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REAPPRAISAL OF GEARS LUBRICATION IN LIGHT OF ELASTOHYDRODYNAMIC THEORIES

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ABSTRACT

The present work comprises a computer-aided analysis of the lubrication performance of Involute and Novikov gears. Due to the complexity of gears geometry and their kinematics, gears meshing is simulated by equivalent cylinders loaded and moving in such a way to fairly describe gears load sharing effect and the combined rolling/sliding speeds along the line of action.

Different theories to describe Elastohydrodynamic lubrication (EHD) have been considered. Both oil film thickness and coefficient of friction are chosen to represent the lubrication performance characteristics due to their paramont effect on major modes of gear failure. The trend to increase power and efficiency, especially in military applications, has created the need for a more accurate, efficient and quality gears. One of such development is the introduction of circular-arc gears, known as Novikov/Wildhaber or Circ-arc gears. This type of gears has been herein analysed.

Results show that efficient lubrication (maximum oil film thickness and minimum frictional loss) could be attained whenever the involute gears geometry renders higher pressure angles and smaller tooth modules. However, the Novikov gears are more advantageous over the common involute gears as they render oil film thickness ten times greater and a relatively reduced coefficient of friction under the same loading conditions. Lower values for both pressure angles and helix angles are prefered in case of Novikov gears.

INTRODUCTION

The problem of gears lubrication was treated by many investigators either by hydrodynamic theory or by elastohydrodynamic theory. The most early solutions to the equivalent problem of

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the lubrication of a cylinder near a plane was presented by Martin [1] considering Reynolds equation for rigid solids and an incompressible isoviscous lubricant. Later, Peppler [2] was concerned with the maximum oil film pressure which could occur in a contact between gears teeth lubricated by a fluid of constant viscosity. Meldahl [3] examined the effect of high pressure on film shape and pressure distribution for a constant viscosity lubricant. Gatcombe [4] examined theoretically the influence of the viscosity pressure characteristics of a lubricant on film formation. However, his solution was not enough to allow the prediction of satisfactory fluid film between gear teeth. An extremely valuable work had been carried out by Grubin [5] . He examined the problem by assuming that the surfaces of the bounding solids would adopt the shape produced by dry contact. His analysis allowed for the effect of pressure upon lubricant viscosity. He derived an approximate film thickness equation for highly loaded elastic contacts. Hersey and Lowdenslager [6] completed an investigation of the film thickness between rigid gear teeth lubricated by a fluid characterised by a parabolic viscosity-pressure relationship. The change in theoretical load capacity from the isoviscous prediction was found to be very similar to that calculated by Gatcombe for an exponential relationship.

Petrusevich [7] obtained solutions satisfying the governing elastic and hydrodynamic equations. The three main conclusions of his work, which now characterise the highly loaded lubricated contacts, are an almost parallel oil film within the contact zone followed by a local restriction near the trailing end, a Hertzian pressure curve over most of the contact and a local second pressure peak of considerable value near the outlet end of the contact zone. Portisky [8] reported an attempt to find a pressure distribution which satisfies the conditions of the lubricated contacts. Dorr [9] re-examined the elastic contacts lubricated by an isoviscous fluid. He developed an iterative method to calculate the pressure distribution and surface displacement. Osterle [10] analyzed the involute gears lubrication problem in order to make a correction to the conventional parabolic arc film thickness.

Dowson and Higginson [11] presented accurately a formula for the minimum film thickness in EHD lubricated contacts. Their formula was then applied for the analysis of lubrication of rollers and gears. Dudley [12] summerized some observations and examples of gear teeth EHD lubrication. One of the most important observation he got was the existance of an appreciable oil film seperating the teeth and ranges from 0.25 to 25 microns. He also introduced a correlation between gear teeth failure and both oil film thickness and teeth surface finish. Dowson and Whitaker [13] discussed the isothermal lubrication of cylinders. They introduced the valid range of application of the "rigid" and "elastic" film thickness relationships. They defined an "intermediate" range of conditions between the rigid and the elastic regimes.

Recently, Mokhtar [14] has confirmed experimentally, and defined the boundaries between rigid and elastic lubrication regimes.

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Radzimovisky and Wayne [15] presented a theoretical work on spur gears lubrication. They concluded that, for a constant load applied to a constant speed driving gear, the film thickness varries in a continuous cyclic manner, and that the squeezing motion has a definite significance on the oil film load carrying capacity. Cheng [16] , [17] developed a numerical solution of EHD film thickness in an elliptical contact and solved the two dimensional Reynolds equation in the inlet region by a finite difference method. He presented an isothermal EHD solution for the full range of pressure viscosity coefficient. Gu [18] published an investigation about EHD lubrication of involute gears taking into account the load sharing between the engaged teeth. A value of 2 to 2.5 microns for the film thickness was recorded.

Wellauer and Holloway [19] presented an analysis for the application of EHD theory to calculate gear teeth oil film thickness, specific film thickness and the probability of tooth surface distress. Mokhtar [20] , [21] published a new approach to gears and cams lubrication giving a complete analytical solution of rigid disks operating under dynamic loading. The approach introduced the developed hydrodynamic pressure due to squeeze action under sliding and rolling motions which represent the conditions of gears operation. Rao [22] showed that the maximum coefficient of friction during the meshing cycle is about 1.5 times greater than the average value. Jackson & Chapman [23] and Jackson & Rowe [24] summerized the EHD lubrication of heavily loaded gears and provided equations for determination of the lubricant film thickness by introducing a Lubricant Parameter(LP) as a lubricant property for EHD lubrication. Wang and Cheng [25] reported a theoretical study to predict the variation of the dynamic load, the surface temperature and the lubricant film thickness along the path of contact during the engaging cycle of a pair of involute spur gears.

About tribology of Novikov gears, little has been published as regards both analytical and experimental. Kumar and Bremble [26], [27] have studied the basic requirements for simulation of contact conditions between their teeth. They measured the oil film thickness between a crowned cylinderical disc rolling inside a hollow cylinder by measuring the capacitance .

From the previous review, it could be seen that the full analytical study of gears lubrication has not been attained. In the analysis to come, the geometrical parameters (pressure angle and tooth module for involute profile and pressure angle, helix angle and tooth module for Novikov gears profile) will be investigated. The analysis will consider the EHD lubrication performance of gears as measured by the oil film thickness and the coefficient of friction.

GOVERNING EQUATIONS

Simulation Of Gears:

As shown in fig.(1), the path of contact C is divided into two parts namely the length of approach L_a and the length of recess L_r. For a contact point at a distance X from the pitch point, the radii of curvature are;

$$R_{c1} = R_1 . \sin\phi + X$$

$$R_{c2} = R_2 . \sin\phi - X$$

and the equivalent radius of curvature R is;

$$1/R = 1/R_{c1} + 1/R_{c2} \qquad (1)$$

The velocities at the point of contact are;

$$u_1 = \omega_1 . (R_1 . \sin\phi + X)$$

$$u_2 = \omega_2 . (R_2 . \sin\phi - X)$$

$$U = (u_1 + u_2)/2$$

$$= V.(\sin\phi + (R_2 - R_1).X/(2R_1R_2)) \qquad (2)$$

If W is the load per unit face width of the gear teeth, the shared load W_D acting on one pair of the engaged teeth is;

$$W_D = W \qquad \text{for} \quad (P_b - L_a) \geq X \geq -(P_b - L_r)$$

and $$W_D = W/2 \qquad \text{otherwise} \qquad (3)$$

where P_b is the base pitch.

In Novikov gears, fig.(2), the teeth profile of the pinion is convex while that of the gear wheel is concave. The working surface of the teeth is circular helical and the contact lime (ellipse under load) sweeps across the face width in the axial direction.

The lengthwise radii of curvature in the normal section[29] is given by;

$$R_{1N} = \frac{R_1 .(1 + \tan\beta . \cos\phi)^{3/2}}{\tan^2\beta . \sin\phi(1+R_1 . \sin\phi/\rho_1)}$$

$$R_{2N} = \frac{R_2 .(1 + \tan\beta . \cos\phi)^{3/2}}{\tan^2\beta . \sin\phi(1+R_2 . \sin\phi/\rho_2)}$$

and the equivalent radius of curvature R_e is given by;

$$1/R_e = 1/R_{1N} + 1/R_{2N} \qquad (4)$$

The axial overlap factor or contact ratio is expressed as;

$$\xi = b . \sin\beta / m_n > 1 \qquad (5)$$

and concequently, the load will be transmitted to the wheel by

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one pair and two pairs of teeth. The latter case (2 pairs) occures on the portions of the wheels nearest to the end faces and having the length $(\xi - 1)b$.

The velocity of contact travel along the axis of the gears is;

$$U_R = V.\cot\beta \qquad (6)$$

and this rolling velocity depends mainly on the helix angle.

EHD Lubrication Theories:

The oil film thickness depends upon the combined action of several factors such as the oil viscosity, the teeth curvature, the acting load, the entraining velocity and the elastic properties of teeth materials. The oil film thickness between gear teeth at any point along the path of contact can be predicted from the existing EHD lubrication theories as expressed in dimensionless form by the following formula;

$$(h/R) = K.(\alpha E)^{e1}(\eta U/ER)^{e2}(W/ER)^{e3} \qquad (7)$$

where;

K is a constant (given in the following table).

U is the entraining velocity $(U=(u_1+u_2)/2)$.

Values of e_1, e_2, e_3 are also given in the table.

Theory	K	e_1	e_2	e_3
Grubin [5]	1.950	0.730	0.730	-0.091
Dowson & Higginson [11]	1.600	0.600	0.700	-0.130
Cheng(Line Contact) [17]	1.984	0.740	0.740	-0.110
Cheng(Elliptical Contact) [16]	1.176	0.688	0.688	-0.033

Since this equation is dimensionless, it could be transformed to the dimensional form with any consistent set of units.

RESULTS AND DISCUSSIONS

In case of involute gears, the oil film thickness is calculated according to Cheng's theory [17] for line contact and the coefficient of friction is calculated according to Buckingham's formula [28] based on the sliding velocity. The effect of pressure angle on oil film thickness is illustrated by fig.(3) and fig.(4). The increase in oil film thickness with the increase in pressure angle can be attributed to the corresponding increase in both, equivalent radius of curvature and entraining velocity, equations (1),(2). It is also found that small pressure angles result in uniform nondisturbed film thickness, and the load sharing occures at big portions of contact in case of small pressure angles, which explains the smoothness in operation for gears of low pressure angles. The effect of pressure angle on the sliding friction is shown in fig.(5). It is clear that higher friction coefficients are at smaller pressure angles, which could be referred to the effect of the pressure angle on the sliding velocity.

The effect of tooth module on oil film thickness is shown in fig.(6),(7). It is found that smaller modules result in higher film thickness with a small disturbance near the pitch point. The disturbance shown at the middle is contributed to the effect of load sharing. Fig.(8) illustrates the effect of tooth module on the friction coefficient. It is found that the coefficient of friction gets higher values at larger modules, which is referred to the increase in sliding velocity at higher values of tooth modules.

In case of Novikov gears, the oil film thickness is calculated according to Cheng's theory[16] for elliptical contact and the friction coefficient calculation is based on Crook's theory[2] for rolling traction. Due to load sharing, the oil film thickness is slightly decreased in the middle part of the tooth face and the coefficient of friction is greatly reduced (about 50%). The oil film thickness is found to be greatly decreased with the increase in helix angle as shown in fig.(9),(10), which is attributed to the decrease in rolling velocity and equivalent radius of curvature, equations(6),(4). As the rolling friction is a function of the oil film thickness, the same effect is found as shown in fig.(11).

The effect of pressure angle on oil film thickness is shown in fig.(12). Higher values for oil film thickness are attributed to high equivalent radius of curvature and pure rolling situations with high rolling velocity. The decrease of oil film thickness with the increase in pressure angle is due to the decrease in the equivalent radius of curvature. The coefficient of friction also decreases with the increase in pressure angle as shown in fig.(13).

The tooth module is found to have no effect on both oil film thickness and rolling friction, since there is no motion along the tooth profile.

CONCLUSIONS

For involute profile gears, higher pressure angles and smaller tooth modules are preferred for efficient lubrication, while for circ-arc Novikov gears smaller pressure angles and smaller helix angles are preferred.

Compared with involute gears, the oil film thickness between Novikov gears teeth is about ten times greater than that between involute teeth at the same operating and geometrical conditions, while the coefficient of friction is about one half of its value in case of involute gears.

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NOMENCLATURE

b	Gear tooth width.
C	Length of contact path., $(C=L_a+L_r)$.
E	Equivalent modulus of elasticity.
f	Coefficient of friction.
h	Oil film thickness.
L_a, L_r	Length of approach and recess respectively.
m_n	Gear tooth normal module.
R_1, R_2	Pitch radii of gears, pinion and wheel respectively.
R_{c1}, R_{c2}	Radii of curvature for equivalent discs.
U_R	Rolling velocity.
U_s	Sliding velocity.
u_1, u_2	Linear velocity of contact surfaces.
V	Pitch line velocity.
X	Distance from the pitch point to the point of contact along the line of action for involute gears. Or the distance from the face end to the point of contact for Novikov gears.
α	Pressure-viscosity coefficient.
β	Helix angle.
η	Lubricant viscosity at inlet temperature.
ρ_1, ρ_2	Profile radii for pinion and gear wheel.
ϕ	Pressure angle.
ω	Angular speed.
ν	Poisson's ratio.
ξ	Overlap or contact ratio.

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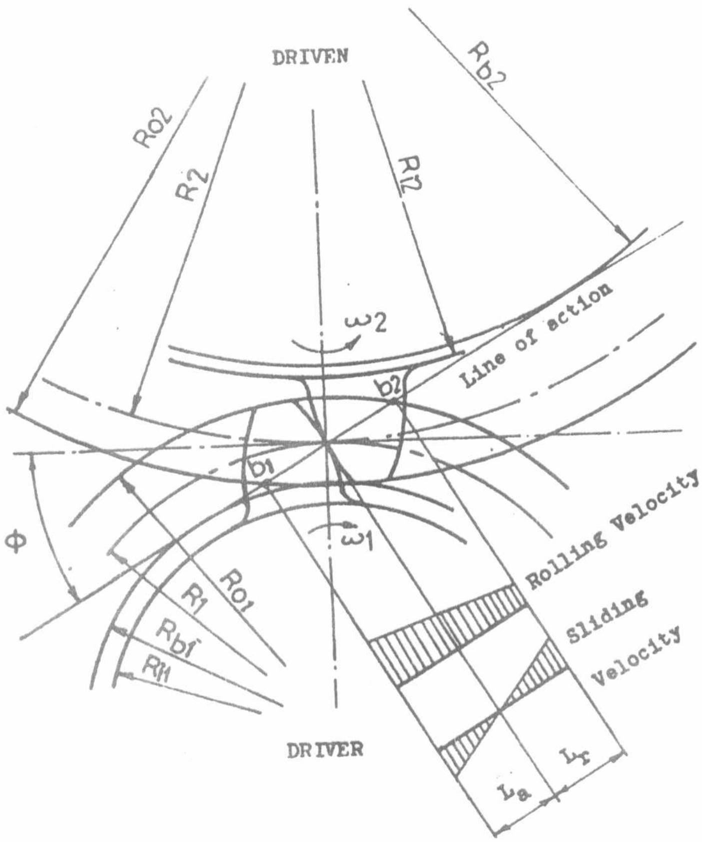

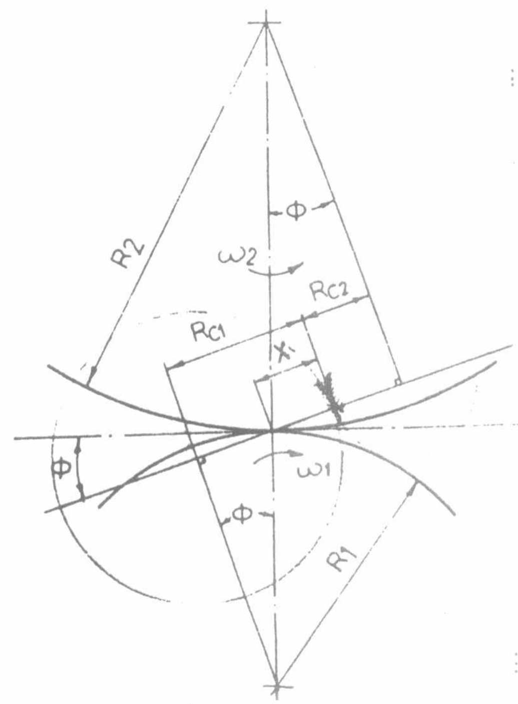

Simulation of Spur Gear Contact.

Fig.(1): Sliding and Rolling Velocities.

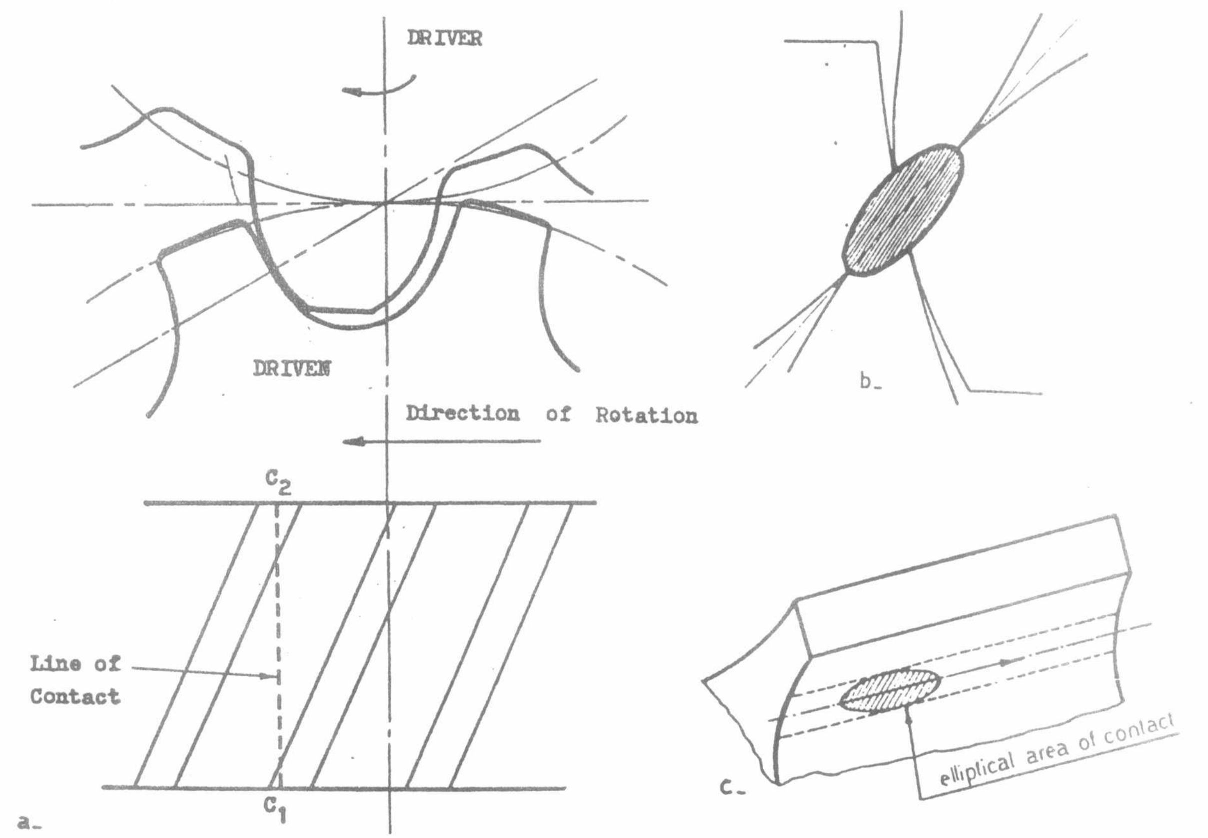

Fig.(2): Novikov Gear Teeth Contact

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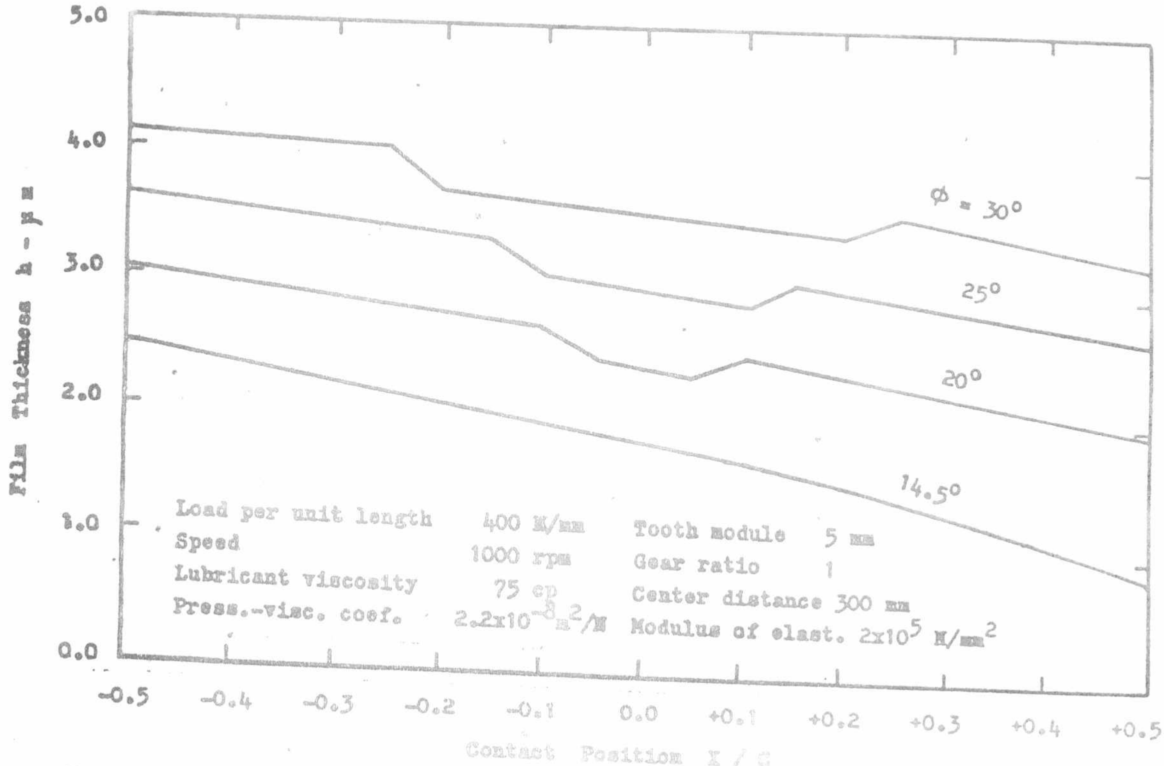

Fig.(3): Effect of pressure angle on film thicknesss variation.

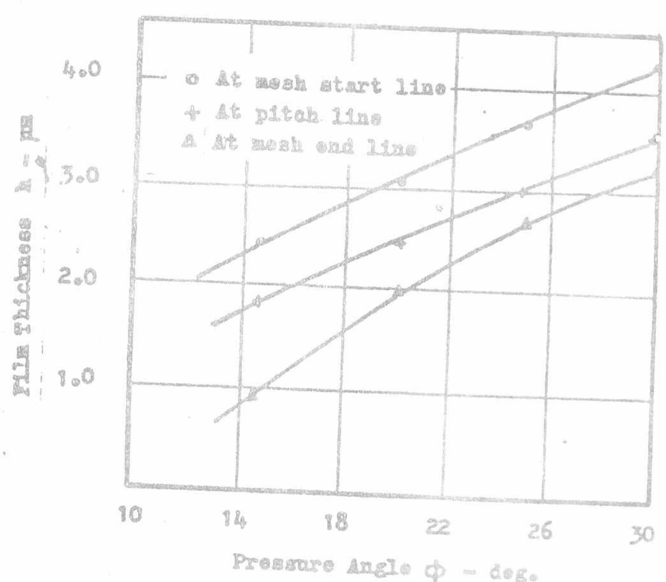

Fig.(4): Effect of pressure angle on film thickness.

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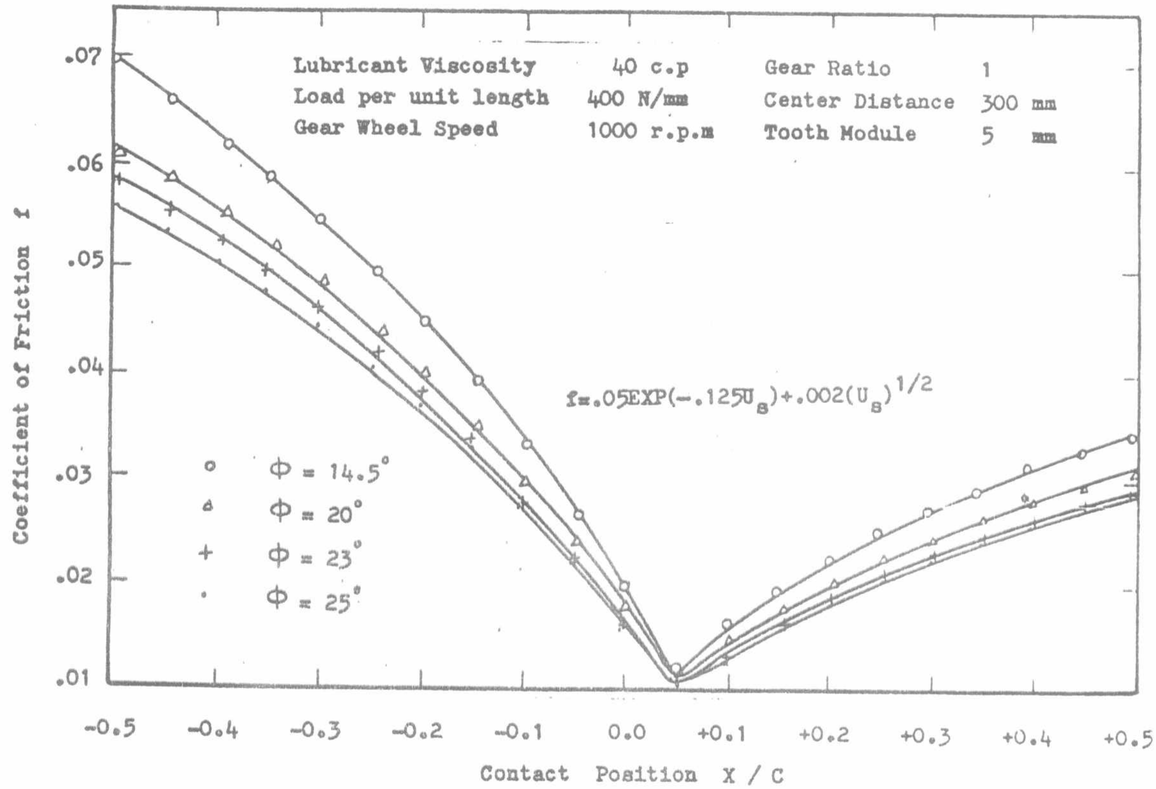

Fig.(5): Effect of Pressure Angle on Coefficient of Friction in spur gears.

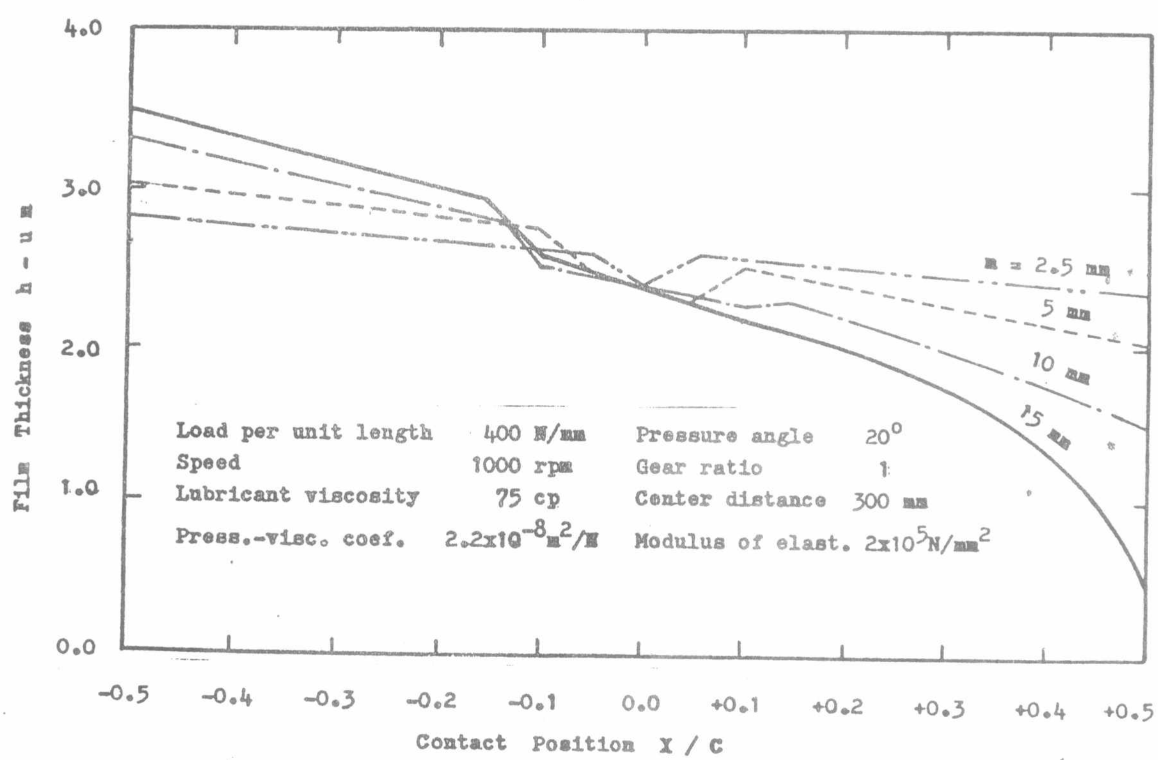

Fig.(6): Effect of tooth module on film thickness variation.

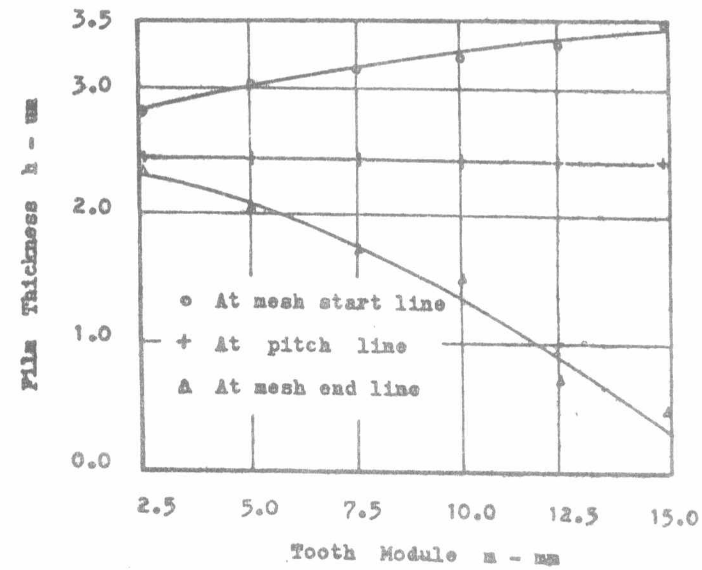

Fig.(7): Effect of Tooth Module on film thickness.

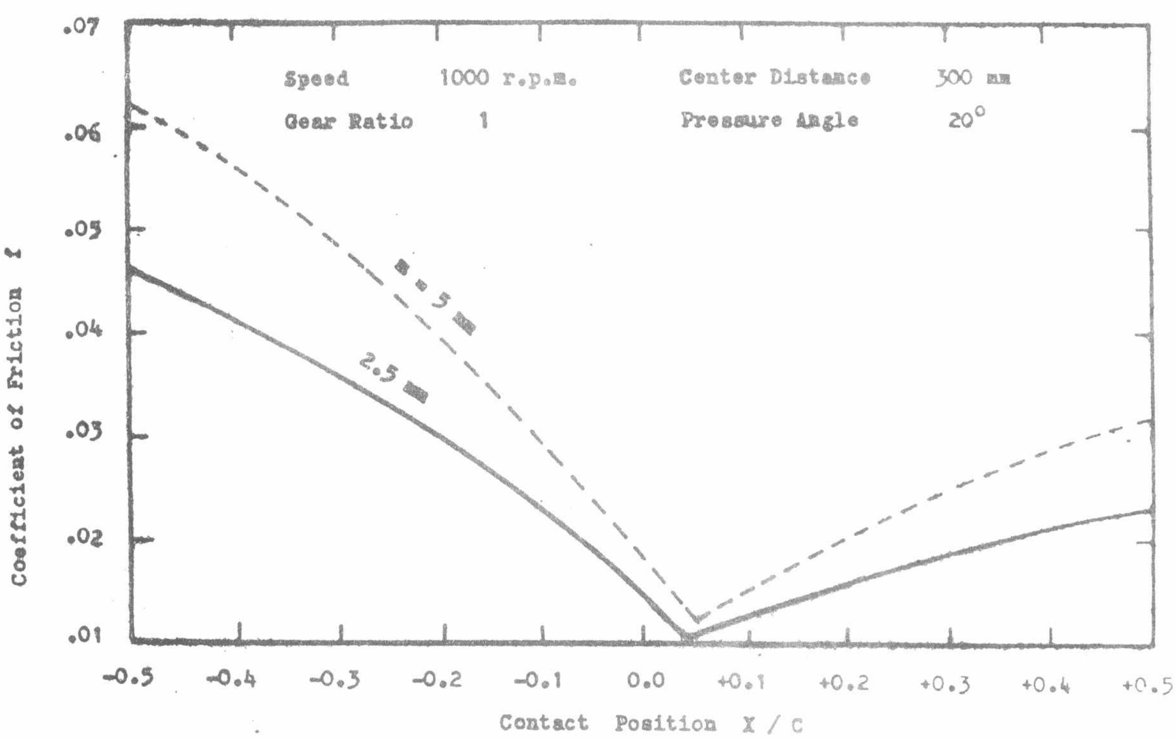

Fig.(8) : Effect of Tooth Module on Variation of Coefficient of Friction in spur gears.

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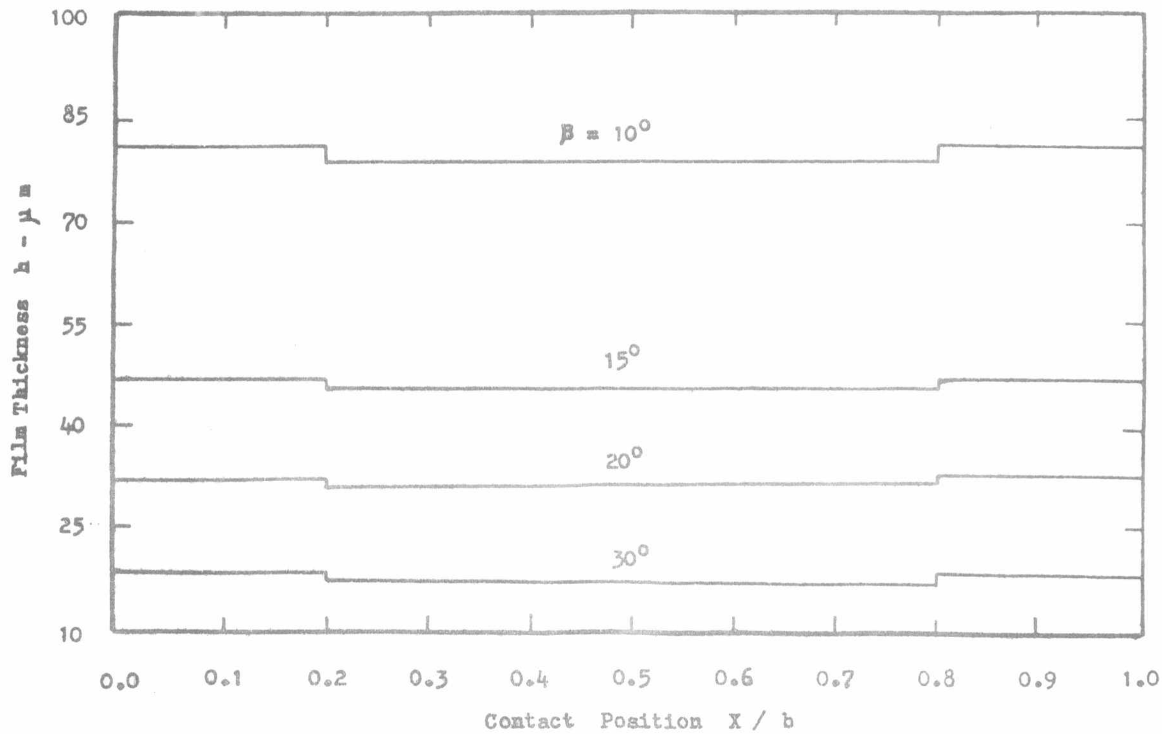

Fig.(9): Effect of Helix Angle on Variation of Film Thickness across the Tooth Width.

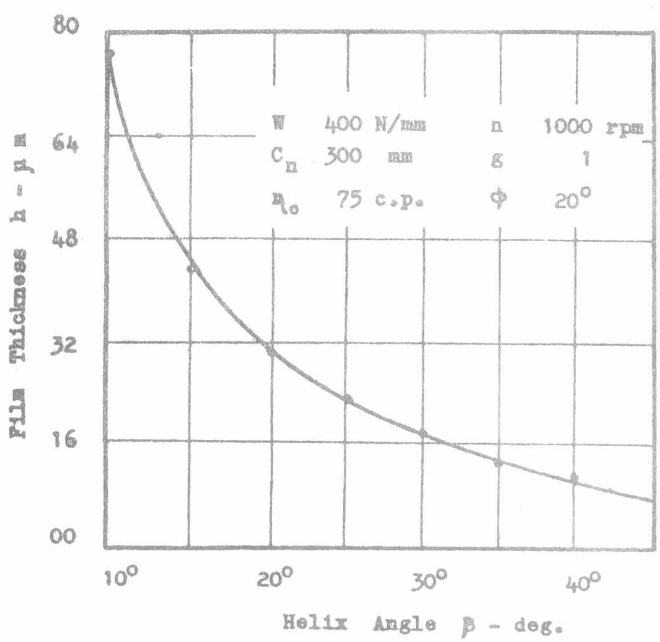

Fig.(10): Effect of Helix angle on Film thickness.

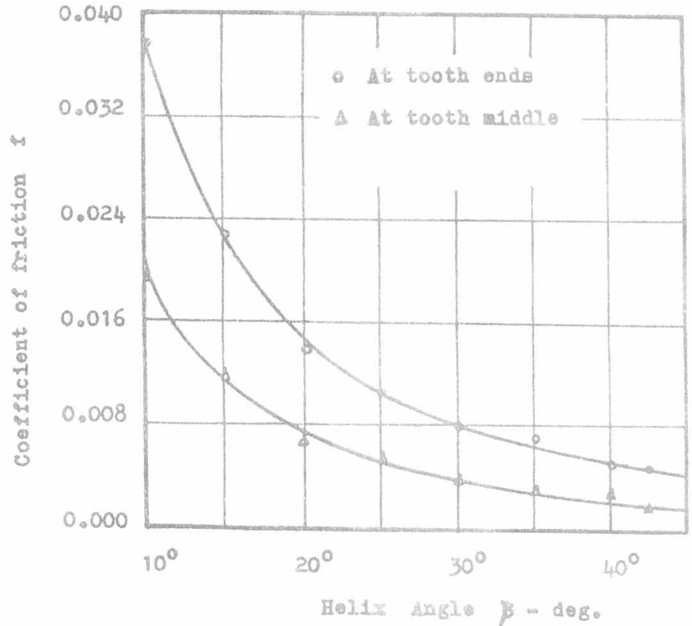

Fig.(11): Effect of Helix Angle on Coefficient of friction.

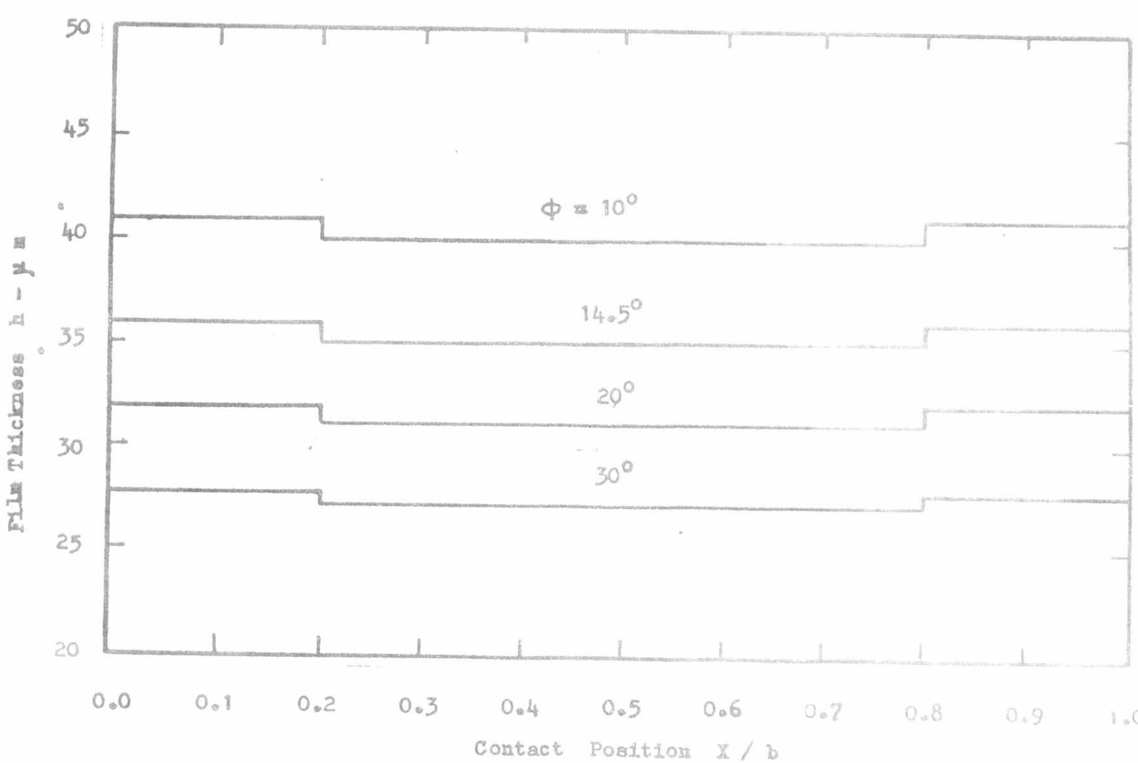

Fig.(12): Effect of Pressure Angle on Variation of Film Thickness across the Tooth Width.

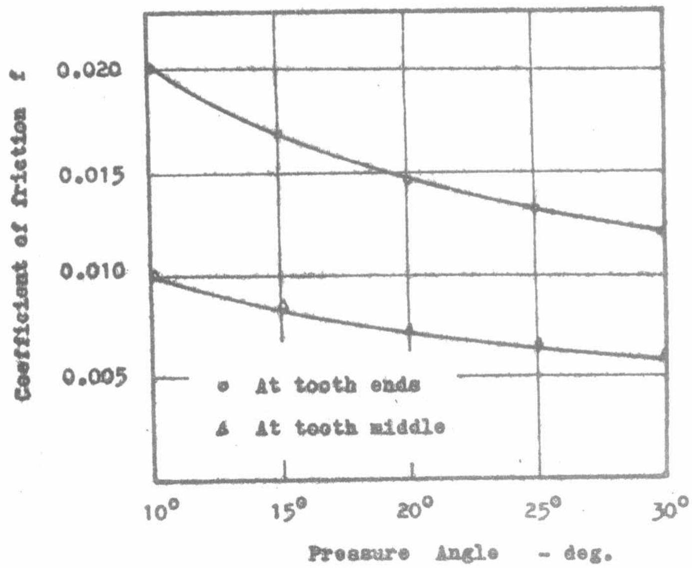

Fig.(13): Effect of Pressure Angle on the Coefficient of friction

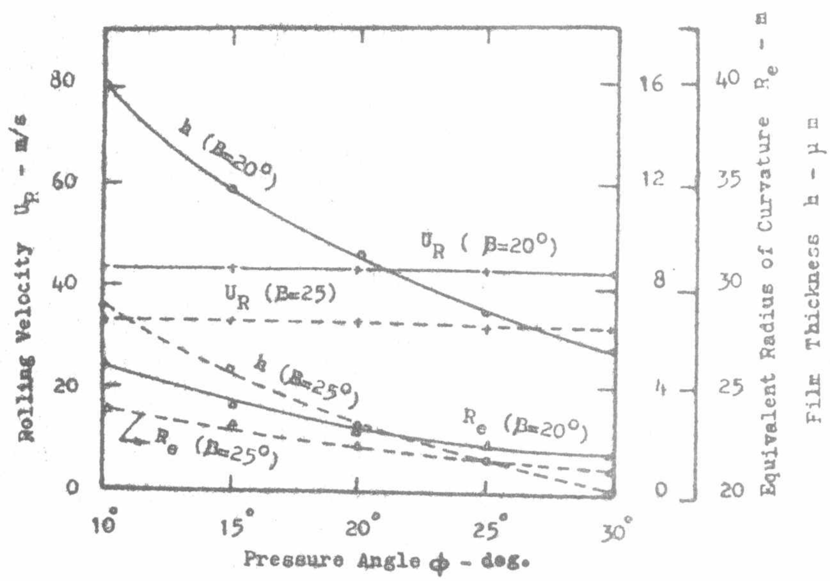

Fig.(14): Effect of Pressure angle on Film thickness, Rolling velocity and Equivalent radius of curvature.