL IMPLEMENTATION OF THE PROPOSED EQUATION

The effectiveness of equation(3) in representing the results for two load level experiments conducted on different steel alloys is shown in figs 3 to 6. The results for SAE 4130 steel and Maraging 30 CVM steel.

The plots show the effectiveness of the proposed formulas for representing the experimental results than the linear damage as well as double linear damage rule. The power factor $\alpha = \sqrt{S_1/S_2 \, . \, N_1/N_2}$ is more effective in representing the remaining cycle ratio n_2/N_2 versus the applied cycle ratio n_1/N_1

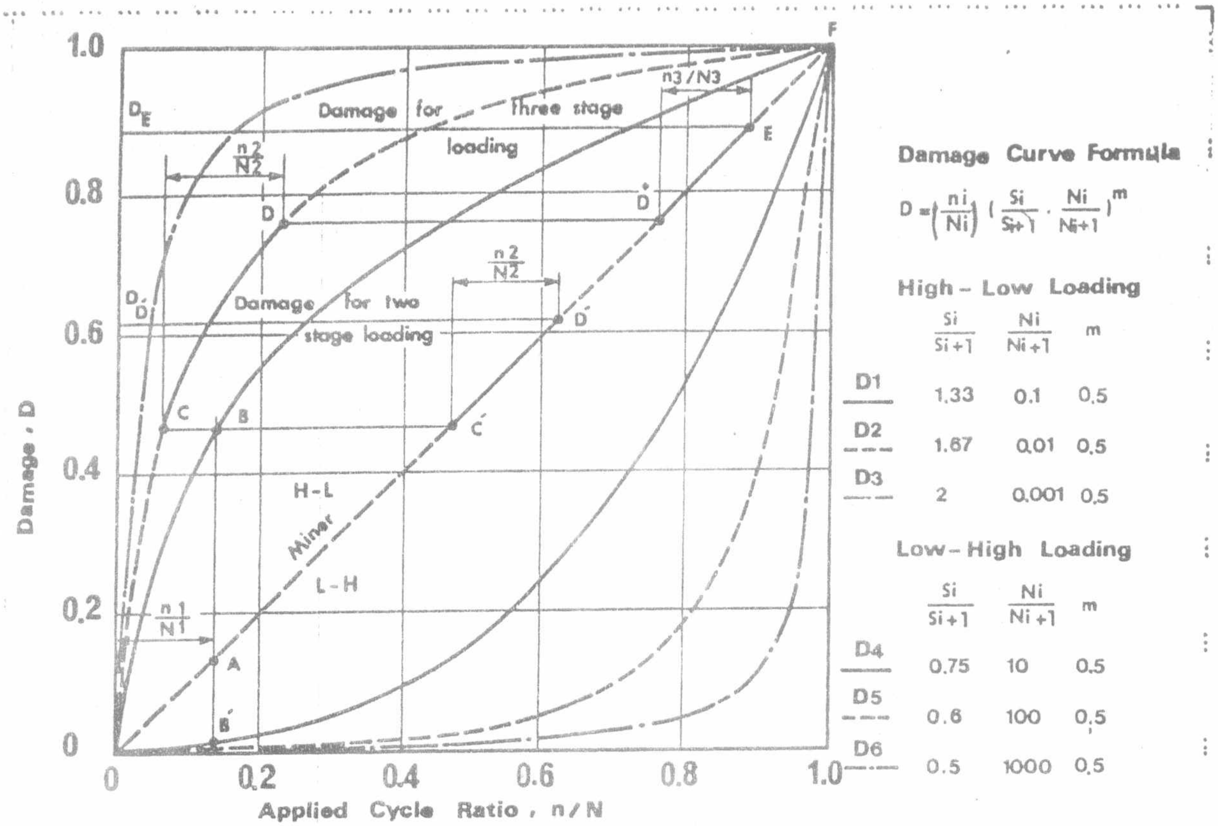


Fig. 1 Schematic of Damage Curve Concept of Summing Cumulative Damage in Complex Loading

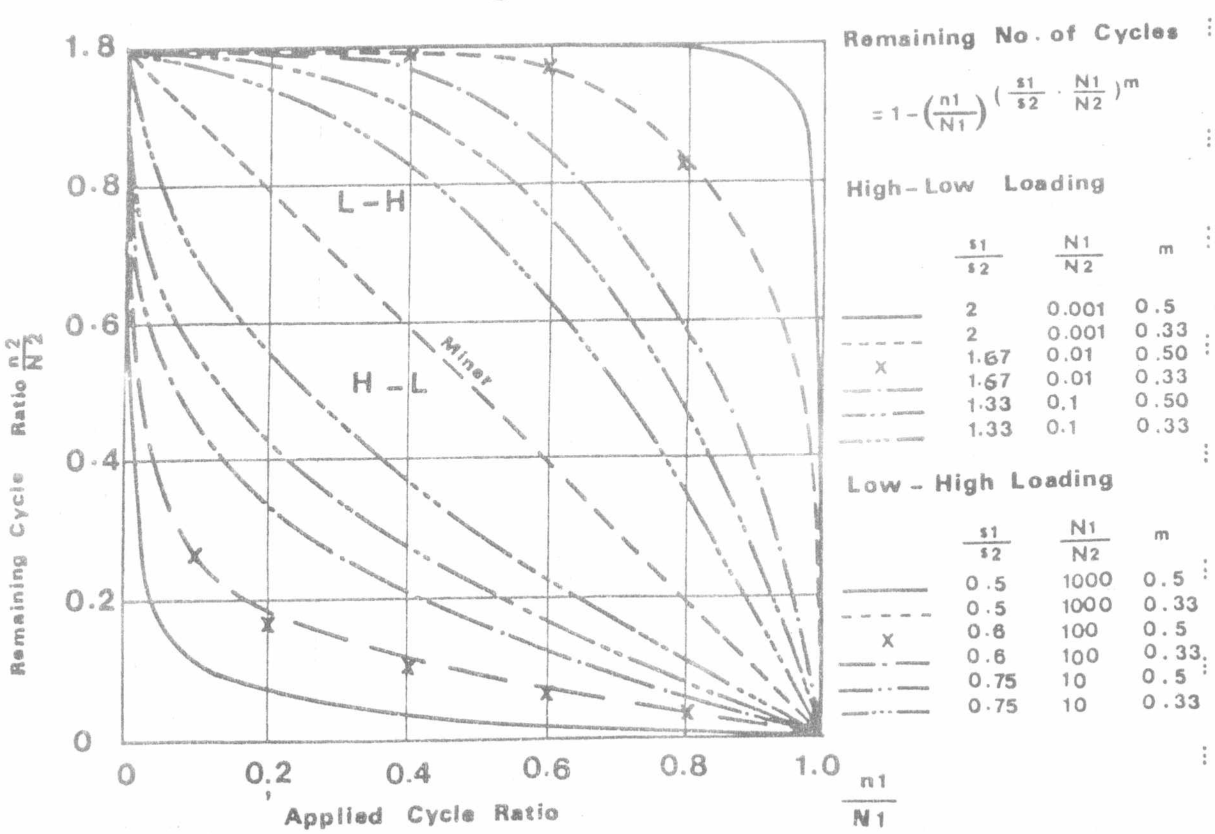


Fig 2 Cycle Ratio Relation ship for two Level Tests as deduced from damage Curve - Present theory

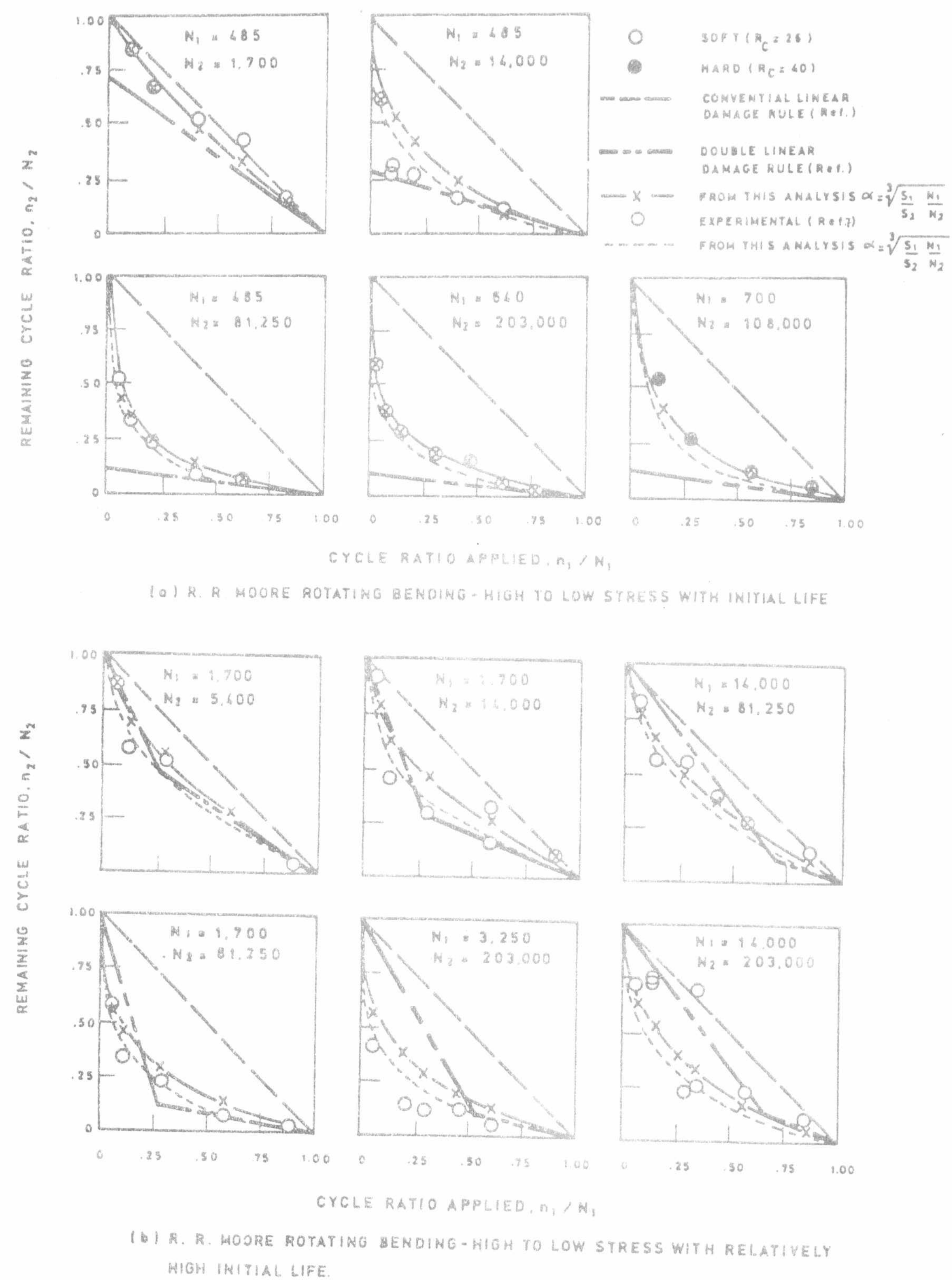
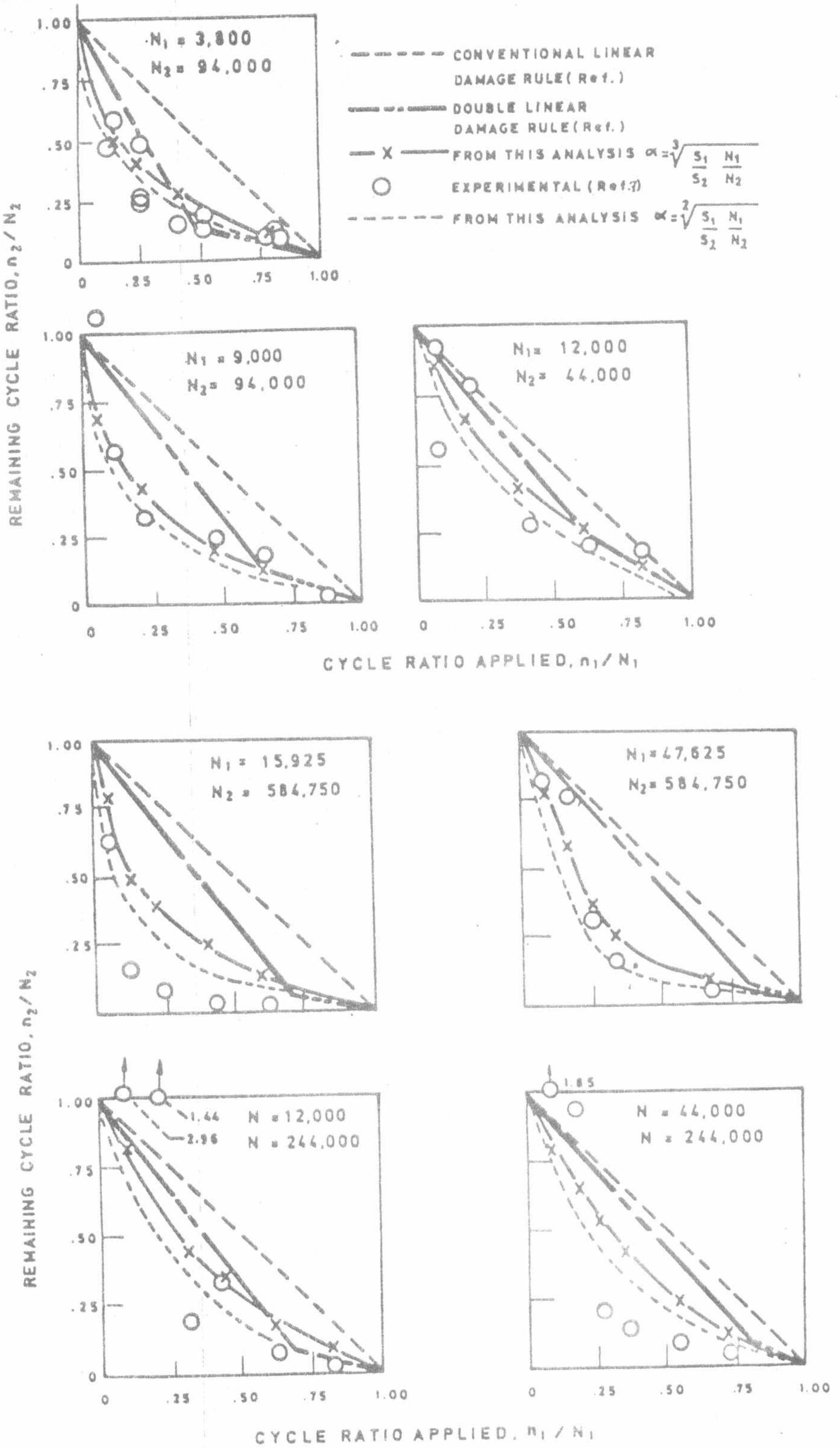
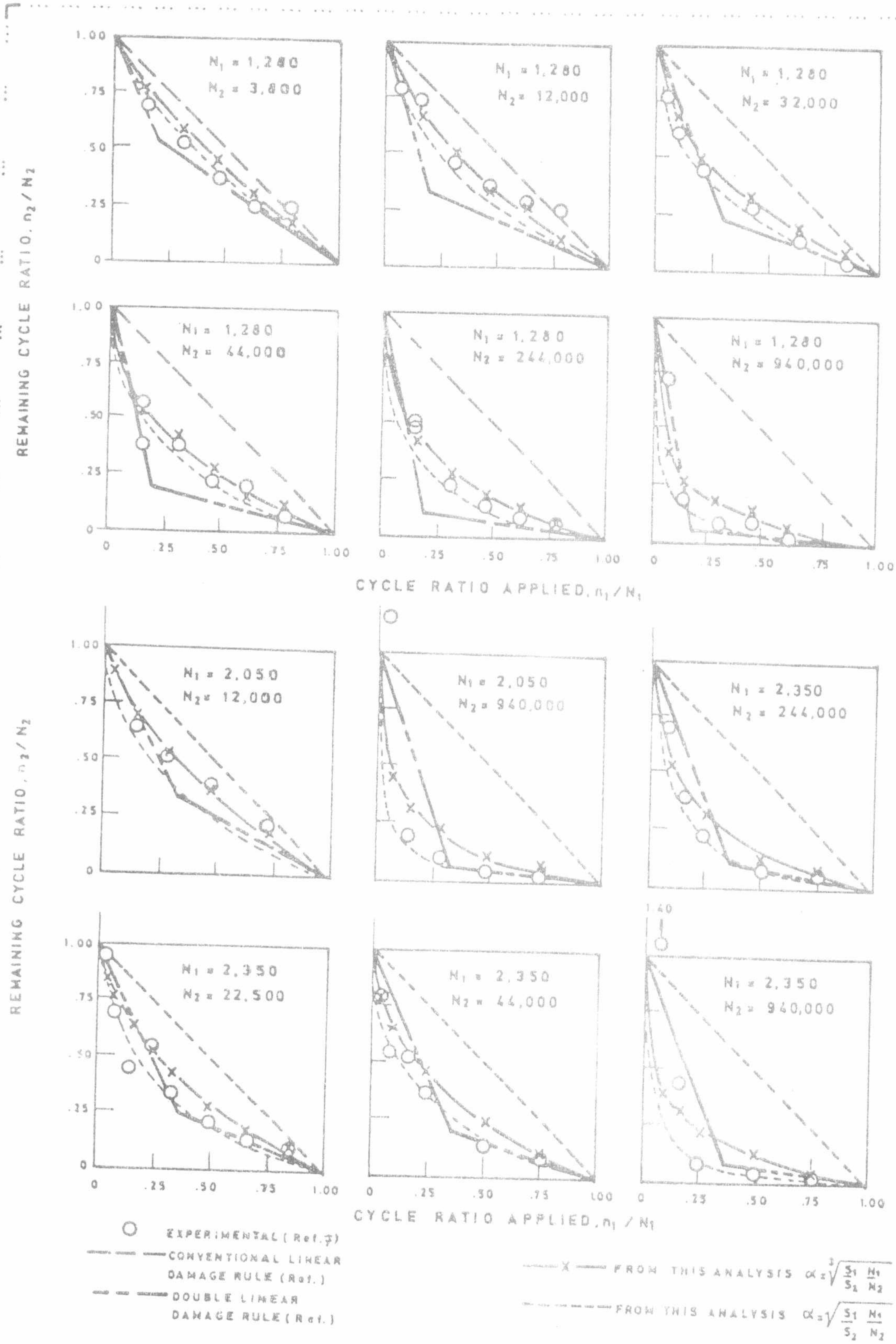


FIG. 3 COMPARISON OF PREDICTED FATIGUE BEHAVIOR BY THIS ANALYSIS, CONVENTIONAL AND DOUBLE LINEAR DAMAGE RULES WITH EXPERIMENTAL DATA FOR TWO STRESS (STRAIN) LEVEL TESTS. MATERIAL, SAE 4130 STEEL.



(b) KRONE ROTATING BENDING-HIGH TO LOW STRESS WITH INITAL LIFE.

FIG.4. COMPARISON OF PREDICTED FATIGUE BEHAVIOR BY THIS ANALYSIS, CONVENTIONAL AND DOUBLE LINEAR DAMAGE RULES WITH EXPERIMENTAL DATA FOR TWO STRESS (STRAIN) LEVEL TESTS_MATERIAL, 300 CVM STEEL.



(b) KRONE ROTATING BENDING - HIGH TO LOW STRESS WITH INITIAL LIFE.

FIG. 5 COMPARISON OF PREDICTED FATIGUE BEHAVIOR BY THIS ANALYSIS, CONVENTIONAL AND DOUBLE LINEAR DAMAGE RULES WITH EXPERIMENTAL DATA FOR TWO STRESS (STRAIN) LEVEL TESTS. MATERIAL, MARAGED 300 CVM STEEL.

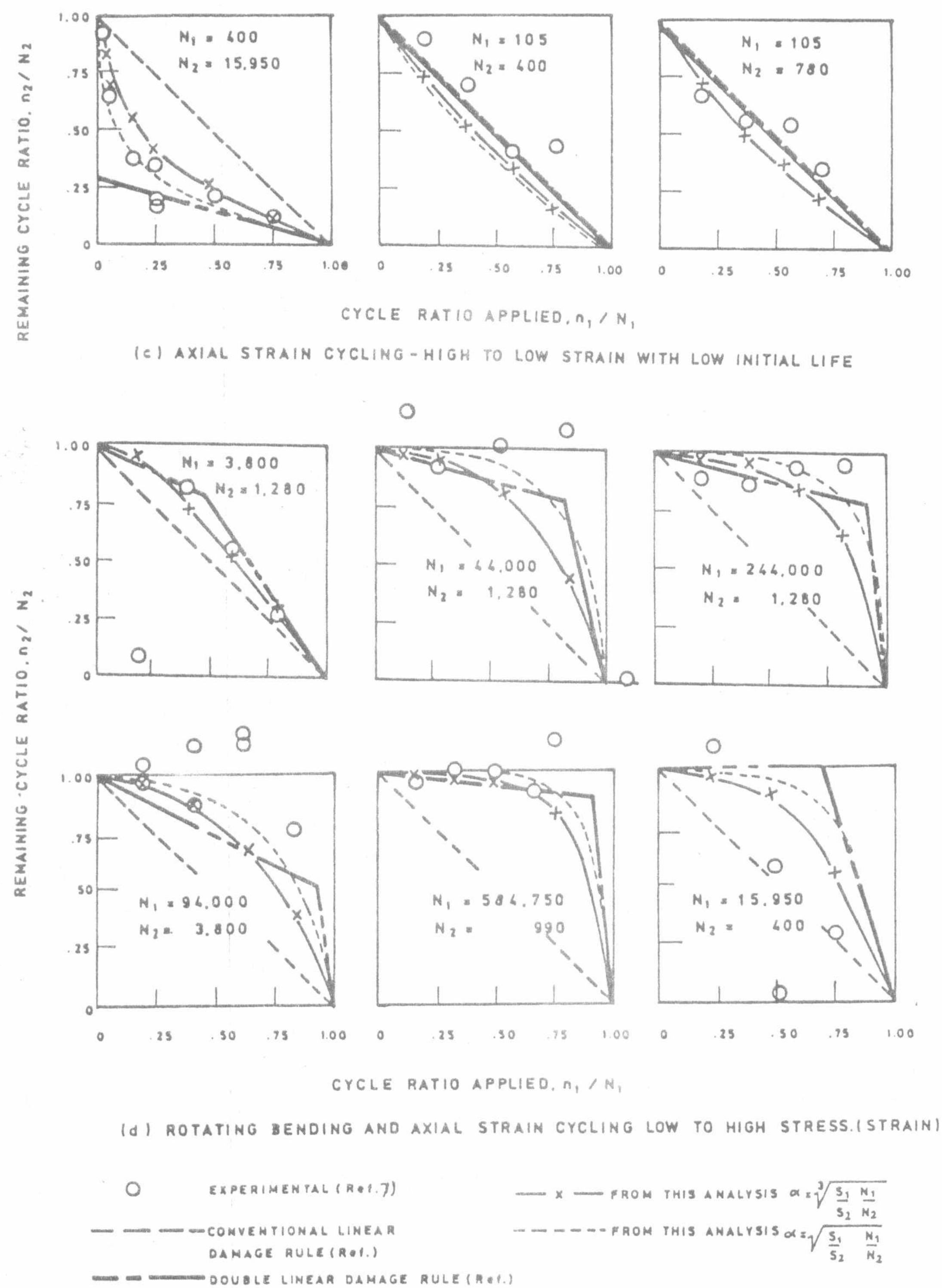


FIG.6 COMPARISON OF PREDICTED FATIGUE BEHAVIOR BY THIS ANALYSIS, CONVENTIONAL AND DOUBLE LINEAR DAMAGE RULES WITH EXPERIMENTAL DATA FOR TWO STRESS (STRAIN) LEVEL TESTS. MATERIAL 300 CVM STEEL

rather than $\alpha = \sqrt[3]{S_1/S_2 \cdot N_1/N_2}$. Thus it is more appropriate to use m = 1/2 rather than 1/3.

Comparison of linear damage rule and double linear damage with the proposed equation, it produces better life predictions than LDR as well as DLDR. on average the proposed equation with $\alpha = S_1/S_2 \cdot N_1/N_2$ provide estimates for remaining life about 15% high while using $\alpha = S_1/S_2 \cdot N_1/N_2$ provide estimates of 8% high or lower depends on the nature of load applied first H-L, or L-H. Thus the proposed equation agrees quite well with the experimental results, the difference may be due to the nature of scatter occurs in fatigue testing.

CONCLUSION

This approach consisting of attempts to assess the damage produced by cycling and its accumulation due to changing amplitude. The proposed rational form give a considerable deviation from Miner's rule. This deviation depends on the order of loading, the stress level and life time - number of cycles to failure.

Comparison of the results obtained from the proposed rational damage equation with the analytical results of the most commonly known methods of LDR, DLDR and the experimental results available in the literature for SAE 4130 steel and Maraging 300 CVM steel, proved its validity. It is seen that agreement with experiment is from fair to excellent and difinitely much better than agreement with Miner's rule or DLDR.

On average its estimate for life prediction approximately ± 8% than experimental while the DLDR provide estimates of 12% high and LDR provide estimates 37% high. It should be emphasized that the proposed equation is based on rational coefficients whose general validity for more than two stage loading has not been established. Hence more study is needed to obtain the good perspective of its merits and its validity for endurance limit predictions.

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