MILITARY TECHNICAL COLLEGE

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COMPARISON OF METHODS OF SELECTING GEAR RATIOS OF AUTOMOBILE TRANSMISSION

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ABSTRACT

Inspite the fact that conventional gear boxes of automobiles have several disadvantages, they can be made more efficient if the number of gear ratios and their selection are properly chosen. Inereasing the number of speeds improves the vehicle's dynamic performance which is expected to be optimal with stepless continuously variable speed drive. However, for a fixed number of speeds, selection of individual gear ratios plays an important role in improving the vehicle's dynamic performance.

Selection can be done according to various mathematical progressions such as: arithmatic, harmonic, geometric with constant and increasing roots.

The goal of this work is to compare these commonly used methods. Comparison is based on calculating the wasted energy represented by the difference between areas under the curves of discret and continuous power transmission. The wasted energy corresponding to each progression was calculated by a special computer Fortran program considering the technical data of a Jeep car as a real example.

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INTRODUCTION

Automobile transmission is required to provide the vehicle with tractive effort-speed characteristics suitable for the largely variating load conditions. Between the many types of transmission, the mechanical with stepped gear ratios is still widely used.

Mechanical gear box has a highest, a lowest and intermediate gear ratios. The highest is determined from the condition for maximum tractive effort i.e maximum gradability specified or lowest speed required. On the other hand, the lowest ratio is determined knowing the maximum required vehicle speed. The intermediate ratios are chosen according to different methods or mathematical progressions.

METHODS OF SELECTION OF THE GEAR RATIOS

The following mathematical progressions are commonly used for determining the gear ratios of the automobile transmission:

1. Arithmatic
2. Harmonic
3. Geometric
4. Geometric with increasing root

Slection can also be done using two of any of these progressions. Choice of the suitable progression depends upon vehicle type, specific power and operational demands. For example, for passenger cars having high specific power, higher gears (i.e lower ratios) are used. For heavy duty vehicles where load conditions are more severe, low gears are used.

In arithmatic progression, high gears are widely spaced and low gears are more close to each other, Fig. 1a. In harmonic progression, the high gears are more spaced and the low gears are widely spaced, Fig. 1b. Therefore, the geometric progression stands as a compromise between them, Fig. 1c. Geometric progression with increasing root is a compromise between the geometric and the harmonic, Fig. 1d.

The relationships for calculating the individual ratios according to the mentioned progressions are as follows:
Arithmatic.

$$i_1 - i_2 = i_2 - i_3 = \ldots\ldots = i_{n-1} - i_n = \text{constant}$$

Where, $i_1, i_2, i_3, \ldots\ldots$ are the ratios of 1st, 2nd and 3rd speeds
and i_n, i_{n-1} are the ratios of top and before the top speeds

Harmonic.

$$\frac{1}{i_2} - \frac{1}{i_1} = \frac{1}{i_3} - \frac{1}{i_2} = \ldots\ldots = \frac{1}{i_{n-1}} - \frac{1}{i_n} = \text{constant}$$

Geometric .

$$\frac{i_1}{i_2} = \frac{i_2}{i_3} = \ldots\ldots\ldots = \frac{i_{n-1}}{i_n} = \text{constant}$$

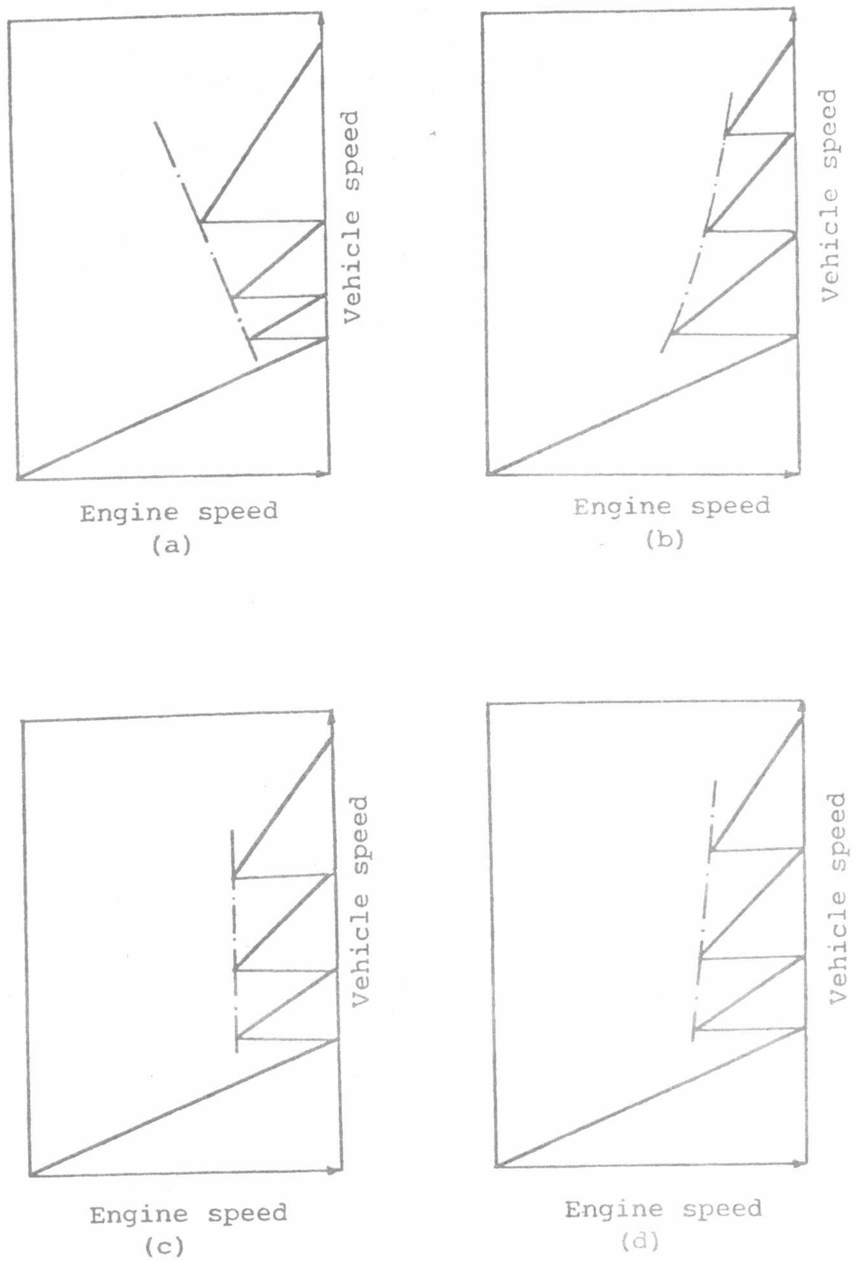

Fig.1. Engine and vehicle speed relativships with gear boxes spacing according to:

(a) Arithmatic progression.
(b) Harmonic progression
(c) Geometric progression
(d) Geometric progression with increasing root.

Geometric with increasing root.

$$\frac{i_{n-1}}{i_n} = q_1 \quad \frac{i_{n-2}}{i_{n-1}} = q_2 \text{ , ... , } \frac{i_1}{i_2} = q_{n-1}$$

and

$$\frac{q_2}{q_1} = \frac{q_3}{q_2} = \ldots = \frac{q_{n-1}}{q_n} = \text{constant}$$

Where, q_1, q_2, ... q_n are the values of the increasing root.

CHARACTERISTICS OF THE MECHANICAL TRANSMISSION

During gear chaning in the mechanical gear box, a part of energy is wasted due to stepped power transmission. Increasing the number of gear ratios minimizes this energy waste and makes the tractive effort-speed diagram more close the ideal one representing a continuous power transmission, this is schematically shown in Fig. 2.

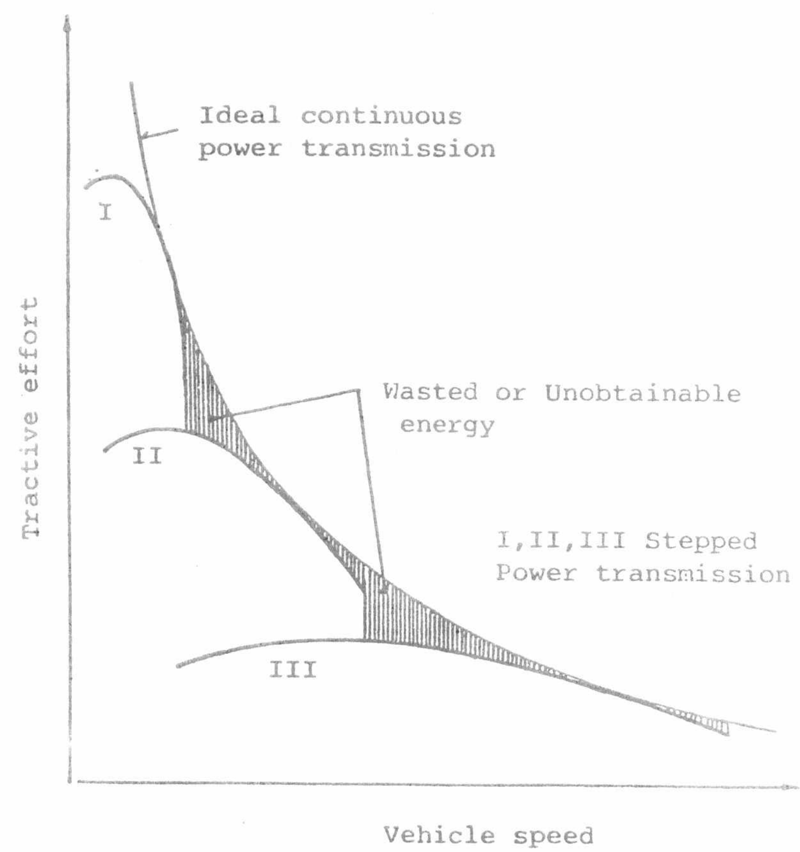

Fig.2. Traction-Speed relationships for stepped and continuous power transmission.

With a mechanical transmission, if two gears are widely speced, a gap of tractive effor would appear when chaning from a higher to a lower speed as Fig. 3a,b shows. In this case, the low gear permits to accelerate the vehicle on a specified grade and the higher gear when shifted would not enable overcoming the same grade. A speed gap may also occur causing the utilization of the engine in its unstable speed range, Fig. 3 c,d.

COMPUTATION OF THE WASTED ENERGY

The wasted energy is represented by the difference of areas nunder the curves of tractive effort with ideal and stepped power transmission. To compute these areas, the equations of tractive effort-speed curves should be known.

Equation of the ideal curve is written as:

$$F_t . V/2700 = P_{e\,max} \cdot \eta_t = \text{constant} \tag{1}$$

Where:

F_t = available tractive effort at wheels , N
V = vehicle speed, km/h
$P_{e\,max}$ = maximum engine power, HP
η_t = total mechanical effeciency of the running gear.

Equation of the tractive effort for stepped power transmission is written as:

$$F'_t = M_e\, i_t . \eta_t / r_d = K_1\, M_e\, i_t \tag{2}$$

Where:

M_e = engine torque, N.m
i_t = total transmission ratio
r_d = wheel's dynamic radius

Equation of engine torque can be expressed in terms of engine power (P_e) and speed (n_e) as :

$$M_e = 7162\, P_e / n_e = K_2\, P_e / n_e \tag{3}$$

Relationship between engine power and speed can be expressed by the following equation:

$$P_e = P_{e\,max} \left[A_1 \left(\frac{n_e}{n_N}\right) + A_2 \left(\frac{n_e}{n_N}\right)^2 - A_3 \left(\frac{n_e}{n_N}\right)^3 + A_4 \left(\frac{n_e}{n_N}\right)^4 \right] \tag{4}$$

Where: A_1, A_2, A_3, A_4 = constants determined by fitting the actual engine power curve.
n_N = engine speed at maximum power.

Vehicle speed can also be expressed as a function of engine speed as follows:

$$V = 0{,}377\, n_e\, r_d / i_t \quad \text{(km/h)}$$

hence,

$$n_e = V.\, i_t / 0{,}377\, r_d = K_3\, i_t\, V \tag{5}$$

Substituting P_e , n_e from equations 4,5 into equation 3, and the obtained expression for M_e into equation 2, the equation for F'_t is obtained.

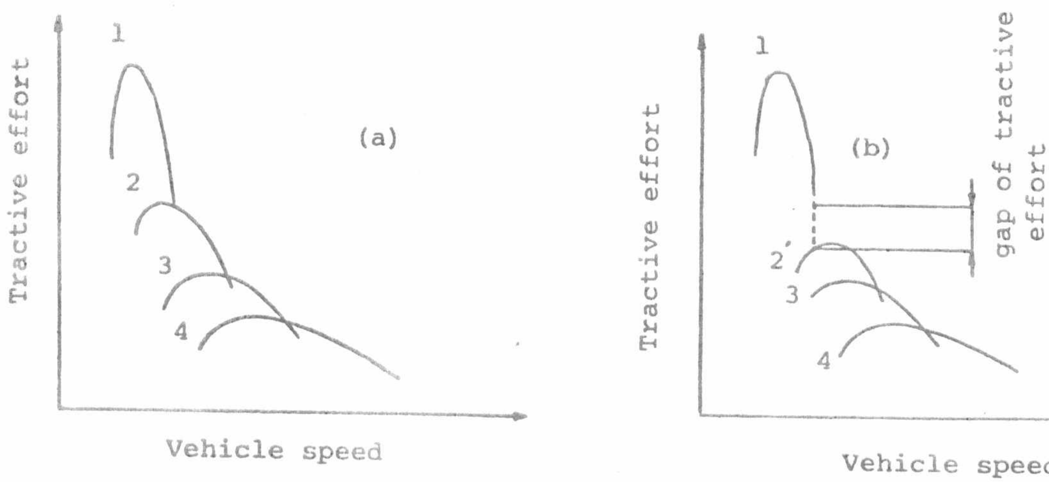

Fig.3. (a) Gear ratios are suitably spaced.
(b) Gear ratio (2) replaced by (2') caused the appearance of a gap of tractive effort.

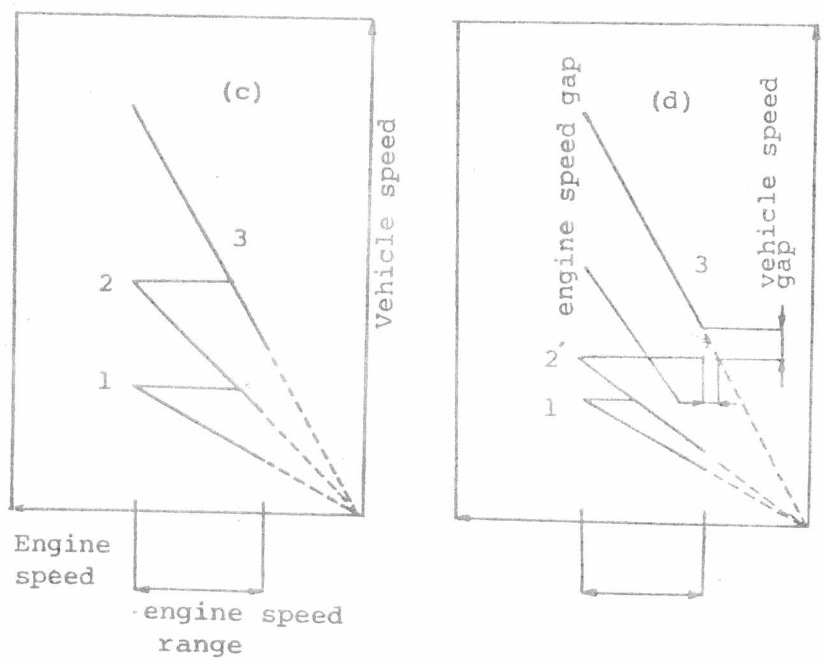

Fig.3. (c) Gear ratios suitably spaced.
(d) Gear ratio (2) replaced by (2') caused the appearance of speed gaps.

$$F'_t = K_1 K_2 P_{e\,max} \left[A_1/n_N + A_2 A_3 i_t V/n_N^2 - A_3 K_3 i_t^2 V^2/n_N^3 + A_4 K_3 i_t^3 V^3/n_N^4 \right]$$

The wasted energy ΔE is then calculated as follows:

$$\Delta E = \int_{V_o}^{V_{max}} F_t \, dV - \int_{V_o}^{V_{max}} F'_t \, dV \qquad (6)$$

Where: V_o = Vehicle speed at which the traction forces of ideal and 1st speed curves are equal.

V_{max} = maximum vehicle speed. V_o, V_{max} in Km/h

Computation of the energy wasted for the cases where gears are sclected according to the mentioned progressions is calculated using a special Fortran computer program enclosed as appendix.

The running gear of a Jeep car is considered as an actual example for comparing the different methods of gear ratios selection. The car has the following main data:

Main gear box

	1st speed ratio	3,1:1
	2nd speed ratio	1,612:1
	3rd speed ratio	1:1

Auxiliary gear box

	high range ratio	1:1
	low range ratio	2,03:1

Axle ratio 3,73:1

Calculated wheels dynamic radius $r_d \doteq 0,35$ m

The engine has torque and power characteristics shown in Fig.4.

If individual gear ratios of a four speed transmission are selected according to the mentioned progressions keeping the ratios of the top and 1st speeds as those for the actual gear box, they would have the following values:

	1st	2nd	3rd	4th
Arithmatic progression	3,1	2,4	1,7	1
Harmonic progression	3,1	1,823	1,291	1
Geometric;constant root	3,1	2,13	1,46	1
increasing root	3,1	2,019	1,386	1

The traction-speed curves corresponding to each of these progressions and the actual curve of the Jeep car are shown in Figs 5,6,7,8,9.

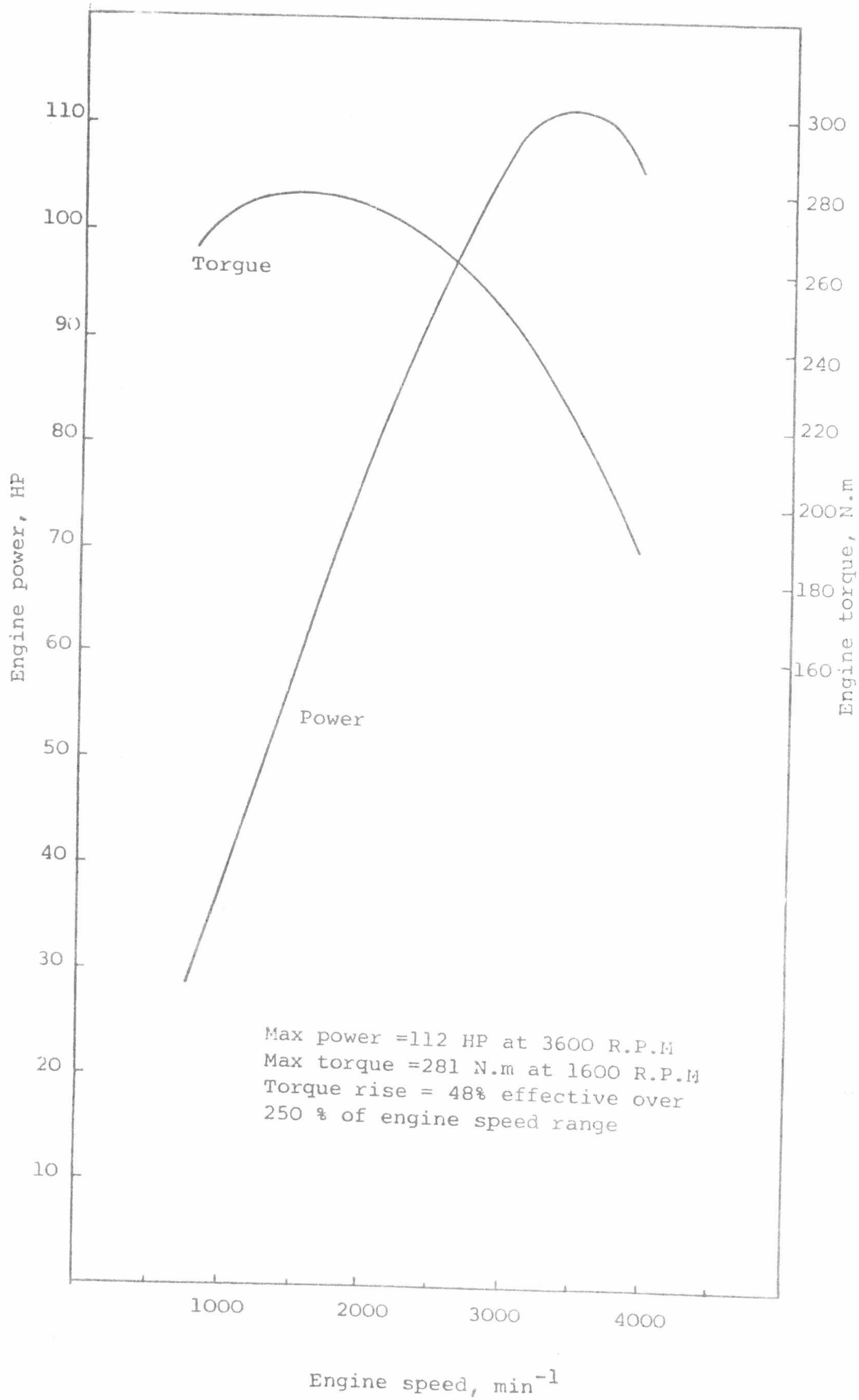

Fig.4. Engine characteristic power and torque curves.

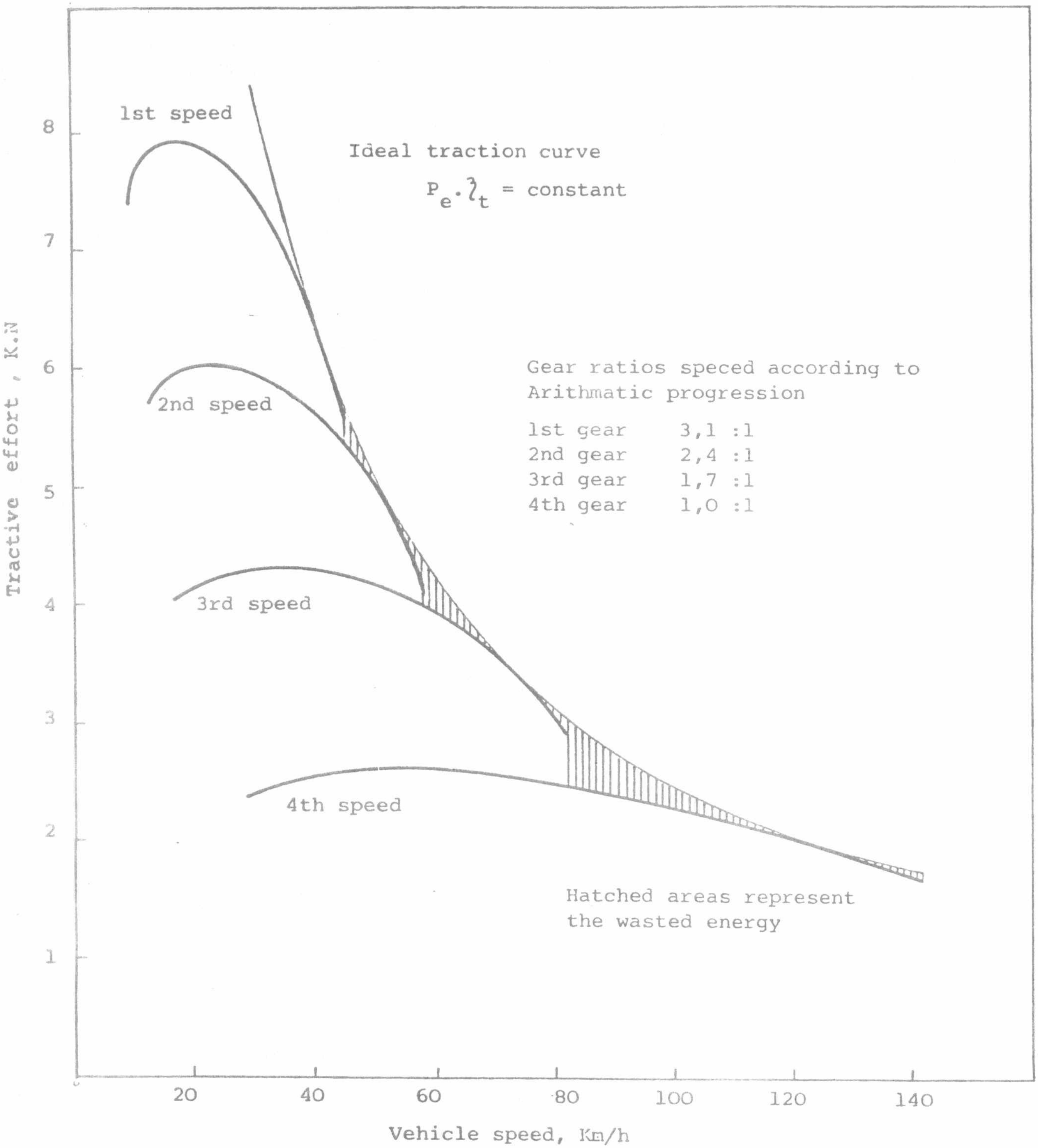

Fig.5. Traction-speed diagram of the Jeep car provided by using a four speed gear box.

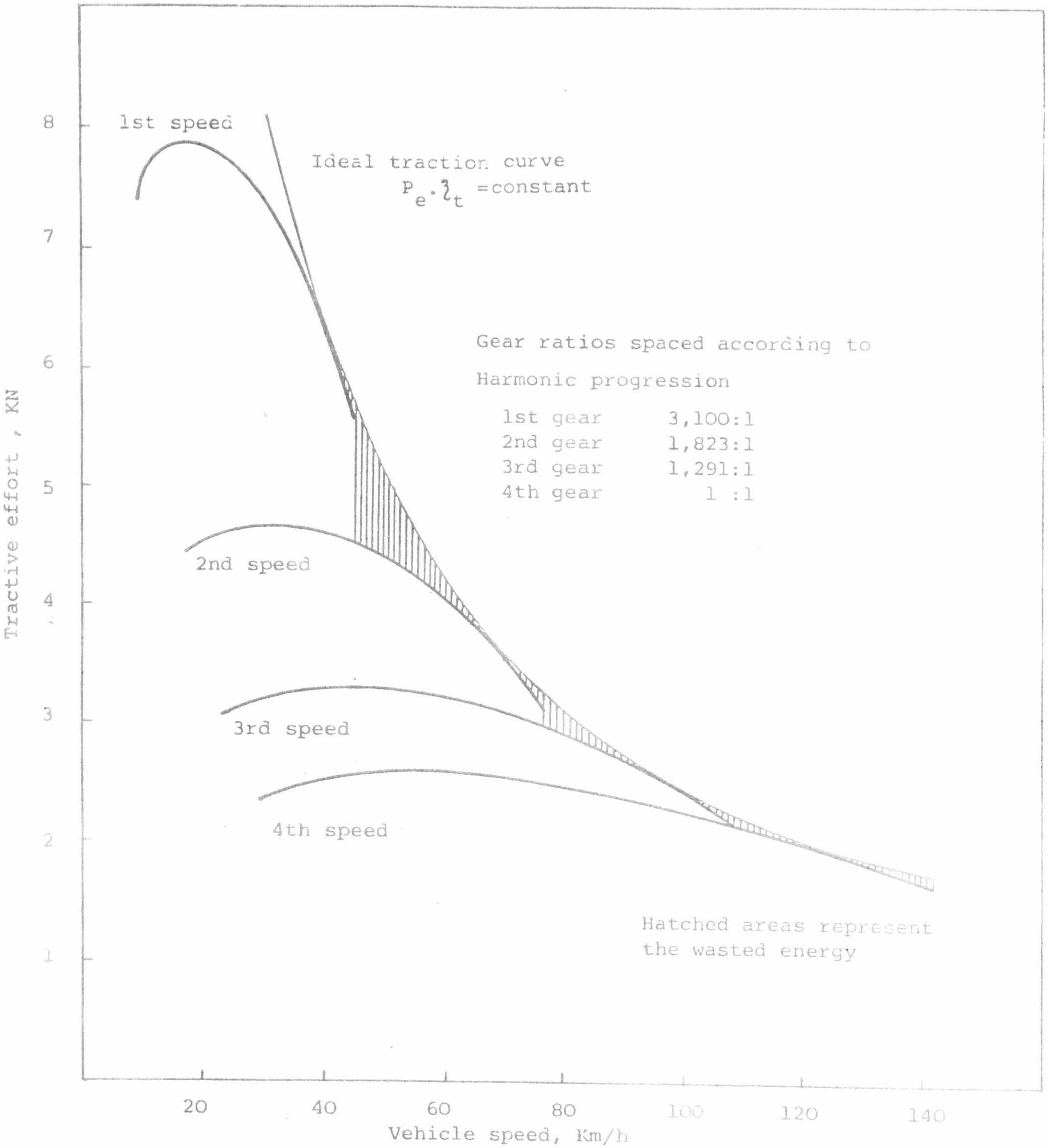

Fig.6. Traction-speed diagram of the Jeep car provided by using a four-speed gear box.

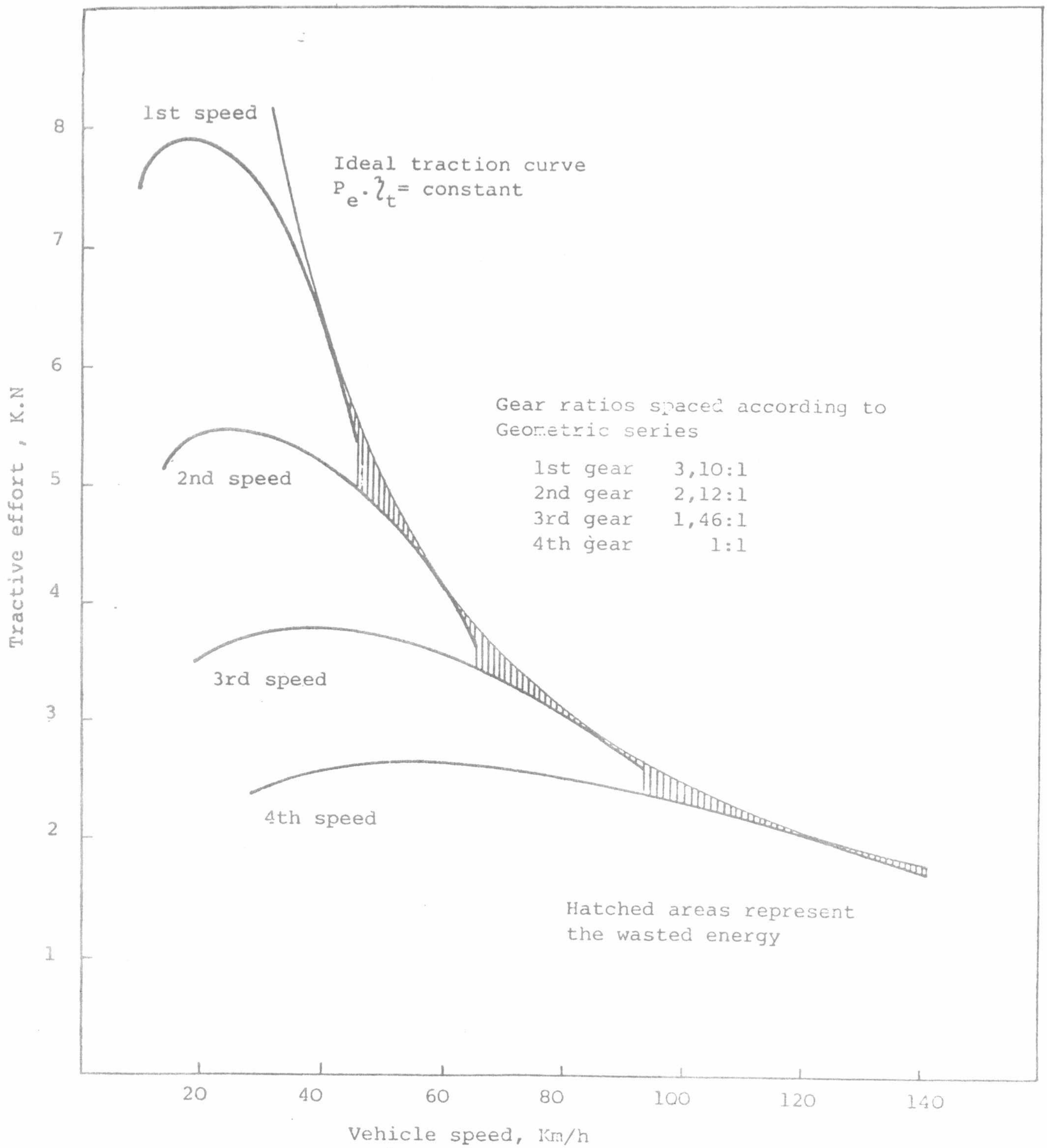

Fig.7. Traction-speed diagram of the Jeep car provided by using a four-speed gear box.

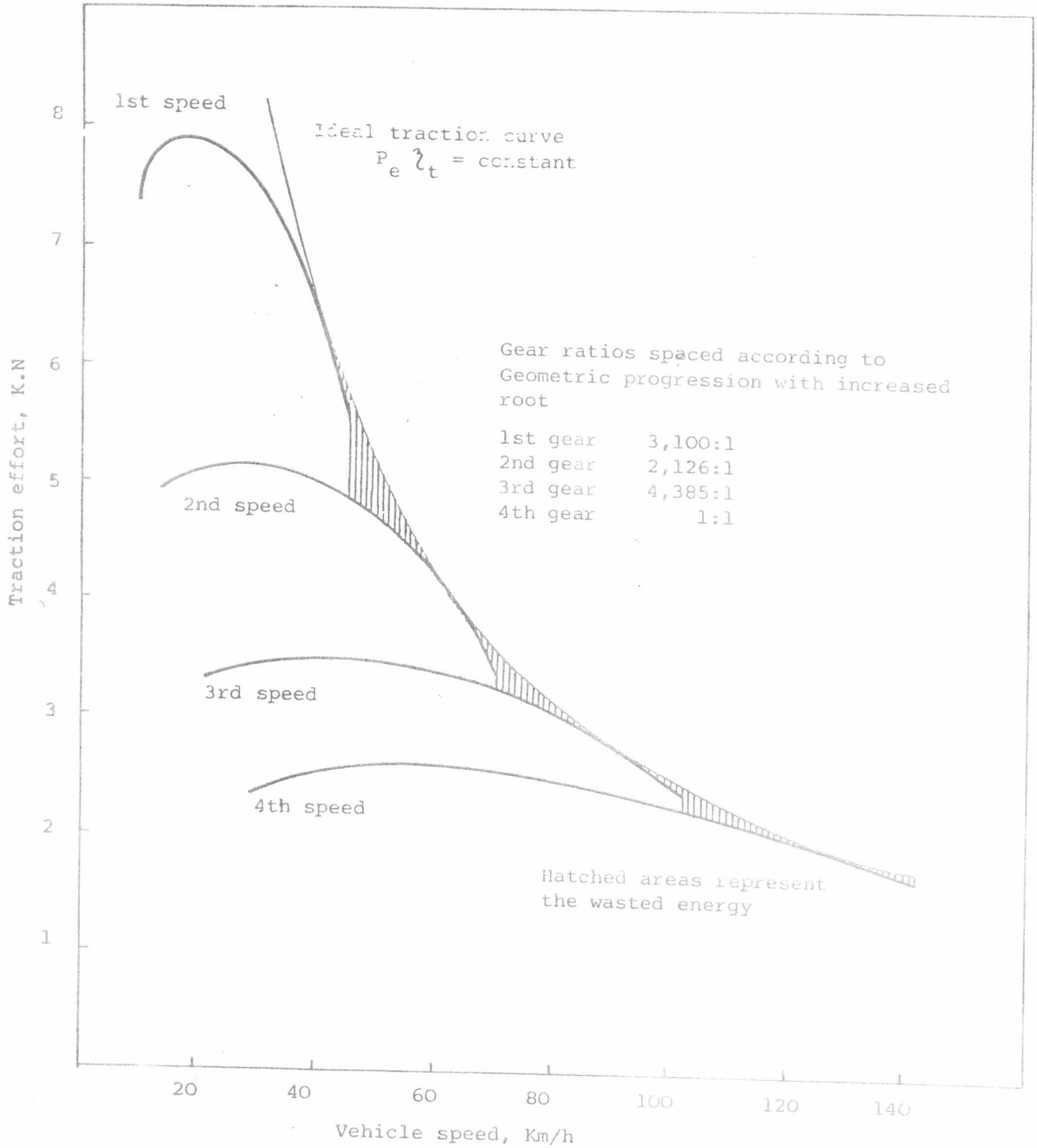

Fig.8. Traction-speed diagram of the Jeep car provided by using a four-speed gear box.

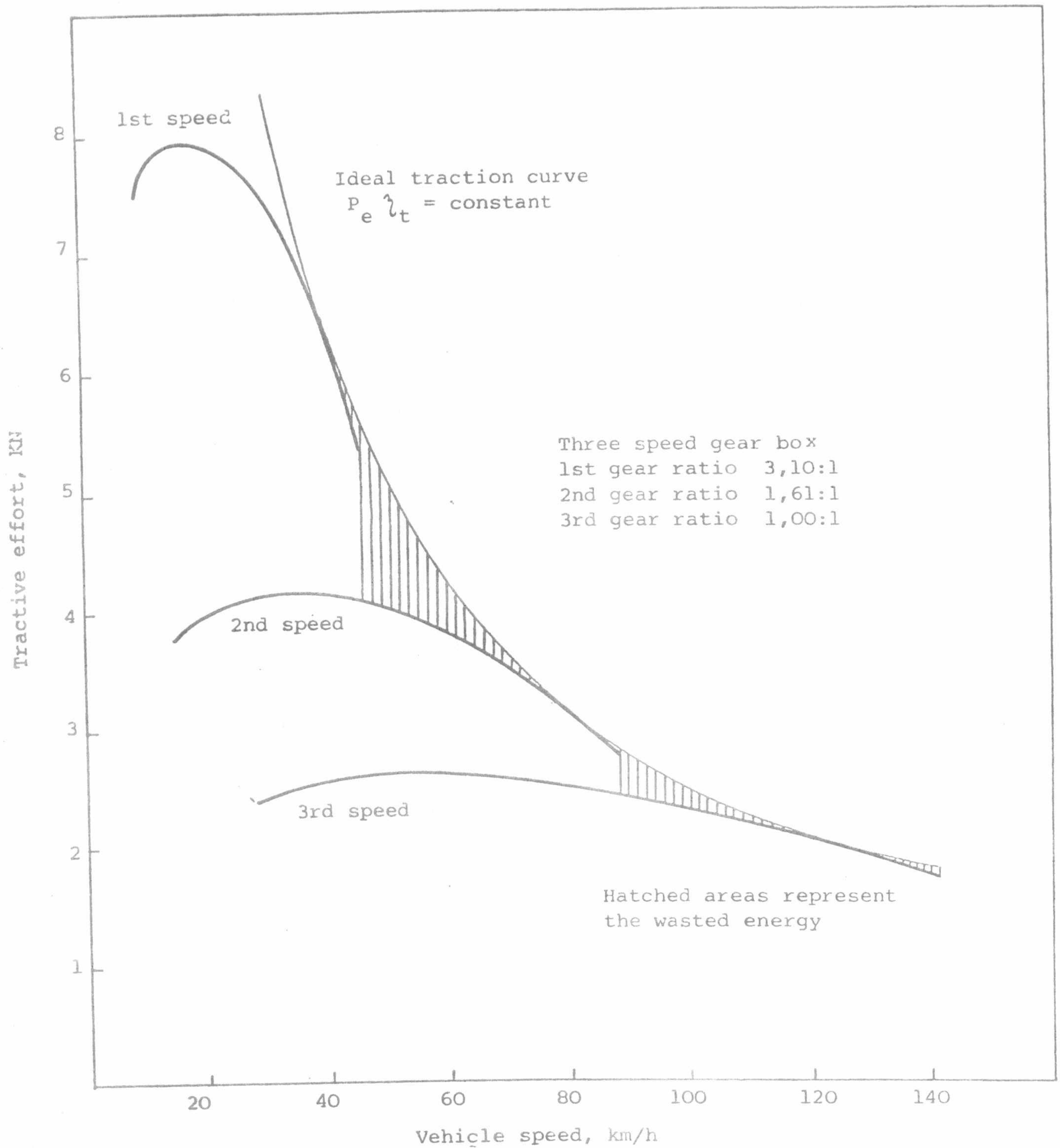

Fig.9. Actual traction-speed diagram of the jeep car.

RESULTS

Table 1 represents the results of computation of the predicted energy waste when individual speeds are engaged and also their total values. The energy waste is calculated as the difference between great and small areas which in turn correspond to the ideal and stepped traction speed curves.

Table 1. Predicted energy losses

GREAT AREA	SMALL AREA	DIFFERENCE
2708.174072	2685.552979	22.621094
9659.296875	9390.898438	268.398438
9689.128906	9416.507813	272.621094
9776.738228	9499.230469	277.507813
GEOMETRIC LOSSES=	841.148439	
2708.174072	2685.552979	22.621094
13665.78 906	12823.816406	841.972656
8874.820313	8675.082203	199.738281
6584.550781	6512.171875	72.378806
HARMONIC LOSSES=	1136.710937	
2708.174072	2685.552979	22.621094
6534.835938	6463.261719	71.574219
8798.121094	8597.070313	201.050781
13792.203125	12929.796875	862.406250
ARITHMETIC LOSSES=	1157.652344	
2708.174072	2685.552979	22.621094
11064.101563	10641.890625	422.210938
9629.289063	9361.660166	267.628906
8431.769531	8265.394531	166.375000
MOD.GEOMETRIC LOSSES=	878.835938	

CONCLUSION

Comparison of methods of selecting the gear ratios of automobile transmission can be done by calculating the wasted energy due to their traction characteristics relative to the e ideal one with continuous power transmission.

The presented example considering the data of a Jeep car showed that the geometric progressions with constant and increased roots give the least energy waste while the arithmatic and harmonic give approximately 30 % higher energy waste.

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APPENDIX

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C     APROGRAM TO COMPARE METHODS FOR
C       GEAR RATIOS CALCULATIONS .
      READ(5,100)A,B1,B2,B3,B4,NG
      WRITE(3,200)
      DO 10 I=1,4
      SUM=0.0
      DO 20 L=1,NG
      READ(5,300)V2,V1,GR
      F1=A (ALOG(V2)-ALOG(V1))
      F2=B1 GR (V2-V1)+
     1B2*GR**2/2*(V2**2-V1**2)+
     2B3*GR**3/3*(V2**3-V1**3)+
     3B4*GR**4/4*(V2**4-V1**4)
      DIF=F1-F2
      WRITE(3,400)F1,F2,DIF
   20 SUM=SUM+DIF
      GO TO(110,120,130,140),I
  110 WRITE(3,1100)SUM
      GO TO 10
  120 WRITE(3,1200)SUM
      GO TO 10
  130 WRITE(3,1300)SUM
      GO TO 10
  140 WRITE(3,1400)SUM
   10 CONTINUE
  100 FORMAT(F10.2,4F10.7,I5)
  200 FORMAT(5X,'GREAT AREA',10X,'SMALL AREA',10X,'DIFFERENCE')
  300 FORMAT(2F10.2,F10.3)
  400 FORMAT(5X,3F15.6)
 1100 FORMAT(//5X,'GEOMETRIC LOSSES=',F15.6)
 1200 FORMAT(//5X,'HARMONIC LOSSES=',F16.6)
 1300 FORMAT(//5X,'ARITHMETIC LOSSES=',F14.6)
 1400 FORMAT(//5X,'MOD.GEOMETRIC LOSSES=',F12.6)
      STOP
      END
```