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DECOUPLING AND OPTIMUM CONTROL OF A CLASS OF MULTI-INPUT MULTI-OUTPUT SYSTEMS

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ABSTRACT

The design of multi-input multi-output (MIMO) industrial control systems, by use of the modern control theory, usually leads to considerable difficulties as a direct consequence of the following points: the mathematical models are usually of high order and the state feedback schemes are of complicated nature. The present research deals with the problem of simultanious decoupling and control based on the diagonal dominancy over certain frequency range and the stabilization of the inverse Nyquist array (INA) plots at an optimum point. An approximate design technique is proposed for certain class of (MIMO) systems. The effect of emergency outage of one of the controllers is studied. The results have been checked by simulating two-input two-output system on the analog computer.

INTRODUCTION AND PROBLEM FORMULATION

The design of multi-input multi-output (MIMO) industrial control systems (thermal, chemical, mechanical, ... etc.) by use of the modern control theory (state space approach), usually leads to considerable difficulties because: the mathematical models of these processes are of high order, and the state feedback schemes are of complicated nature. The decoupling technique based on the principle of diagenal dominancy which had been established by Rosenbrock [1] - represents one of the practically sucessful approaches for the analysis and design of such systems. The problem of diagonal dominancy of the open loop transfer function matrix had been reviewed by several authers [2], [3]. In the previously given works, the problem is treated separately far from the requirements imposed by the designer on the system performance. Moreover, no sufficient attension has been addressed to the concept of diagonal dominancy about certain

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frequency which has a significant role in determining the performance of the system.

This research deals with the problem of simultanious decoupling and control based on the diagonal dominancy over certain frequency range and the stabilization of the inverse Nyquist array (INA) plots at an optimum point. The frequency range at which complete decoupling exists at certain frequency and diagonal dominancy occurs at other frequencies, represents the most significant range for determining the performance of the MIMO system. The frequency at which complete decoupling exists had been obtained from the condition of optimum performance (minimization of the dispersion of the error signal) of a family of INA similar plots for the different components of the open loop transfer function matrix. An approximate design technique is proposed and Gershgorin bands are plotted to check the diagonal dominancy at the choosen frequency range. The effect of emergency outage of one of the controllers of the MIMO system is studied. The results have been checked by simulating two-input two-output system on the analog computer.

TUNING OF A CLASS OF SINGLE-INPUT SINGLE-OUTPUT (SISO) SYSTEMS.

A class of SISO systems has been invistigated in this section in order to compare the INA of these systems when optimaly tuned. Each SISO system consists of a plant $W_p(s)$ and a controller $W_c(s)$. Eight types of plant transfer functions are considered:

$$\frac{1}{(Ts+1)^2}\ ;\quad \frac{e^{-\tau s}}{(Ts+1)}\ ;\quad \frac{e^{-\tau s}}{(Ts+1)^2}\ ;\quad \frac{1}{(Ts+1)^3}\ ;$$

$$\frac{1}{s(Ts+1)^2}\ ;\quad \frac{e^{-\tau s}}{s(Ts+1)}\ ;\quad \frac{e^{-\tau s}}{s(Ts+1)^2}\ ;\quad \frac{1}{s(Ts+1)^3}\ .$$

The previously given transfer functions can represent a wide class of thermal, chemical and mechanical systems. The controller transfer function of each plant is obtained using Wiener-Hopf optimum controller [4] in the form:

$$W_c(s) = \frac{W_1(s)}{W_p(1 - W_1(s))} \qquad (1)$$

The controller transfer functions can be approximated in the region of lower frequancies in the following form:

$$W_c(s) = \frac{1}{\tau s\, W_p^o(s)} \qquad (2)$$

where

τ - the pure time delay of the plant

$W_p^o(s)$ - the transfer function of the plant without pure time delay

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In the common practical situations, some standard types of controllers such as P, PI, PID controllers are usually used. Each SISO systems is tuned such that the following conditions are satisfied:

a) the resonant peak is maintained at a value $M = 1.62$.
b) the dispersion or the square of the error signal is minimized.

The INA of the open loop transfer functions of the optimally controlled systems are plotted in Fig. 1. It is clear from Fig. 1 that all systems have approximately equal INA plot at a point having coordinates (1.25, 150°). In the consequent analysis, this point will be choosen as a center for complete decoupling, while a suitable range about this point will be choosen such that the diagonal dominancy conditions for a MIMO system will be satisfied.

DECOUPLING AND DIAGONAL DOMINANCY ABOUT CERTAIN FREQUENCY

Fig. 2 shows a MIMO system, the plant of which has a transfer function matrix in the form:

$$W_p(s) = \begin{bmatrix} W_{11}(s) & W_{12}(s) & \cdots & W_{1n}(s) \\ W_{21}(s) & W_{22}(s) & \cdots & W_{2n}(s) \\ \vdots & \vdots & & \vdots \\ W_{n1}(s) & W_{n2}(s) & \cdots & W_{nn}(s) \end{bmatrix} \quad (3)$$

where $W_{ij}(s)$ belongs to the class of the previously given transfer functions. The controller transfer function matrix is considered to be diagonal matrix in the form:

$$W_c(s) = \begin{bmatrix} W_{c11}(s) & & & \\ & W_{c22}(s) & & 0 \\ 0 & & \ddots & \\ & & & W_{cnn}(s) \end{bmatrix} \quad (4)$$

where $W_{cij}(s)$ belongs to the class of Wiener-Hopf optimum controllers previously given by expressions (1), (2); or to the class of standard P, PI, PID controllers (widely used in industry).

Refering to the results given in the previous sections, the INA common point (1.25, 150°) corresponds to a point (0.8, -150°) on the common Nyquist plot. Accordingly, the decoupling problem can be formulated as follows: find a matrix [K] such that the open loop transfer function matrix is a diagonal matrix in the form:

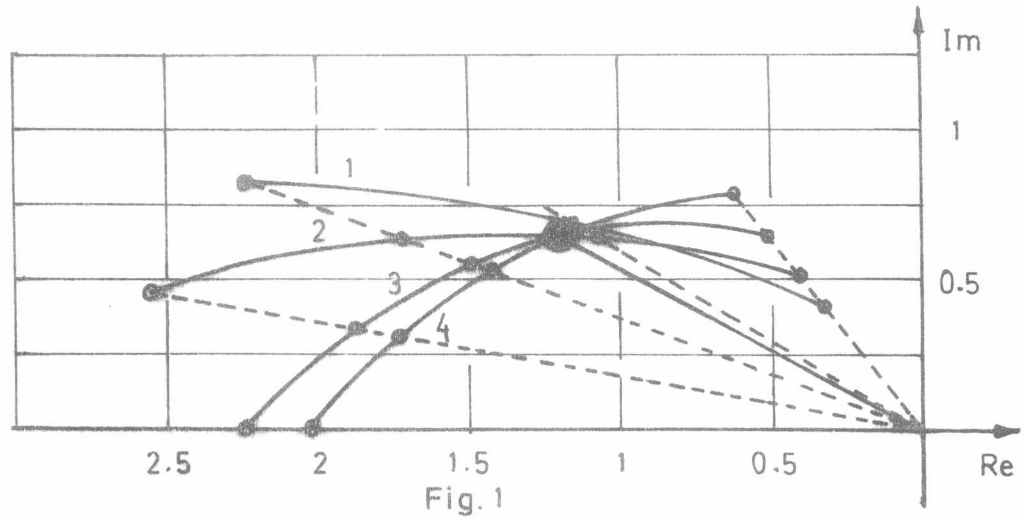

Fig. 1
The INA of the SISO systems

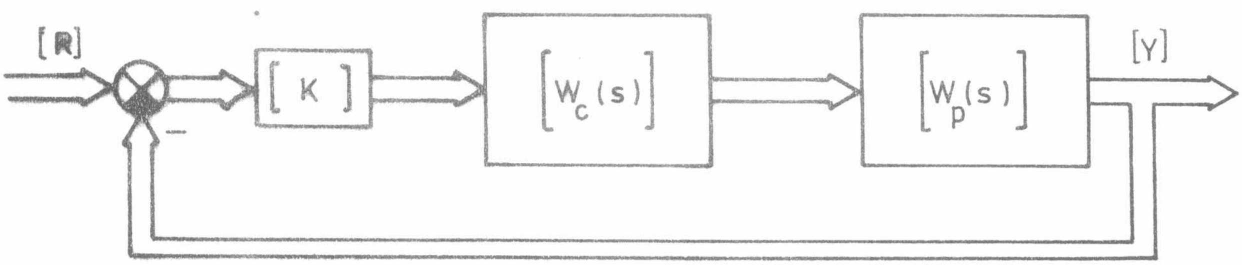

Fig. 2
The MIMO system

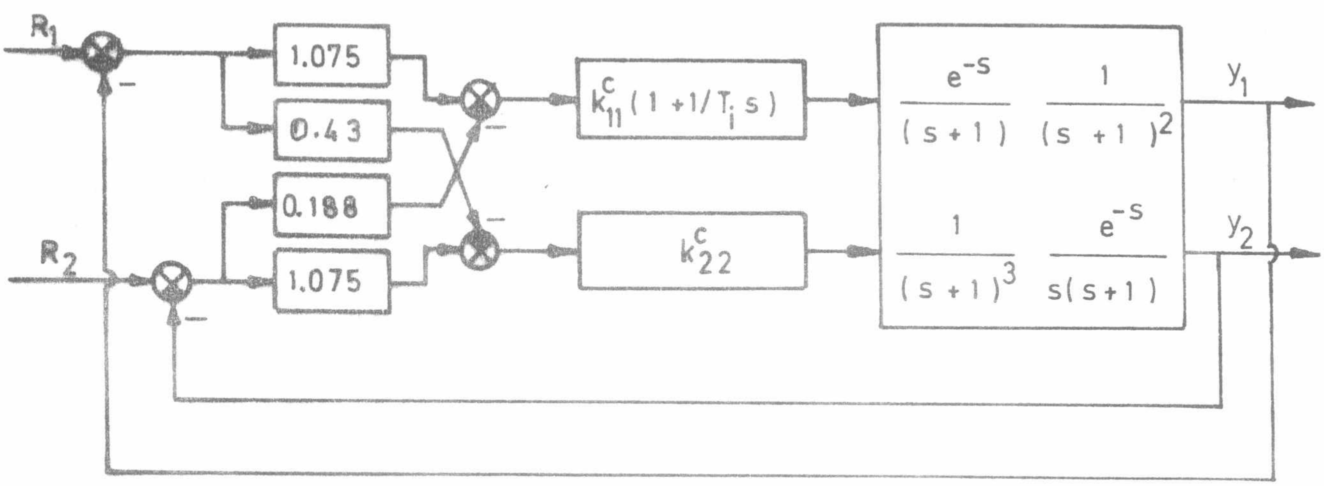

Fig. 3
Simulation of two-input two-output systm

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$$[K][W_c(s)][W_p(s)] = \begin{bmatrix} 0.8 & & & 0 \\ & 0.8 & & \\ 0 & & \ddots & \\ & & & 0.8 \end{bmatrix} \qquad (5)$$

To achieve diagonalization at a point of the INA (1.25, 150°) the matrix $[W_c(s)][W_p(s)]$ can be expressed in the form:

$$[W_c(s)][W_p(s)] = \begin{bmatrix} 0.8 & a_{12} & \cdots & a_{1n} \\ a_{21} & 0.8 & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & 0.8 \end{bmatrix} \qquad (6)$$

i.e. the diagonal elements are equal to 0.8 (the point at which complete decoupling occurs). The off-diagonal elements a_{ij} are obtained as follows:

a) find the optimum values of the controller parameters corresponding to controller $W_{cii}(s)$ and a plant $W_{pii}(s)$ connected as SISO system.
b) generate a group of SISO systems consisting of controllers $W_{cii}(s)$ (the optimum parameters of which are obtained in the previous step) and plants $W_{ij}(s), i \neq j$.
c) find the modulia of the open loop frequency responses of the previously generated systems corresponding to a phase angle of -150°
d) the modulia obtained in step (c) represent the values of a_{ij} given in matrix (6).

The diagonalization of the matrix $[W_c(s)][W_p(s)]$ may lead to the following cases:

a) some distinct real eigen values
b) some repeated real eigen values
c) some repeated or conjugate complex eigen values.

The previously given cases can lead to the following configurations of the decoupled or nearly decoupled systems:

a. completely decoupled systems if there exists some distinct eigen values.
b. one-sided interaction if some roots are repeated and Jordan blocks will appear on the diagenal of the diagonalized matrix.
c. two-sided interaction if complex eigen values appear.

It is clear from the given configurations that the appeared one-sided or two-sided interactions can be considered as an input signal to an individual SISO system, and the corresponding systems can be tuned as a SISO system. This remark can be concluded also from some previous works in the field of large-scale systems.

It is easy to get the effect of the inaccuracy of the INA point (1.25, 150°) on the degree of decoupling. Assume that each system has a deviation of ε_{ii} for each diagonal element then:

$$\begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & \vdots & & \vdots \\ K_{n1} & \cdots & \cdots & K_{nn} \end{bmatrix} \begin{bmatrix} 0.8+\varepsilon_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & 0.8+\varepsilon_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}+\varepsilon_{nn} \end{bmatrix} =$$

$$= \begin{bmatrix} 0.8+K_{11}\varepsilon_{11} & K_{12}\varepsilon_{22} & \cdots & K_{1n}\varepsilon_{nn} \\ K_{21}\varepsilon_{11} & 0.8+K_{AA}\varepsilon_{AA} & \cdots & K_{2n}\varepsilon_{nn} \\ \vdots & \vdots & & \vdots \\ K_{n1}\varepsilon_{nn} & \cdots & \cdots & 0.8+K_{nn}\varepsilon_{nn} \end{bmatrix} \quad (7)$$

The matrix on the righ-hand side of the previous equality leads to the fact that the degree of diagonal dominancy depends on the order (n) of the system and the deviation ε_{ii}.

As the proposed technique relies on the concept of diagonal dominancy rather than complete diagonalization, Gershgorin bands are recommended to be used for checking the dominancy at any situation.

THE EFFECT OF EMERGENCY OUTAGE OF A CONTROLLER

If the controller $W_{ii}(s)$ of the MIMO system is in a situation of emergency outage, some loops of the system may suffer from instability, and the decoupling property or even the diagonal dominancy will be lost. The effect of outage of controller $W_{ii}(s)$ may be studied by the following matrix manipulations:

$$[K][W_c(s)][W_p(s)] = [K] \begin{bmatrix} W_{11}W_{c11} & W_{12}W_{c11} & \cdots & W_{1n}W_{c11} \\ W_{21}W_{c22} & W_{22}W_{c22} & \cdots & W_{2n}W_{c22} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \\ W_{n1}W_{cnn} & W_{n2}W_{cnn} & \cdots & W_{nn}W_{cnn} \end{bmatrix} \leftarrow i^{th}\text{ row}$$

As the i^{th} row of the previous matrix vanishes, then manipulating the matrix $[K][W_c(s)][W_p(s)]$, the diagonalization property or even the diagonal dominancy will not exist. In a great number of cases the system will suffer from instability, the matter which can be avoided by introducing a computational unit to manipulate and reset another new values of the matrix [K]. This scheme is easy to fulfill either by installing on-line computational unit or by suitable alarming and manual resetting. In such situations, Ostrowski bands are suitable to use for estimating the stability margins and the effect of changing [K] on the stability of the system.

SIMULATION RESULTS

Two-input two-output control system was simulated on an analog computer. The schematic diagram of the simulated system is given in Fig. 3. Two types of controllers (P, PI) are used and optimally tuned according to the previously given procedure. The matrix [K] was calculated by the diagonalization procedure at INA point (1.25, 150°). The step responses of the different channels of the system had been recorded and given in Fig. 4. The numerical values of the simulated system are related to the following example about the decoupling and optimum tuning of two-input two-output system.

EXAMPLE

Consider the system given in the previous section and which was simulated on the analog computer where:

$$W_p(s) = \begin{bmatrix} \frac{e^{-s}}{s+1} & \frac{1}{(s+1)^2} \\ \frac{1}{(s+1)^3} & \frac{e^{-s}}{s(s+1)} \end{bmatrix} \quad ; \quad W_c(s) = \begin{bmatrix} K^c_{11}(1+\frac{1}{T_{11}s}) & 0 \\ 0 & K^c_{22} \end{bmatrix}$$

To get the matrix [K], we have:

$$[K][W_c(s)][W_p(s)] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} 0.8 & a_{12} \\ a_{21} & 0.8 \end{bmatrix}$$

To get a_{12}, a_{21}, the values of K^c_{11}, T_{11} which optimizes the SISO system having plant transfer function $W_{11}(s)$ should be obtained. Also, the value of K^c_{22} which optimizes the SISO system having plant transfer function $W_{22}(s)$ must be obtained. Performing the procedure given in the previous sections, it is easy to get the following results:

$$K^c_{11} = 0.56 \quad ; \quad T_{11} = 0.7 \text{ sec.} \quad ; \quad K^c_{22} = 0.54$$

Then, plot the Nyquist plot of the transfer function $W_{12}(s)$ as if it is tuned by a PI-controller of $K^c_{11} = 0.56$, $T_{11} = 0.7$ sec. Plot also the Nyquist plot of the transfer function $W_{21}(s)$ as if it is tuned by a P-controller of $K^c_{22} = 0.54$. Find the modulia of the previous plots at phase angle of -150°, which will represent the values of a_{12}, a_{21}. These

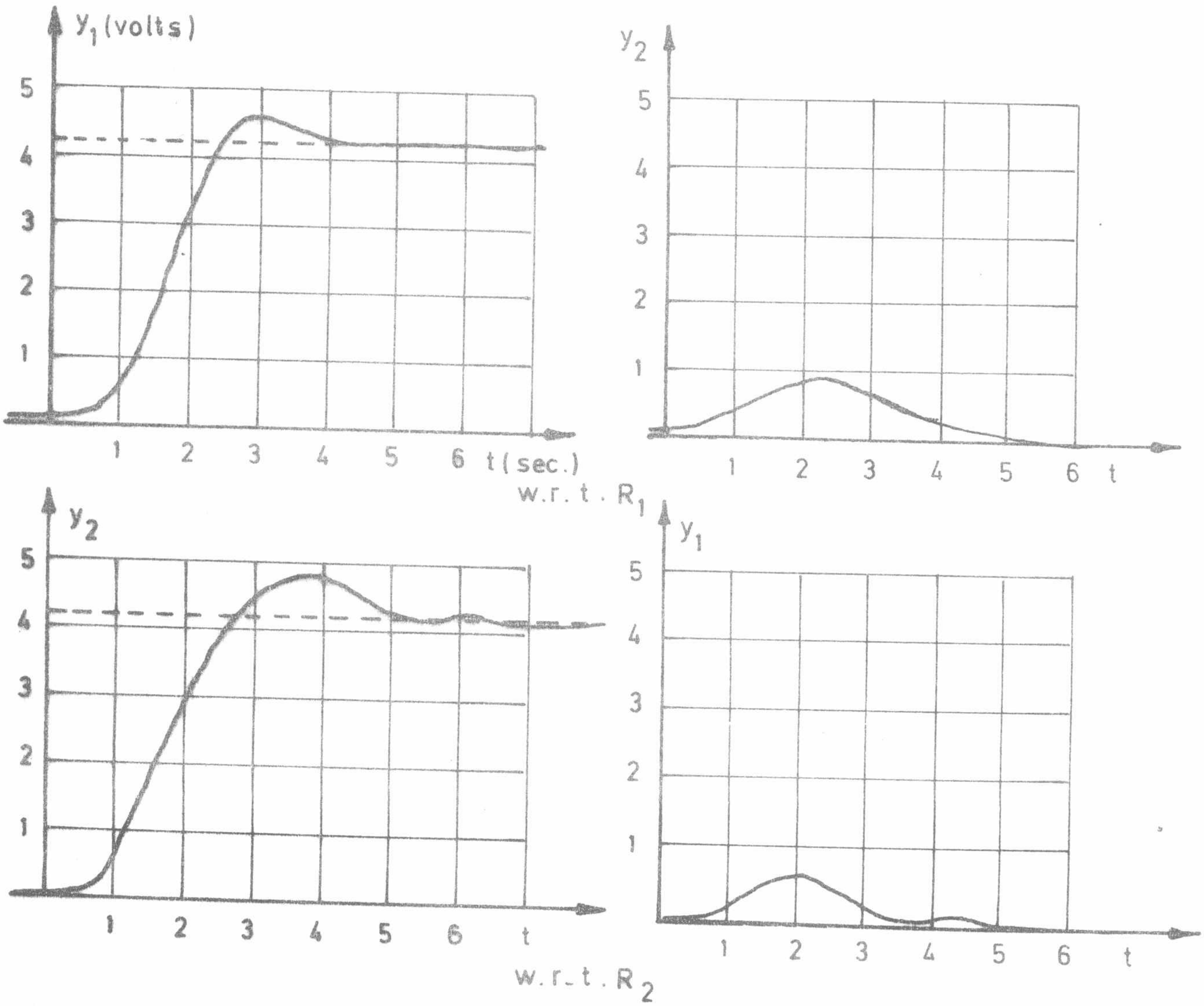

Fig. 4
Step responses

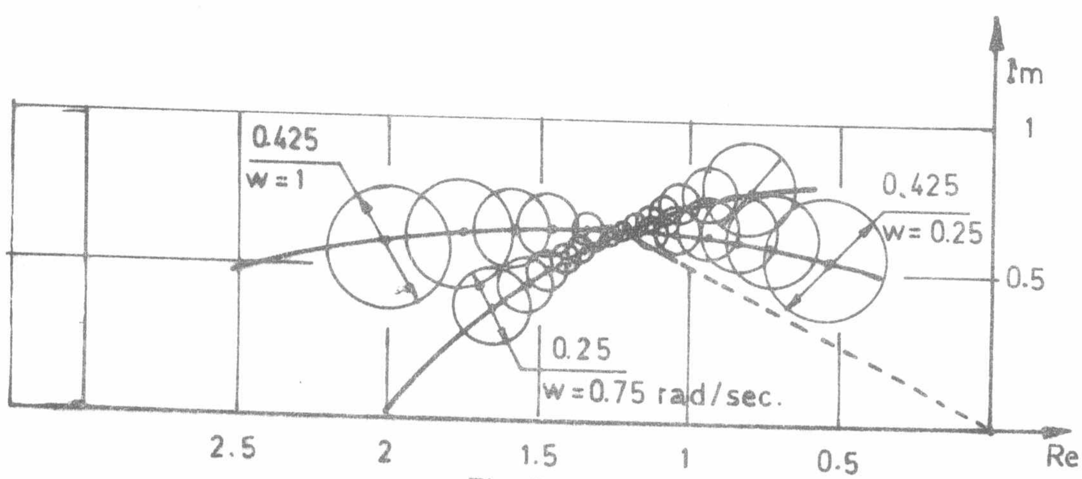

Fig. 5
Gershgorin bands of the optimum system

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values are: $a_{12} = 0.32$; $a_{21} = 0.14$. Accordingly we have:

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} 0.8 & 0.32 \\ 0.14 & 0.8 \end{bmatrix} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$$

It is easy to get the matrix K in the form:

$$[K] = \begin{bmatrix} 1.075 & -0.43 \\ -0.188 & 1.075 \end{bmatrix}$$

Then, it is possible to deduce the schematic diagram of the optimally tuned and decoupled system as shown in Fig. 3. Fig. 5 shows the Gershgorin bands to check the dominancy of the transfer function matrix of the open loop system. The nearly decoupled system had been checked by the results previously obtained by simulation on the analog computer, as shown in Fig. 4.

CONCLUSIONS

The given procedure is mainly relevant to the practice in the field of process control rather than the extensive theoretical leterature widely spread as a research work in this area. The main concept of the procedure is based on the fact that: all optimally tuned systems related to the considered class of plants have nearly the same INA plots about a point laying in a region which is very significant for the optimum tuning of such systems. The given procedure has the following merits:

a. The decoupling procedure is considerably simplified, as the matrix manipulations are related to some numerical values but not to Laplace operators.
b. It is easy to implement some units to work against the subsidery effects of the emergency outage of some controllers.
c. The procedure is applicable for the majority of the industrial plants and is compatible with the process control engineering practice.

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