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MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

IMPROVING THE DYNAMIC PERFORMANCE OF MACHINES VIA MODAL TESTING AND CAD APPROACH

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ABSTRACT

Dynamic performance has become a significant criterion in the design of many machines. In this paper an integrated approach utilizes modal testing results and finite element modelling into a system analysis routine to give the engineer a usefull tool to select design modifications and to predict the dynamic performance of a new machine. This approach can be used for redesign of existing machines or the development of new systems. A specific application of the method on a harvester shaker is described.

INTRODUCTION

There are several ways to estimate the dynamic characteristics of a machine from purely analytical investigations. The most practical analytical method for predicting the dynamic behaviour of a complex machine, before it exists, is through the application of FE(finite element) technique. Argyris (1) and Turner (2) dealt with complex systems by subdividing them into components that were treated separately to produce results that were synthesized to obtain dynamic properties of the complete system. A primary emphasis is directed toward force methods, and the problem of dealing with the connection system among the components is handled by considering the equilibrium of the interaction force system. However, displacement methods are considered also, and in these methods it is suggested that the interconnection problem may be resolved by equating matching boundary displacements (3). Studies by Turner, Martin, and Weikel (4) consider the analysis of complex systems by a mixed stiffness and/or displacement methods.

From an experimental standpoint, the most convenient method of determining the dynamic characteristics of a machine, when it exists, is to use modal analysis technique. With the presently available instrumentation modal parameters of a machine can be identified by recording its response characteristics at some selected points after it is subjected to an artificial excitation force (5). For R&D problems in many machines experimental techniques minimize the amount of assumptions and idealizations

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and prove superior for defining the dynamic behaviour. For a complex machine, where many modes must be documented with many points, the testing time involved is great. However, the development of digital FFT (Fast Fourier Transform) modal analysis [6] have allowed dynamic testing to be speeded up.

This paper introduces a hybrid method, which combines modal tests, FE, and system analysis approach to study dynamics of machines. It presents some theoretical background and illustrates, by example, use of the method to predict a machine performance for the development or troubleshooting redesign. In the proposed method the experimental modal analysis technique provides dynamic information about machine elements, which already exist and will not be modified, FE approach provides similar information about other elements, and system analysis is used in understanding how forcing functions interact with elements dynamics to produce an adequate machine response.

BASIC PRINCIPLES AND CONCEPTS

To model properly the dynamics of a machine, which assumed to have n degrees of freedom, it is required to define or introduce at least an n number of generalized coordinates x's. The equations of motion for the overall machine (assuming a linear system with viscous damping), written in terms of these generalized coordinates, have the following matrix form:

$$[M]\,[\ddot{X}] + [C]\,[\dot{X}] + [K]\,[X] = [F(t)] \qquad (1)$$

Equations of similar form in terms of a set of local generalized coordinates y's apply to each separate machine element. For example, the following equation applies to the r-th element:

$$[m]_r[\ddot{y}]_r + [c]_r[\dot{y}]_r + [k]_r[y]_r = [p(t)]_r \qquad (2)$$

Certain elements, perhaps those from previous design, may be modelled through modal tests while new or modified elements may be defined analytically using FE technique. Therefore, equations similar to eq.(2) are deduced for all elements of the machine and all of them can be combined in the following matrix form:

$$[m]\,[\ddot{Y}] + [c]\,[\dot{Y}] + [k]\,[Y] = [P(t)] \qquad (3)$$

Eq. (3) can be considered as a set of equations of motion for the group of elements before assembly. Assembly is a physical process of connecting these elements which gives rise to equations of constraint among the components of the vector [Y]. Now if there are m components in the vector [Y] and k equations of constraint relating them, then there will exist a subset of these components containing n=m-k displacements that are independent. This subset may be related to that set by a linear transformation

$$[Y] = [B]\,[X] \qquad (4)$$

The present analysis is limited to small displacements, so that the linear assumption is not violated. The construction of the matrix [B] requires knowledge of the constraints imposed on all elements by the system of connections. For example, at certain connection a constraint may exist which requires that the translations of a common point on the r-th and s-th elements be equal

i.e. $y_{is} = y_{jr}$

Another contraint may require that a small rotations be equal:

$$\phi_r = \phi_s$$

In such cases, the treatment of all displacement constraint at the points of connections leads to a set of linear constraint equations among y's which can be expressed in matrix form

$$[T]\,[Y] = 0 \qquad (4)$$

Where [T] is a $k \times m$ rectangular matrix of constant coefficients. This matrix may be partitioned in the following form: $[T] = [G \;\; H]$. Where [G] is a square nonsingular matrix of the order k. Thus, eq.(4) can be expanded to

$$[G]\,[Y]_d + [H]\,[Y]_n = 0 \qquad (5)$$

Therefore, the dependent set may be written in terms of the independent set as

$$[Y]_d = -[G]^{-1}\,[H]\,[Y]_n \qquad (6)$$

From this, an equation can be derived that relates the complete vector [Y] to the independent subset

$$[Y] = \begin{bmatrix} Y_n \\ Y_d \end{bmatrix} = \begin{bmatrix} I \\ -G^{-1}H \end{bmatrix} [Y_n] = \begin{bmatrix} I \\ -G^{-1}H \end{bmatrix} [S][X] = [B]\,[X] \qquad (7)$$

Where [S] is another linear transformation matrix introduced for convenience to relate the vector of $[Y_n]$ to the generalized displacement vector [X] of the machine. Substitution of eq.(4) into eq.(3), followed by premultiplication by $[B]^T$ yields

$$[B^T][m][B][\ddot{Y}] + [B^T][c][B][\dot{Y}] + [B^T][k][B][Y] = [B^T]\,[P(t)] \qquad (8)$$

Comparing eq.(8) with eq.(1), the following identities are noted

$$M = B^T m\,B\,,\; K = B^T k\,B,\; C = B^T c\,B,\; F(t) = B^T P(t) \qquad (9)$$

Introducing the concept of the modal coordinates of the machine q's, then, another set of coordinates(q's) exists which, if used instead of the generalized coordinates x's, would yield a set of n individual equations each with one unknown, rather then the coupled set of equations [6]. The modal coordinates of a machine is related to the generalized coordinates by the equation

$$[q] = [A] [X] \tag{10}$$

The rows of [A] indicate the amount of each component of X which forms one modal coordinate and is called a mode shape vector. The matrix [A] is determined through the modal analysis by the requirements that it decouple eq.(1). The uncoupled equations of motion in terms of only modal coordinates can be written as

$$\lceil M \rfloor \{\ddot{q}\} + \lceil C \rfloor \{\dot{q}\} + \lceil K \rfloor \{q\} = \{Q(t)\} \tag{11}$$

Eq.(11)represent the standard form of the overall machine model implemented in the present approach.

SYSTEM ANALYSIS

Once the overall machine model has been developed, it can be used to study forced response by applying external loads to any D.O.F. in the model. The loads applied can take the form of a force, displacement, velocity, or acceleration. Alternatively, the machine model can be manipulated to yield an eigenvalue solution; that is, a modal data base for the characteristics of the machine. This data base can be modified to predict the effect of altering elements' configuration,constraint equations, stiffness, mass, and damping properties. The results of this analysis can in turn be used to refine elements design as shown in Fig. 1.

The analysis procedures is standardized so as to facilitate programming for solution by digital computers. Computer program allows an interactive mode in predicting best solution and guides the user through various input and output options. Once the best solution has been implemented through hardware modifications, the machine may be tested again to verify acceptable machine performance.

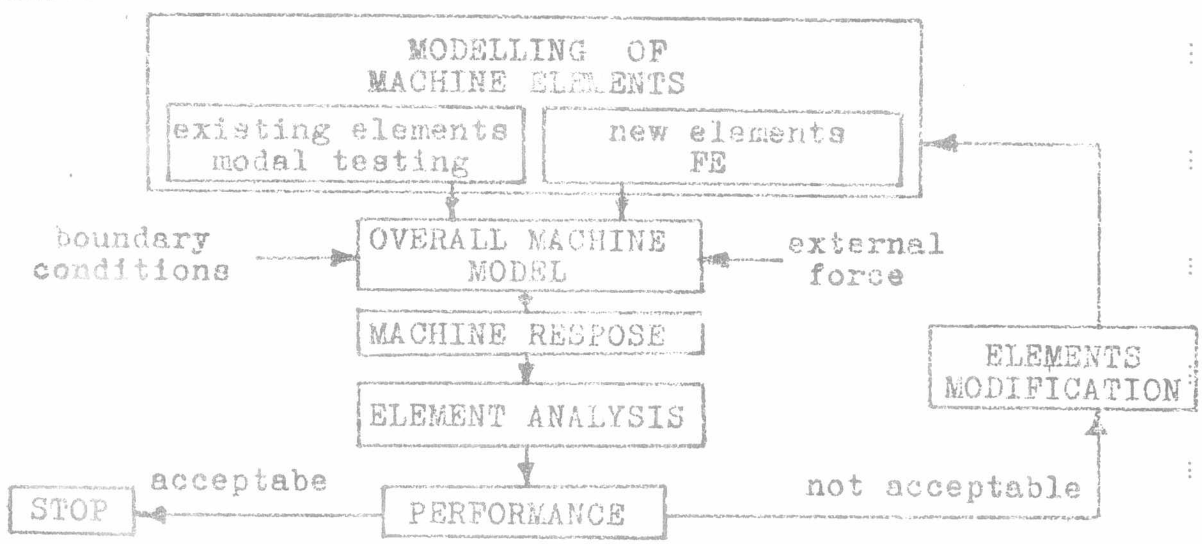

Fig. 1 System analysis procedure to refine elements design.

CASE STUDY

Now that the basic principles and concepts of the present app-

roach have been developed, this section will deal with a practical application to illustrate the steps in the procedure.
6 A manufacturer desired to reduce the amplitude of vibrations of a harvester shaker in certain directions to avoid their damaging effects. According to the proposed approach the development project steps were: (1) Obtain modal model for existing well behaved elements using modal testing; (2) Deduce analytical model for other (new) elements; (3) Build machine model based on 1 & 2 above; (4) Develop machine performance and diagnose troubeshooting problems; (5) Simulate suggested modifications and obtain best solution; (6) Implement the recommended solution and verify the performance of the modified machine through modal testing.

To accomplish the above objectives a global cartesian coordinate system is defined for the machine as shown on the skeleton model of Fig.2. Another local coordinate system may be defined for each element when it is convenience. A modal survey is then performed for machine base and shaking mechanism using the following hardware setup: (1) PCB piezotronics instrumented 1.1 kg. hammer kit model K291A; (2) Brüel and Kjœr triaxial accelerometer model 4321.; (3) Brüel and Kjœr charge amplifiers type 2635; (4) Brüel and Kjœr 4 channel FM tape recorder model 7003; (5) GenRad model 2507 minicomputer and data acquisition system.

Modal Test

The choice of number of test stations for modal test depends on the complexity of the tested element and the end use of the information. For example, the response of the shaker base is measured by an accelerometer at 12 stations. Each of these stations has 3 degree of freedom so that the obtained model of the base has 36 DOF. A multiple impacting in the +x and -z directions on a rubber pad at station No. 3 is used to excite it. Although the noise to signal ratio using this excitation method was relatively low, it was greatly enhanced by averaging 20 averages. Recycling the data is used when the frequency window become too small to obtain enough averages during a single run. The use of a triaxial accelerometer and a 4-channel recorder allowes us to simultaneously record the data required to estimate the transfer function of the 3 DOF at each point, thus reducing the number of tests by a factor of 3 (in the field). Data processing is performed at a later time (in the laboratory) using the GenRad data acquisition system. Typically, this is done by first qualifying all system parameters i.e. sampling rate, triggering level, channels scale, bandwidth..etc. The digitization rate necessary to properly define the waveform is taken2-3times [7], the highest frequency of interest. The procedure of modal parameters calculation from digital data are automated with an FFT program. Inertance response plots are developed for the element under test and significant resonant frequencies are noted and designated for automatic amplitude computation by the FFT program. The analytical expression for TF and modal coefficints are obtained by the implementation of a "weighted" least squares curve fitting technique. The validity of the model is checked by reconstruction of functions from the modal data base. Fig. 3 compares measured frequency respose function H_{35} to synthesised one on Bode plot of amplitude and phase. Fig. 4 shows typical mode shape for the frame.at 14.2 Hz.

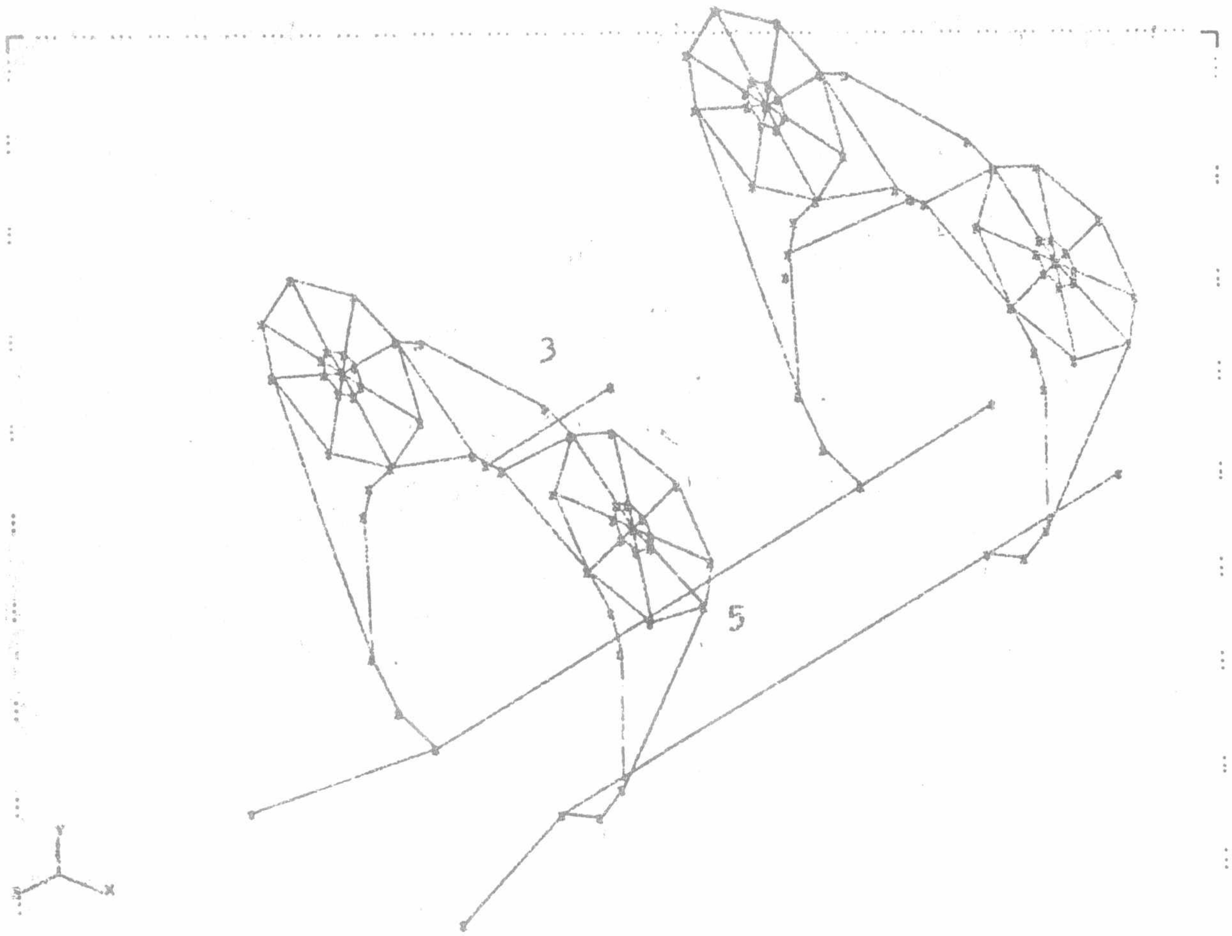

Fig.2 Machine skeleton model

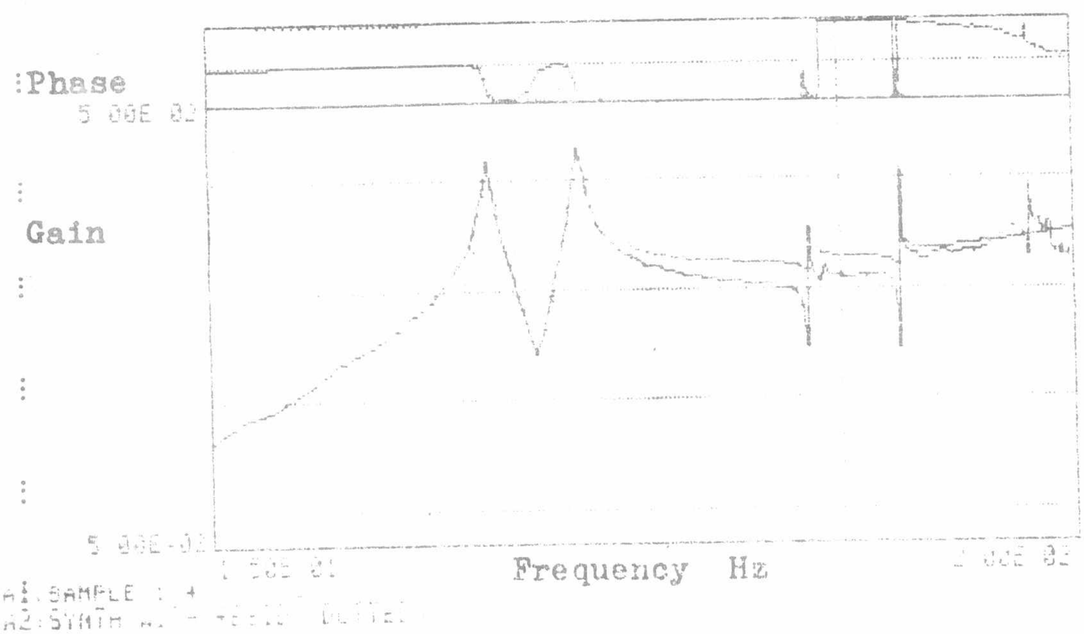

Fig. 3 Frequency response function H_{35} (measured versus estimated or Synthesized).

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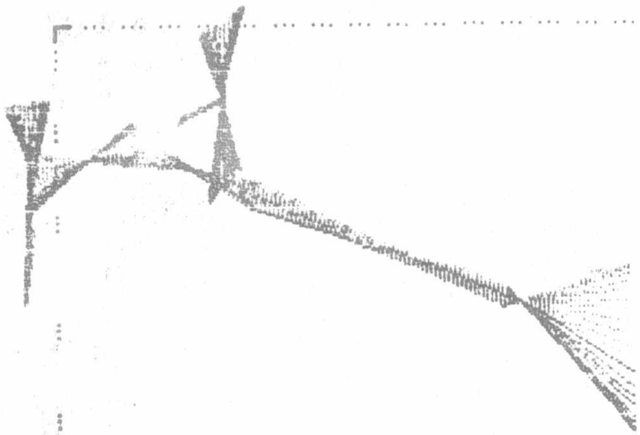
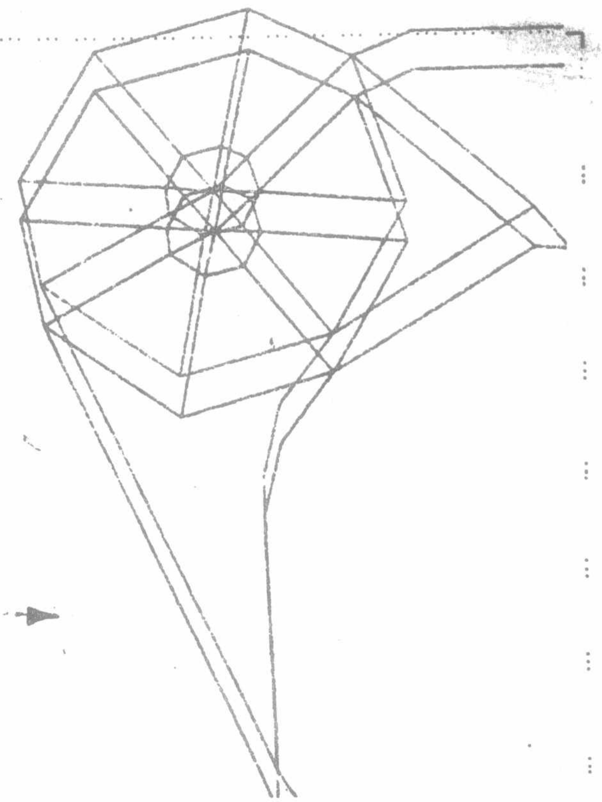


Fig. 4 Typical 3rd mode shape measured for the frame.

Fig. 5 Finite element mesh for one of the oscillating arms.



Analytical Modelling

The finite element technique is used to describe the dynamic characteristics of the shaking heads, Fig 5. The matrix equations has been generated and reduced using Guyan reduction technique. Main emphasis is given to the sufficient matrix size to ensure that the dynamic range accurately spans the frequency range of interest in the problem.

Assembly of Models

The proposed method allowes the dynamic representation of the overall machine to be obtained from element information. After determining the property matrices for each element, the analysis program assembles them in the machine dynamic stiffness, mass, and damping matrices by considering constraint and transformation matrices [T] and [S]. Also, element forces are transformed into machine forces by the same transformation. Thus, a set of equations of motion related to the machine is formulated.

System Analysis

The dynamic characteristics of the machine is then developed by solving the above set of equations. Through the forementioned transformation the element responses are then derived. Following this, each element may be analyzed separately to determine all constraint forces imposed upon it. Also, dynamic stresses, strains, deflections, or other responses may be determined as desired. The machine model is then used to simulate analytically the effects of different changes to the shaker design. Many changes are evaluated following a step-by-step procedure before any hardware modifications are recommended. The order in which these changes are studied is based upon the ease of implementation by the manufacturer. In each case a comparison is made to the original modal model. To improve the performance of the shaker it is recommended that the total machine stiffness to ground be increased in a series of steps: (1) Stiffening the suspension in the vertical direction; (2) Increasing wheel (and/or pivot) bearings stiffness by using different set of bearings. The analysis also reveals that machine resonances existed both above and below current operating speed after the modifications. Any changes in operating speed would need to be evaluated carefully before implementation.

Results Verification

The modified harvester shaker is tested using modal analysis method and the identified machine characteristics is almost identical to the estimated one. Deviation observed in natural frequencies is less than 0.5 percent, for example. The forced respose of the shaker in operation showed a reduction of about 90 percent in the amplitude in the required directions. Fig. 6 shows the prototype of the harvester shaker after modifications.

CONCLUSIONS

A method is developed for analyzing the dynamic behaviour of a complex machine that can be dismounted into interconnected elements. It combines experimental and analytical techniques into system analysis routine to provide an important information for designing superior mechanical products. In the proposed method the generalized mass, stiffness, and damping matrices are determined, either analytically or experimentally, for each machine element, as are generalized forces. The requirement of the system continuity gives rise to equations of displacement compatibility at the connections. These equations of constraint are used to determine a set of machine generalized coordinates equal in number to the total number of element coordinates minus the number of equations of constraint. Constraint equations are used to construct a transformation relating element local coordinates to machine global coordinates. This transformation is used to drive machine characteristics and forces from element characteristics and forces. Overall machine equations of motion are formulated and solved to determine machine performance and responses. Element responses are found using the transformation. Connection forces are computed from the element equations. Each element can then be isolated and treated separately.

This method allowes an engineer to comprehend how variuos machine elements interact, even before the physical product is available or assembled, and this can serve as a base line for future modifications or designs. Using this hybrid method in troubleshooting problems eliminates many cut-and-try steps in deriving an optimal solution. The information gained in the analysis can identify not only a number of areas that would improve system performance, but also those that would have no effect or actually worsen performance. Finally, the method can be used by designers at any step for correlation and verification of deduced analytical models.

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NOMENCLATURE

B	transformation matrix
C, c	square matrix of machine (element) generalized damping coefficient.
F(t)	vector of time-dependent machine generalized forces.
G	square nonsingular transformation matrix.
H	transformation matrix.
K k	square matrix of machine (element) generalized stiffness.
M m	square matrix of machine (element) generalized mass.
P(t)	vector of time-dependent element generalized forces.
S	transformation matrix
X,X,X	vector of machine generalized displacements, velocities, and accelerations.
Y,Y,Y	vector of element generalized displacements, velocities, and accelerations.
Y_d	vector of dependent variables.
Y_n	vector of independent variables.

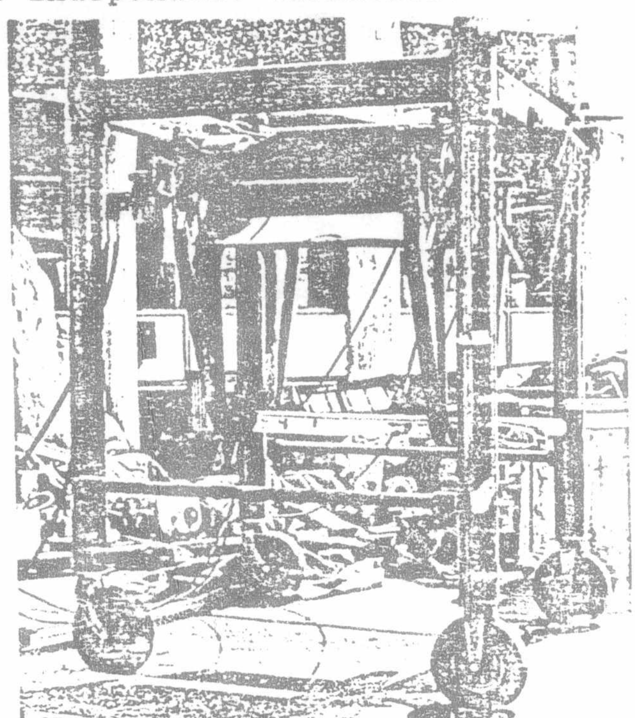

Fig. 6 Prototype of the developed harvester shaker.