MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

1 # VIBRATION OF A VARIABLE CROSS SECTION SHAFT
2 # CARRYING MULTI-DISCS WITH ANY NUMBER OF EXCITATIONS

3 H.M. METWALLY* M.EL-SAYED**

4 ## ABSTRACT

This paper presented the systematic procedures for finding the critical speed and the bending moment in each segement of the vibratory variable cross-section shaft carrying multi discs. Lateral vibrations result from transversal dynamic excitations. The influence coefficients are obtained by using "Castigliano- Theorem". Samples of data for the obtained results are presented for a shaft carrying three discs with an eccentricity on the second disc for both cases of end supported and over hanged shafts on rigid support including the variation of disc position. The results of shaft amplitudes and natural frequencies are obtained by computer analysis.

INTRODUCTION

For about hundered years the problems of rotating shafts was considered scientific workers with a variety of aims and procedures. The problem of critical speed of rotating shaft was studied by Stodola[1], Timoshenko[2] and Den Hartog[3]using the simple theory of statics. The case of variable cross-section when the shaft has a disc or many discs and is supported in ideal rigid bearings are obtained by Metwally and Parszewski[4] by the extantion of the Myklested[5,6] method for a variable cross-section shaft. In this paper it is another method to obtain the bending stress and the critical speed of a variable cross-section shaft carrying three discs, due to lateral vibrations, resulting from a dynamic excitation. In this method can be applied for any number of excitations. The critical speeds and the natural frequencies are obtained by the solution of the series of the corresponding modes of the rotor in ideal rigid bearings can be applied for the case of ainsotropic flexible bearings or supporting structure. The model for the machine which has a variable cross-section shaft and carries many discs in two cases i) Case of end-supported rotor. ii) Case of over-hanged rotor.

THE CHARACTERISTIC EQUATIONS OF THE SYSTEM

The laterial vibration of the variable cross-section shaft carries three discs each has an excitation as shown in Fig. (1).

Each of the two shafts is carrying three discs, these discs are

5 * Associate Professor, Mechanical Engg. Dept., Faculy of Engineering, Alexandria Univ. Alexandria, Egypt.
** Teaching Assistant, Mechanical Engg. Dept, Faculty of Engg. El-Mansoura University, El-Mansoura, Egypt.

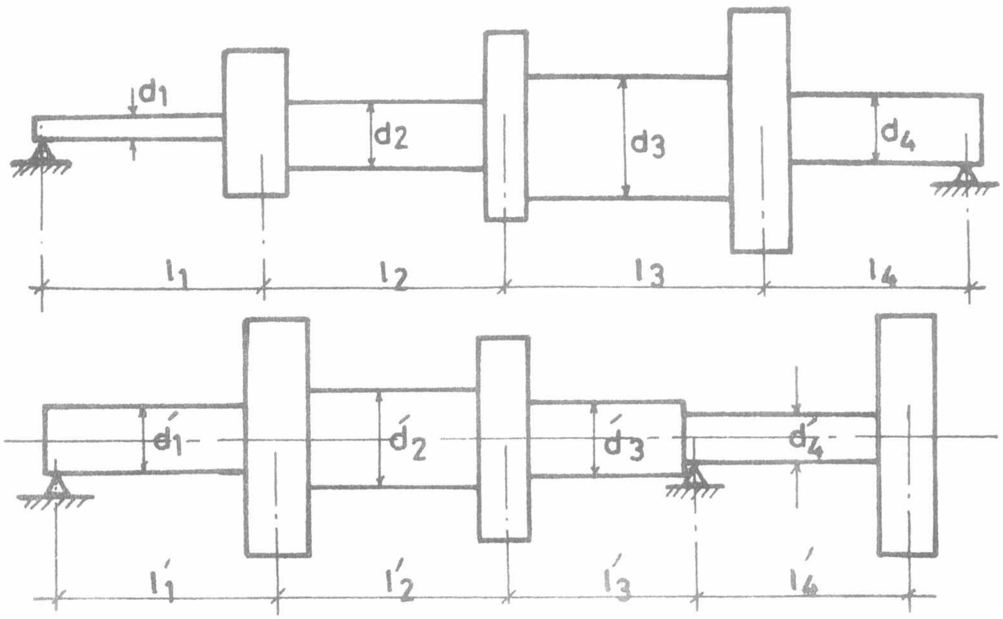


Fig. (1) End Supported and Over-Hanged Rotor

concentrated at positions (1), (2) and (3) as shown in Fig. (2).

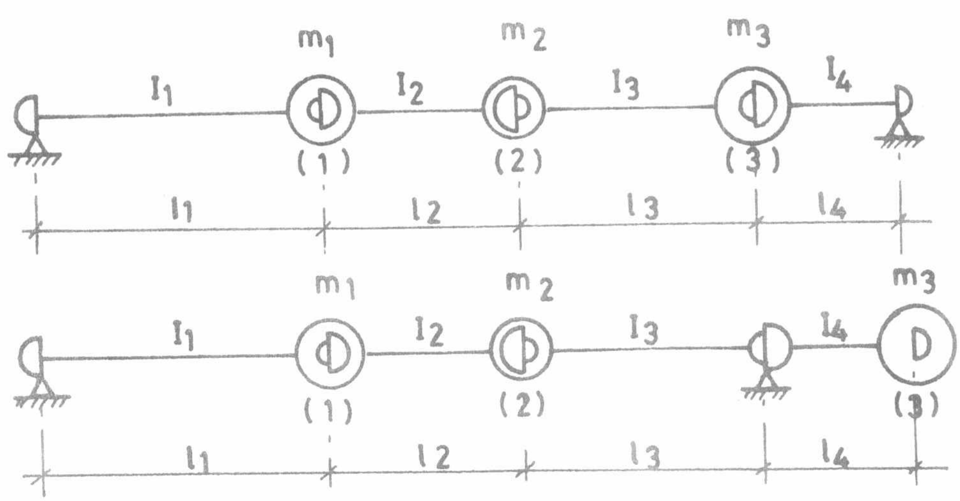


Fig. (2) Simulated Shaft

The variable cross-section shaft is simulated by a shaft having the same number of stations of the original shaft with concentratedmasses conected with each other by a weightless shafts. The mass distribution is carried out so that the simulated shaft has the same total mass, and the center of gravity of each section is the same as the equivalent section of the original shaft. The concentrated masses can be calculated as follows:

$$m_1 = \tfrac{1}{2} m_{s1} + \tfrac{1}{2} m_{s2} + m_{d1}$$

$$m_2 = \tfrac{1}{2} m_{s2} + \tfrac{1}{2} m_{s3} + m_{d2} \qquad (1)$$

$$m_3 = \tfrac{1}{2} m_{s3} + \tfrac{1}{2} m_{s4} + m_{d3}$$

where, m_{s1} , m_{s2} , m_{s3} are the masses of the segments of the shaft for diameters d_1, d_2 and d_3 and m_{d1}, m_{d2} and m_{d3} are the masses of the discs at stations (1), (2) and (3). These method can be applied for any shaft lengthes and many discs.

The system undergo forced vibration due to the eccentricity of one of the discs. Let us assume the eccentricity at position (2), the dynamic force is P_o

$$P_o = m_2 \, e \, w^2 \qquad (2)$$

where m_2 : is the mass of the second disc

e : is the eccentricity of the second disc

w : is the angular velocity of the rotating shaft.

the deflection at the three positions due to lateral vibrations will be y_1, y_2 and y_3 respectively. These deflections can be obtained, using the influence coefficients method, Jacobsen and Ayre[7]. For the case of (n) degrees of freedom system we have:

$$\text{The numbers of influence coefficients} = \frac{n(n+1)}{2} \qquad (3)$$

For the case of three concentrated masses the number of coefficients is equal to six, and these coefficient can be obtained according to the following steps

Case of End-supported shaft

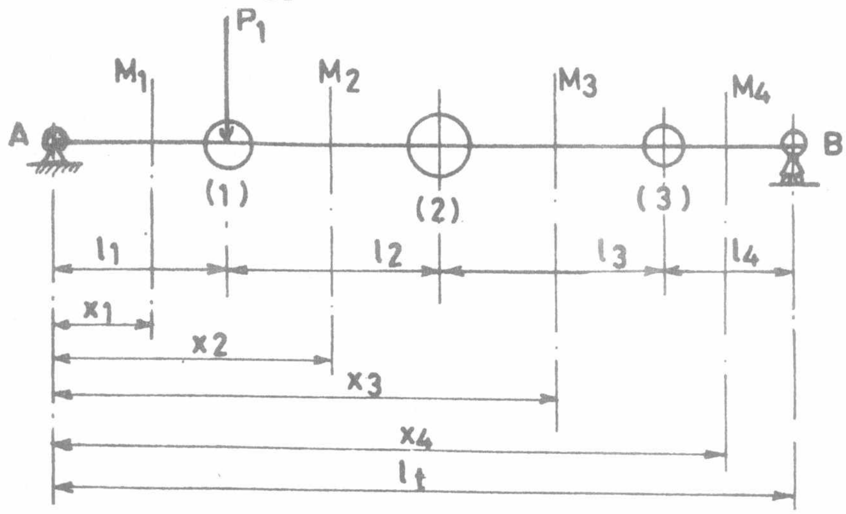


Fig. 3 End Supported Simulated Shaft

For a variable cross-section shaft Fig. 3 consider the exciting force P_1 due to rotation has a bending moment effect relatively large than the discs weight and shaft.

The reaction at the supports A and B are:

$$R_A = P_1 \cdot \frac{l_2 + l_3 + l_4}{l_1 + l_2 + l_3 + l_4} = P_1 \frac{l_2 + l_3 + l_4}{l_t} \qquad (4)$$

$$R_B = P_1 \frac{l_1}{l_1 + l_2 + l_3 + l_4} = P_1 \frac{l_1}{l_t}$$

For finding the deflection at position (1), caused by the force P_1 we can apply Castigliano's theorem, which states that ,

$$\Delta_p = \frac{\partial}{\partial P} \int_0^l \frac{M_x^2}{2EI}\, dx \qquad (5)$$

where : Δ_p = the deflection under the load P.

M_x = the bending moment, at distance x .

E = Modulus of elasticity

I = Moment of inertia of shaft cross-section.

For the variable cross-section shaft considered the deflection Δ_p

$$\Delta_p = \frac{1}{EI_1}\int_0^{l_1} M_1 \frac{\partial M_1}{\partial P_1} dx_1 + \frac{1}{EI_2}\int_{l_1}^{l_1+l_2} M_2 \cdot \frac{\partial M_2}{\partial P_1} dx_2 + \frac{1}{EI_3}\int_{l_1+l_2}^{l_1+l_2+l_3} M_3 \cdot \frac{\partial M_3}{\partial P_1} dx_3 + \frac{1}{EI_4}\int_{l_1+l_2+l_3}^{l_1+l_2+l_3+l_4} M_4 \cdot \frac{\partial M_4}{\partial P_1} dx_4 \qquad (6)$$

The calculated bending moment for each of the four segments due to the acting force P_1 only will produce a deflection Δ_{11} at position(1). According to the definition of the influence coefficient, we can get α_{11} by putting P_1 is unity in the deflection expression.

$$\alpha_{11} = \frac{1}{EI_1}\left\{\frac{1}{3}\left(\frac{l_2+l_3+l_4}{l_t}\right)^2 l_1^3\right\} + \frac{1}{EI_2}\left\{\frac{1}{3}\left[\left(\frac{l_2+l_3+l_4}{l_t}\right)-1\right]^2 \cdot \left[(l_1+l_2)^3-l_1^3\right] + (l_1)\left[\left(\frac{l_2+l_3+l_4}{l_t}\right)-1\right]\left[(l_1+l_2)^2-l_1^2\right] + l_1^2 \cdot l_2\right\} + \frac{1}{EI_3}\left\{\frac{1}{3}\left[\frac{l_2+l_3+l_4}{l_t}-1\right]^2\left[l_t^3-(l_1+l_2+l_3)^3\right] + l_1\left[\frac{l_2+l_3+l_4}{l_t}-1\right]\left[(l_1+l_2+l_3)^2-(l_1+l_2)^2\right] + l_1^2 \cdot l_3\right\} + \frac{1}{EI_4}\left\{\frac{1}{3}\left[\frac{l_2+l_3+l_4}{l_t}-1\right]^2\left[l_t^3-(l_1+l_2+l_3)^3\right] + l_1\left[\frac{l_2+l_3+l_4}{l_t}-1\right]\left[l_t^2-(l_1+l_2+l_3)^2\right] + l_1^2 \cdot l_4\right\} \qquad (7)$$

The calculation of the other influence coefficients α_{22} and α_{33} applying the above procedure, considering the unit force acting at the position (2) for α_{22} and the unit force acting at the position (3) for α_{33}.

The calculation of the influence coefficient α_{21} for the variable cross-section shaft loaded with the two forces P_1 acting at position (1) and force F_2 at position (2), is carried out by calculating the bending moment due to the forces effect. then find the deflection by using the Castigliano's theorem. Differentiating the moments with respect to F_2 partially and then put F_2 equal to zero and P_1 is unity after integrating each shaft segment. The deflection at position (2) caused by the force P_1 which is acting at position(1) is Δ_{21} . By putting P_1 is unity then Δ_{21} is equal the influence α_{21} , can be expressed as:

$$\alpha_{21} = \frac{1}{EI_1}\left[\frac{1}{3}\left(\frac{l_3+l_4}{l_t}\right)\left(\frac{l_2+l_3+l_4}{l_t}\right)l_1^3\right] + \frac{1}{EI_2}\left\{\frac{l_3+l_4}{l_t}\left\{\frac{1}{3}\cdot\left[\left(\frac{l_2+l_3+l_4}{l_t}\right)-1\right]\left[(l_1+l_2)^3-l_1^3\right] + \frac{1}{2}\left[(l_1+l_2)^2-l_1^2\right]\right\}\right\} +$$

$$+\frac{1}{EI_3}\left\{\frac{1}{3}\left[\left(\frac{l_2+l_3+l_4}{l_t}\right)-1\right]\left[\left(\frac{l_3+l_4}{l_t}\right)-1\right]\left[(l_1+l_2+l_3)^3-(l_1+l_2)^3\right]\right.$$

$$+\left\{\frac{1}{2}\left[\left(\frac{l_3+l_4}{l_t}\right)-1\right] + \left(\frac{l_1+l_2}{2}\right)\left[\left(\frac{l_2+l_3+l_4}{l_t}\right)-1\right]\right\}\left[(l_1+l_2+l_3)^2 -\right.$$

$$\left.\left.-(l_1+l_2)^2\right] + l_1(l_1+l_2)l_3\right\} + \frac{1}{EI_4}\left\{\frac{1}{3}\left[\left(\frac{l_2+l_3+l_4}{l_t}\right)-1\right]\left[\left(\frac{l_3+l_4}{l_t}\right)-1\right]\right.$$

$$\left[l_t^3 - (l_1+l_2+l_3)^3\right] + \left\{\frac{1}{2}\left[\left(\frac{l_3+l_4}{l_t}\right)-1\right] + \frac{l_1+l_2}{2}\left[\frac{l_2+l_3+l_4}{l_t}-1\right]\right\}$$

$$\left.\left[l_t^2 - (l_1+l_2+l_3)^2\right] + l_1(l_1+l_2)\,l_4\right\} \qquad (8)$$

Applying the above procedure for finding the influence coefficients α_{31} and α_{32} , knowing that $\alpha_{12} = \alpha_{21}$, $\alpha_{13} = \alpha_{31}$ and $\alpha_{23} = \alpha_{32}$ according to Maxwell's reciprocal theorem.

The values of the influence coefficients for the over-hanged shaft were found by the procedure.

Deflections equations

Consider the variable cross-section shafts carrying the concentrated masses as shown in Fig. (3) and loaded with the dynamic force

P_o sin wt at position (2) .

The deflections at position (1), (2) and (3) respectively refering to Fig. (2) and the dynamic force are:

$$
\begin{aligned}
y_1 &= -\alpha_{11} m_1 \ddot{y}_1 - \alpha_{12} m_2 \ddot{y}_2 - \alpha_{13} m_3 \ddot{y}_3 + \alpha_{12} P_o \sin wt \\
y_2 &= -\alpha_{21} m_1 \ddot{y}_1 - \alpha_{22} m_2 \ddot{y}_2 - \alpha_{23} m_3 \ddot{y}_3 + \alpha_{22} P_o \sin wt \\
y_3 &= -\alpha_{31} m_1 \ddot{y}_1 - \alpha_{32} m_2 \ddot{y}_2 - \alpha_{33} m_3 \ddot{y}_3 + \alpha_{32} P_o \sin wt
\end{aligned}
\tag{9}
$$

Let $y_i = Y_i \sin wt$

The characteristic equations are;

$$
D = \begin{vmatrix}
1-\alpha_{11} m_1 w^2 & -\alpha_{12} m_2 w^2 & -\alpha_{31} m_3 w^2 \\
-\alpha_{12} m_1 w^2 & 1-\alpha_{22} m_2 w^2 & -\alpha_{32} m_3 w^2 \\
-\alpha_{31} m_1 w^2 & -\alpha_{32} m_2 w^2 & 1-\alpha_{33} m_3 w^2
\end{vmatrix}
\tag{10}
$$

The three amplitude will be :

$$
Y_1 = \frac{P_o \begin{vmatrix}
\alpha_{12} & -\alpha_{12} m_2 w^2 & -\alpha_{13} m_3 w^2 \\
\alpha_{22} & 1-\alpha_{22} m_2 w^2 & -\alpha_{23} m_3 w^2 \\
\alpha_{32} & -\alpha_{32} m_2 w^2 & 1-\alpha_{33} m_3 w^2
\end{vmatrix}}{D}
\tag{11}
$$

$$
Y_2 = \frac{P_o \begin{vmatrix}
1-\alpha_{11} m_1 w^2 & \alpha_{12} & -\alpha_{13} m_3 w^2 \\
-\alpha_{21} m_1 w^2 & \alpha_{22} & -\alpha_{23} m_3 w^2 \\
-\alpha_{31} m_1 w^2 & \alpha_{32} & 1-\alpha_{33} m_3 w^2
\end{vmatrix}}{D}
\tag{12}
$$

$$
Y_3 = \frac{P_o \begin{vmatrix}
1-\alpha_{11} m_1 w^2 & -\alpha_{12} m_2 w^2 & \alpha_{12} \\
-\alpha_{21} m_1 w^2 & 1-\alpha_{22} m_2 w^2 & \alpha_{22} \\
\alpha_{31} m_1 w^2 & -\alpha_{32} m_2 w^2 & \alpha_{32}
\end{vmatrix}}{D}
\tag{13}
$$

Samples of Design Data

Samples of design data are given for three degrees of freedom

6

system to illustrate the previous procedure for both cases of end supported and over hanged rotors. The input parameters are:

length : $l_1 = 254$, $l_2 = 127$, $l_3 = 309$, $l_4 = 200$ mm

diameter: $d_1 = 82.27$, $d_2 = 82,58$, $d_3 = 83.66$, $d_4 = 83.72$ mm

for simple supports

weight of discs : $W_1 = W_2 = W_3 = 1.5$ K.N

eccentricity of the second disc $e_2 = 2.5$ mm

Young's modulus E = 200 G.N./m

rotating speed w = 100 rad/s.

For Over-hanging lengths: $l_1 = 76.2$, $l_2 = 254$, $l_3 = 200$, $l_4 = 359.8$

" " " diameter: $d_1 = 48.19$, $d_2 = 48.43$, $d_3 = 50.16$, $d_4 = 50.16$

Results Obtained

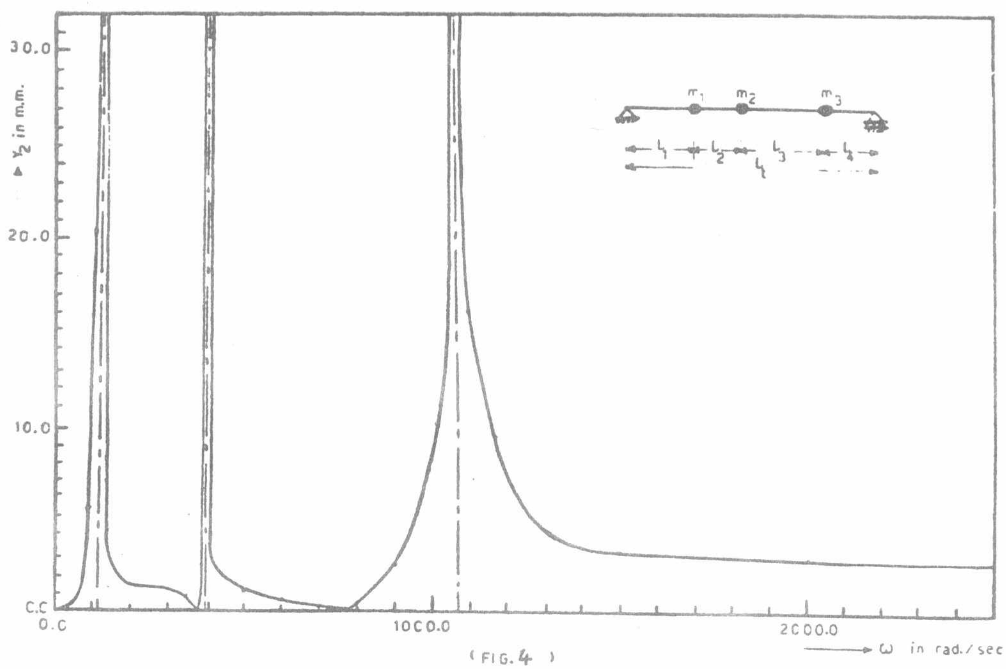
The results are drawn between the frequency w of the rotating shaft and the amplitude of deflection as shown in Fig. (4) and (6) for Y_2, Y_3 and Y_3 for simple supported rotor. The deflection curves for over hanging rotor as shown in Fig. (7), (8) and (9) for Y_1, Y_2 and Y_3 are drawn against the frequency of the rotating shaft w.

Conclusions

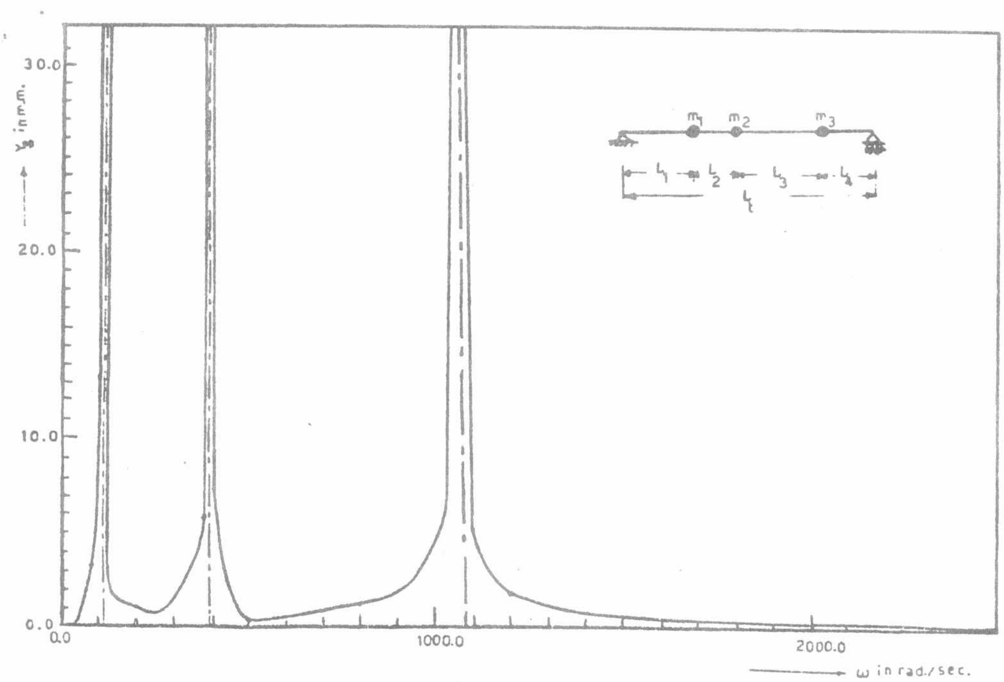
The above method is an another method for determine the natural frequencies of any rotor has a variable cross-sections and carrying many unbalanced discs, also the deflections at discs.

REFERENCES

1. Stodola, "The steam Turbine" 1905
2. Timoshenko, S., Vibration Problems in Engineering, D. Van Norsted, New York, 1955.
3. Den Hartog, J.P., Mechanical Vibration, 4th Ed. Mc Graw-Hill, New York, 1956.
4. Metwally, H.M., Parszewski, Z. "Vibration of Disc Rotors Supported Flexibly" Fourth World Congress on the Theory of Machines and Machinisms, Vol.3, Newcastle upon tyne, England, 1975.
5. Myklestad, N.O. "A New Method of Calculating Natural Modes of Uncoupled Bending Vibration of Airplane Wings and Other types of Beams of Areonautical Sciences, Vol. 11., p. 153, 1944.
6. Myklestad, N.O. "Fundamentals of Vibration Analysis" Mc-Graw-Hill, New York, 1956.
7. Jacobsen, L., and Ayre, R. Engineering Vibrations, Mc-Graw-Hill, Inc., New York 1955.

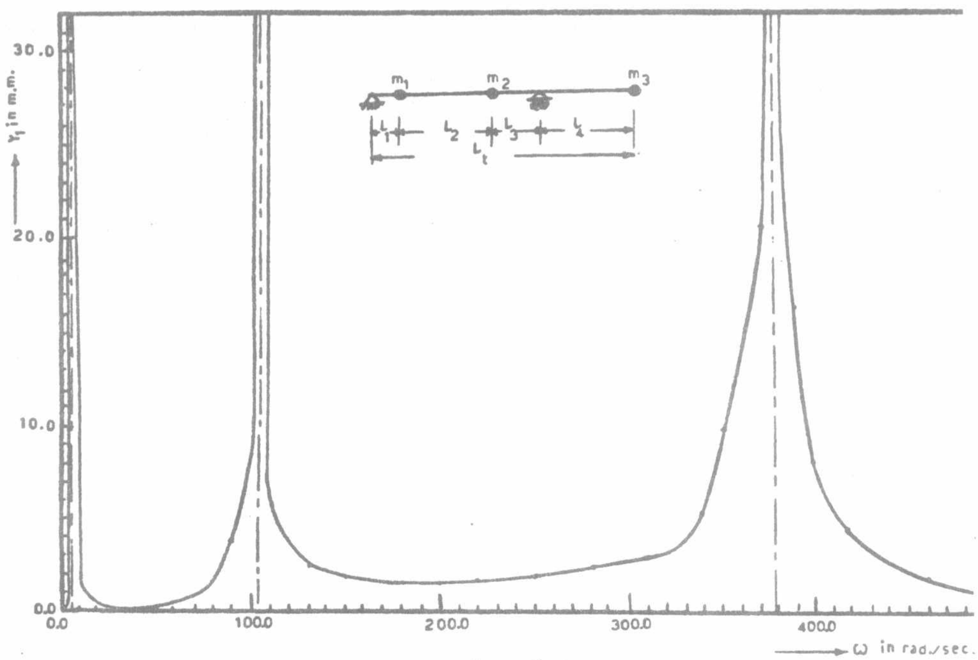

30.0
20.0
10.0
0.0
y_2 in m.m.
0.0
1000.0
2000.0
ω in rad./sec
m_1
m_2
m_3

(FIG. 4)

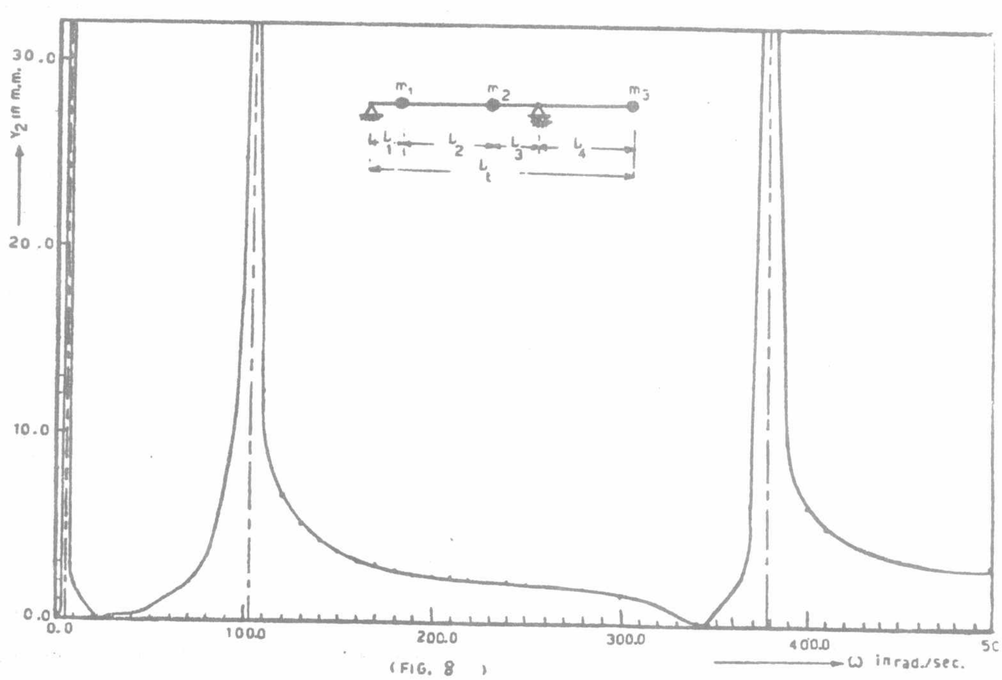

30.0
20.0
10.0
0.0
y_3 in m.m.
0.0
1000.0
2000.0
ω in rad./sec.
m_1
m_2
m_3

(FIG. 6)

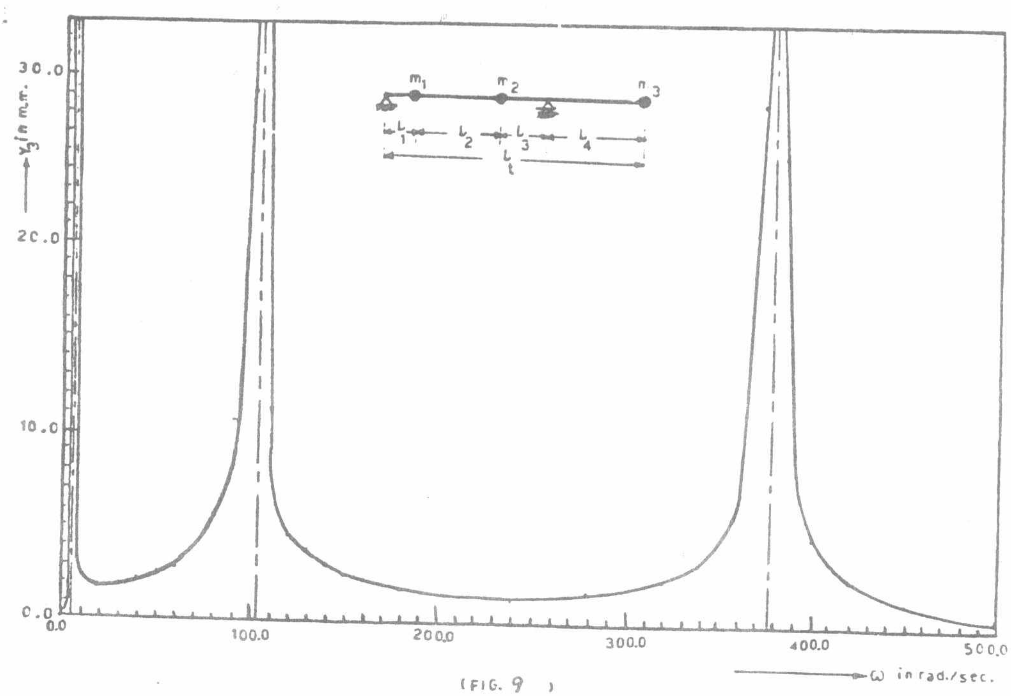
6


30.0
20.0
10.0
0.0
Y_1 in m.m.
0.0
100.0
200.0
300.0
400.0
ω in rad./sec.
m_1
m_2
m_3
L_1
L_2
L_3
L_4
L_t

(FIG. 7)


30.0
20.0
10.0
0.0
Y_2 in m.m.
0.0
100.0
200.0
300.0
400.0
ω in rad./sec.
m_1
m_2
m_3
L_1
L_2
L_3
L_4
L_t

(FIG. 8)



(FIG. 9)