MILITARY TECHNICAL COLLEGE

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EFFECT OF SHELL ENDS ON THE CONTACT PROBLEMS

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ABSTRACT

The effect of shell ends on the contact problems of saddle supporting cylindrical shell is solved using the semi-bending theory of shells. The saddle / cylinder interface pressure and the stress distributions are found for different saddle angles, for both welded saddle and not welded saddle. The min. distance of the saddle position from the shell end is found such that the end has no effect on the stress at the saddle positions. The circumferential stresses at the plane of middle profile of the saddle are found experimentally using strain gauge. The experimental results are in good agreement with those obtained from numerical analysis.

INTRODUCTION

In spite of the importance of the contact problem of supporting shells, it has not been precisely solved. Twin saddle supports are the most frequentely used type of supports for horizontal vessels. The published works treated special cases of this problem by using the generalized bending theory of shells [I,5] or the semi-bending theory of shells [2,3,4] . They consider the shell either infinitly long or finite with clasical boundary conditions. So far semi-empirical methods have been used for design purpose to predict the peak values of stresses[6].These values of stresses depend on the shell parameters, half saddle angle (β), shell end conditions and manner of connections of the saddle to the cylinder. In the present work we consider a semi-infinite long cylindrical shell with free ends rested on two saddles with vertical load (F) at mid length of shell Fig.I. This work studies the effect of the shell ends on the stresses and interface pressure distributions, for welded and not welded saddle, and attempts to pressent a simple analytical solution for this problem. The solution determines the distribution and magnitude of the radial saddle/cylinder interface pressure,

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then the stress resultants, and displacements throughout the vessel are calculated. The solution is based on using the so called semi-bending theory of shells with compressible middle surface for deriving the influence line due to the radial unit load[7]

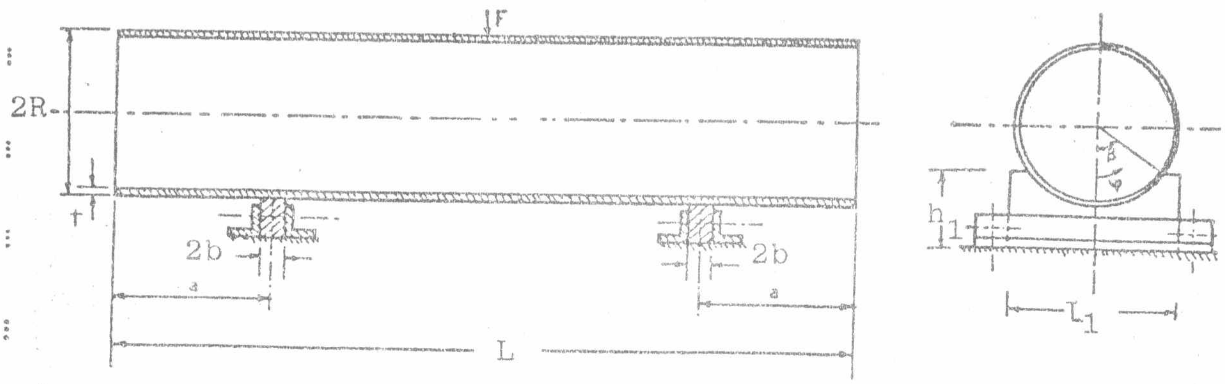


Fig.1 Shell and supports

GOVERINING DIFFERENTIAL EQUATIONS

In the present work the generalized semi-bending theory with compressible middle surface is used in deriving the required equations. The basic assumption of this theory is the negligence of the increment in curvature of the shell in the axial direction. The principle of minimum potential energy is used for deriving the differential equation. By studying the equilibrium of the shell element Fig.2, and the shell deformation, the following differential equation is obtained:-

$$N_n''''(x) - 2a_n N_n''(x) + b_n^2 N_n(x) = c_n \qquad (1)$$

where, $N_n(x)$ is the longitudinal normal force and primes indicate differentiation with respect to x and

$$a_n = \frac{t^2}{R^4} \frac{n^2\left[1+\mu-n^2(2+\mu)+n^4\right]}{12+\delta^2 n^2\left[n^2+\frac{12}{5}(1+\mu)\right]}$$

$$b_n = \frac{t}{R^3} \sqrt{\frac{n^2(n^2-1)}{12+\delta^2 n^2\left[n^2+\frac{12}{5}(1+\mu)\right]}} \qquad (2)$$

$$c_n = \frac{\mu t^2}{\pi R^6} \frac{n^4(n^2-1)^2}{12+\delta^2 n^2\left[n^2+\frac{12}{5}(1+\mu)\right]} \phi^N(x)$$

$$\Phi^N(x) = \S\, T_s^\circ \cos n\psi\, d\psi$$

T_s^o normal force in circumferential direction at n=1.

$\delta = t/R$, t is the thickness of the shell, R is the shell radius and μ is the Poisson's ratio.

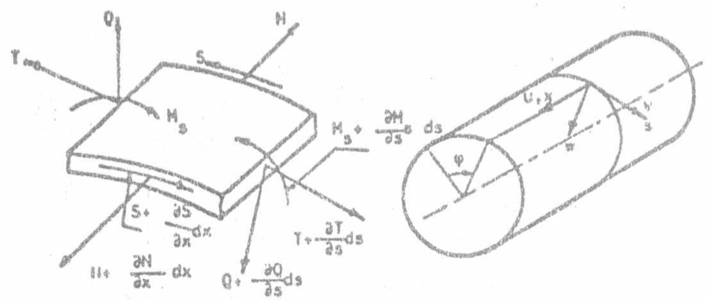


Fig. 2.

SOLUTION OF DIFFERENTIAL EQUATION AND DETERMINATION OF THE INFLUENCE LINES.

Consider a circular cylindrical shell with semi-infinite length subjected to a unit surface load in the interval from $x = a-b$ up to $x = a+b$ as shown in Fig. 3. The surface load has a very small angle. Expressing the load by Fourier series and solving the corressponding differential equation using the boundary conditions of the shell end (free ends), the influence lines for unit radial surface load takes the following forms:-

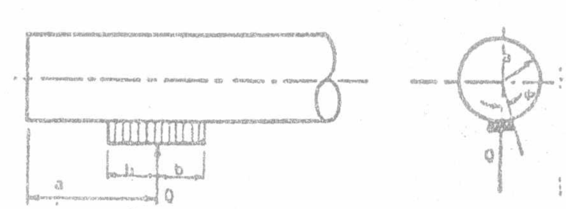


Fig. 3

$$w_R\,(a,\varphi) = \frac{1}{\bar{E}R}\left(\frac{R}{t}\right)^{5/2} F_{WR}$$

$$v_R\,(a,\varphi) = \frac{1}{\bar{E}R}\left(\frac{R}{t}\right)^{5/2} F_{VR}$$

$$\sigma_{sR}\,(a,\varphi) = \frac{1}{t^2}\left(\frac{t}{R}\right)^{\frac{1}{2}} F_{SR}$$

$$\bar{E} = \frac{E}{1-\mu^2} \qquad (3)$$

where, w_R is the radial displacement, v_R is the tangential displacement and σ_{sR} is the circumferential normal stress at the middle profile of the saddle.

For compressible middle surface and $b_n^2 > a_n^2$;

$$F_{WR} = -\frac{6}{\pi K_1 \sin\phi}\sum_{n=2}^{n=\infty}\frac{\sin n\Phi \cos n\varphi}{n(n^2-1)^2}\,F_{1n}$$

$$F_{VR} = -\frac{6}{\pi K_1 \sin\phi}\sum_{n=2}^{\infty}\left[\frac{1+\Lambda n^2(n^2-1)}{n^2(n^2-1)^2}\,F_{1n} - \frac{\mu\delta^2}{12(1-\mu^2)R^2}\,F_{2n}\right]\sin n\varphi\,\sin n\phi$$

$$F_{SR} = -\frac{3}{b\sin\phi}\sum_{n=2}^{\infty}\frac{\sin n\phi \sin n\varphi}{n\,(n^2-1)}F_{1n}$$

$$F_{1n} = -e^{-\omega_1 k_1}\cos r_1 k_1 + \frac{\omega_1^2 - r_1^2}{2\omega_1 r_1}e^{-\omega_1 k_1}\sin r_1 k_1 + \grave{\lambda}_4 + 1,$$

$$\grave{\lambda}_1 = e^{-\omega_1(2k_2+k_1)}\cos r_1(k_2+k_1) - e^{-\omega_1(2k_2-k_1)}\sin r_1(k_2-k_1)$$

$$F_{2n} = \frac{\omega_1^4 - 6\omega_1^2 r_1^2 + r_1^4}{4\omega_1 r_1\,(\omega_1^2+r_1^2)^2}\,(\grave{\lambda}_1\cos r_1 k_2 - 2e^{-\omega_1 k_1}\sin r_1 k_1) + \frac{r_1^2-\omega_1^2}{(r_1^2+\omega_1^2)^2}\,.$$

$$(\grave{\lambda}_2\cos r_1 k_2 + 2e^{-\omega_1 k_1}\cos r_1 k_1 - 2) + \grave{\lambda}_3\,, \qquad (4)$$

$$\grave{\lambda}_2 = e^{-\omega_1(2k_2+k_1)}\cos r_1(k_2+k_1) - e^{-\omega_1(2k_2-k_1)}\sin r_1(k_2-k_1),$$

$$\grave{\lambda}_3 = \left(\frac{\omega_1^4 - 4\omega_1^2 r_1^2 - r_1^4}{2r_1^2\,(\omega_1^2+r_1^2)^2}\lambda_1 + \frac{7\omega_1^4 - 2\omega_1^2 r_1^2 - r_1^4}{4\omega_1 r_1(\omega_1^2+r_1^2)^2}\lambda_2\right)\sin r_1 k_2,$$

$$\grave{\lambda}_4 = (\grave{\lambda}_1\grave{C}_1 - \grave{\lambda}_2\grave{C}_3)\cos r_1 k_2 + (\grave{\lambda}_1\grave{C}_2 + \grave{\lambda}_2\grave{C}_4)\sin r_1 k_2,$$

$$\grave{C}_1 = \frac{12\omega_1^2 r_1^2 - 3\omega_1^4 - r_1^4}{4\omega_1 r_1(\omega_1^2 + r_1^2)}, \qquad \grave{C}_2 = \frac{\omega_1^4 - 5\omega_1^2 + 2r_1^4}{2r_1^2\,(\omega_1^2 + r_1^2)}$$

$$\grave{C}_3 = \frac{4\omega_1^2 - 3r_1^2}{2(\omega_1^2 + r_1^2)}, \qquad \grave{C}_4 = \frac{7\omega_1^4 - 8\omega_1^2 r_1^2 + r_1^4}{4\omega_1 r_1(\omega_1^2 + r_1^2)}, \quad \omega = \sqrt{\tfrac{1}{2}(b_n + a_n)}$$

$$\omega_1 = \frac{R\omega}{\sqrt{\delta}}, \quad r_1 = \frac{R.r}{\sqrt{\delta}}, \quad k_1 = \frac{b}{R}\sqrt{\delta}, \quad k_2 = \frac{a}{R}\sqrt{\delta}, \qquad r = \sqrt{\tfrac{1}{2}(b_n - a_n)}$$

Similar expressions can be derived for $b_n^2 < a_n^2$ and $b_n^2 = a_n^2$,[7]

SOLUTION OF THE CONTACT PROBLEM AND DETERMINATION OF THE INTERFACE PRESSURE

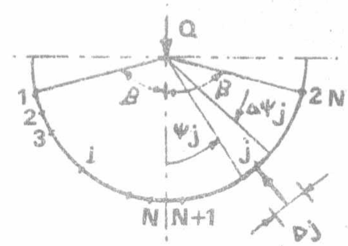
This work will attempt to pressent a simple analytical solution for the contact problem. The solution determines the distribution and magnitude of the radial interface pressure, then the stress and displacement resultants throughout the vessel are calculated. Throughout the analysis, the following assumptions are made:-

1. The saddle is rigid compared with the cylinder

2. The pressure distribution accross the saddle width is constant, see Ref. [I]
3. The self weight of the vessel is small compared with the external applied load and is neglected.

If the required saddles are possitioned equidistance from both ends, with the centres of the saddle arcs on the same cylinder generator, then the distribution and magnitude of the interface pressure is the same for both saddles.

For deriving the continuity equations, the saddle contact surface is divided into 2N rectangular sections, each of axial length 2b equal the saddle width and corresponding to the circumferential length Δ_j with the section of the angle $\Delta\psi_j$,



Radial load in the plane of the saddle profile.

Fig. 4

Fig.4. Each section of area $2b.\ \Delta j$ is loaded by a uniform radial pressure x_j. Then the total radial displacement of the cylinder profile at point i due to all the radial interface pressure is given by :-

$$w_i = \sum_{j=I}^{2N} x_j \cdot w_{Rij} \qquad (5)$$

where, w_{Rij} is the radial displacement of point i on the profile due to a unit radial pressure acting over the area j. Similar expressions exist for all 2N points on the cylinder in the saddle arc region. Assuming that the saddle and the cylinder remain in contact under the action of the saddle reaction (Q). So the resultant radial displacement of the cylinder within the saddle arc, due to the radial interface pressure, must be equal to the radial displacement of the saddle, as a rigid body, relative to the end of the cylinder. Thus the continuity equations or the equations of common radial displacement of all points of the saddle and the cylinder are written in matrix form as:-

$$[W_R]\ [X] = [S_R] \qquad (6)$$

where:-

W_R: 2N x 2N matrix of radial displacement due to unit radial pressure

X : column matrix of saddle radial interface pressure.

S_R: 2N column matrix of saddle radial displacement.

$S_{Ri} = K \cos \psi_i$

K : vertical rigid body displacement of the saddle relative to the shell end.

The equilibrium equation of the vessel Fig. 4. is

$$Q = \sum_{i=I}^{2N} X_i \cdot \cos \psi_i \qquad (7)$$

Solving the continuity equation (6) together with the equilibrium equation (7), the magnitudes of the radial interface pressure can be found, then the displacements and the stress resultants throughout the vessel can be determined. For a crossection passing through the saddle center profile, at any point i on the cylinder, the total radial displacement is given by equ. (5).

The total tangential displacement is:-

$$v_i = \sum_{j=I}^{2N} X_j \, V_{Rij} \qquad (8)$$

and the total circumferential stress is:-

$$\sigma_{si} = \sum_{j=I}^{2N} X_j \, \sigma_{sRij} \qquad (9)$$

Where, V_{Rij}, σ_{sRij} are the circumferential displacement and circumferential stress at point (i) due to a unit radial interface pressure acting over the area at point j on the same profile.

COMPUTER PROGRAM

A program was written for solving the problem using ICL computer with the following steps.

1. Calculation of the influence line, eqn. (3). In this step a quick convergance series representation for the influence lines are used.
2. Formation of the continuity equation (6)
3. Solving the continuity equation together with the equilibrium eqn. (7) to determine the saddle/cylinder interface pressure using Gaussian solution.
4. Calculate the displacements and stresses using eqs.(5,6,7)

Data were prepared for the solution of an example of a model cylinder with the following dimentions:
Radius R = 250 mm, Length L = 2900 mm, Thickness t = 9 mm,

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Shell rigidity $K_I = \frac{b}{R}\sqrt{\delta} = 0.02372$, half saddle angle $\beta=16°$ and $60°$, saddle width $2b = 60$ mm, Poisson's ratio $\mu=0.3$, modulus of elasticity $E = 2 \times 10^3$ MP_a and different shell end factor $k_2 = \frac{a}{R}\sqrt{\delta}$ with various values for the distance "a"

ANALYSIS AND DISCUSSIONS

Studying the interface pressure distribution for different values of K_2, for welded saddle Fig. 5, and for not welded saddle Fig. 6, we conclude that the magnitude of the interface pressure (X) for welded saddle is greater than the corresponding values in not welded saddle, but it has zero values at areas where negative sign appears for welded saddle. To obtain the distribution of the radial interface pressure (X) for not welded saddle, we can use the same computer program for welded saddle, but when the contact is lost at any section the interface pressure must be zero. This condition can be easily incorporated into the program by examining the sign of the resulting interface pressure, where they occured as negative values, remove the corresponding row and column in the matrix $[W_R]$. Set the descret negative load (X) equal to zero in the loading vector [X] and repeat the calculation.

The contact will take place at the saddle horn and middle point of the profile of the saddle for $\beta = 16°$. For $\beta = 40°$ a surface of contact will develop in the central part of the support. For $\beta = 60°$, the contact area in the central part of the saddle arc will increase up to $\psi = 36°$, Fig.7.

The maximum of stress coefficient F_{smax}. decreases with the increase of k_2 upto $k_2 = 0.3$, Fig. 8,9. For $k_2 > 0.3$, F_{smax} has approximately a constant value. Therefore, the circumferential stress σ_s throughout the vessel in the saddle region is not affected with the shell end conditions for $a \geq 0.3R/\sqrt{\delta}$ Fig. 10, shows that the maximum circumferential stress (at the saddle horn) for not welded saddle is less than the corresponding values for welded saddle by 26%.

When the saddle arc is divided into 2N sections with $\Delta\psi j = \beta/N = 4°$ or $1°$, we notice that the numerical results for $N = 1°$ are in good agreement with those experimental results Fig. II . Fig. 12 shows the numerical results of stress for infinitly long shell, Ref. [5,6] , also for semi-infinite length shell; of which we notice that the experimental results are in good agreement with those obtained for semi-infinite length.

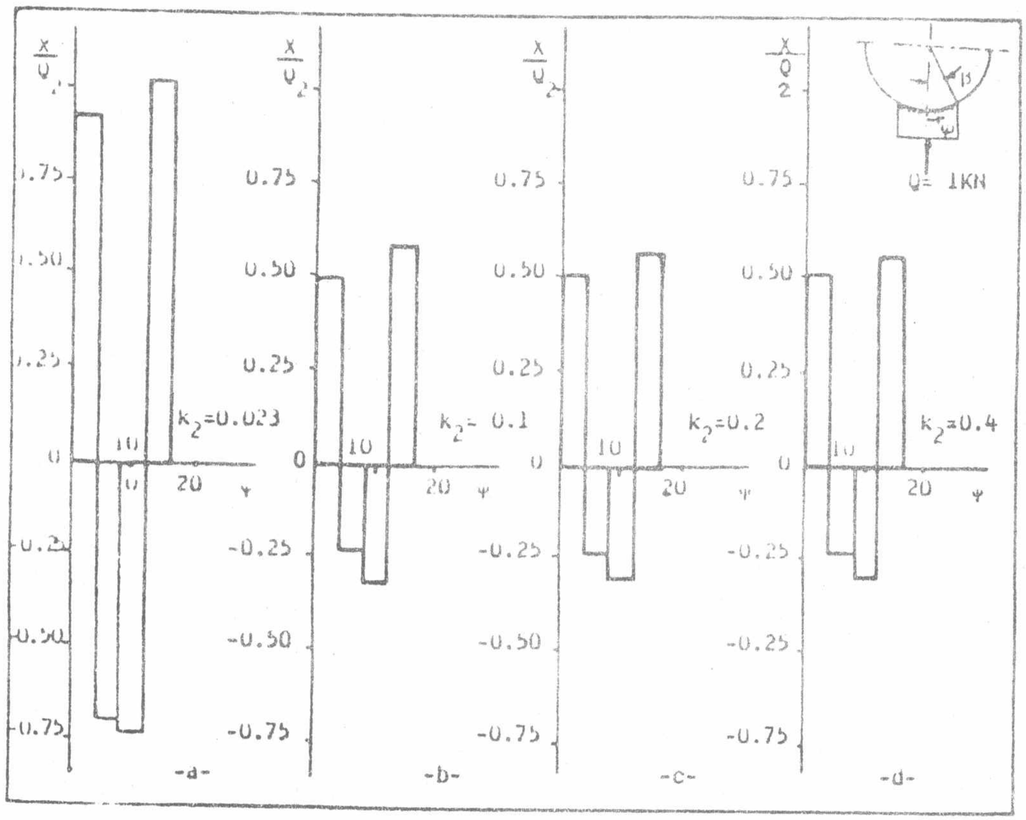

Fig. 5. Interface pressure distribution for welded saddle, different values of K_2 and ($\Delta\phi = 4°$, $\beta = 16°$, $k_1 = 0.02372$, $\delta = 0.036$) due to unit radial load Q= 1 kN.

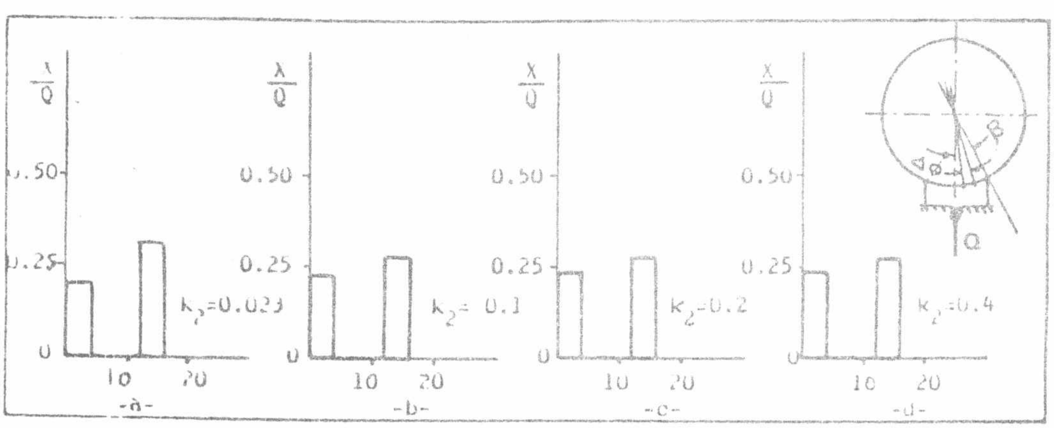

Fig. 6. Interface pressure distribution for not welded saddle, different values of k_2 and ($k_1 = 0.02372$, $\beta = 16°$, $\delta = 0.036$, $\Delta\phi = 4°$) due to unit radial load.

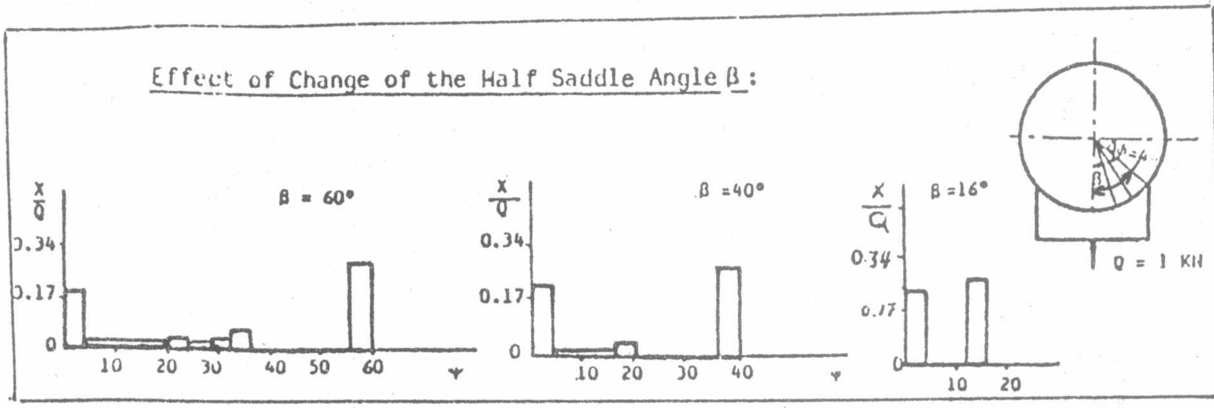

Fig. 7. Interface pressure distribution for not welded saddle and $k_2 = 0.4$, $k_1 = 0.02372$, $\delta = 0.036$) due to unit radial load.

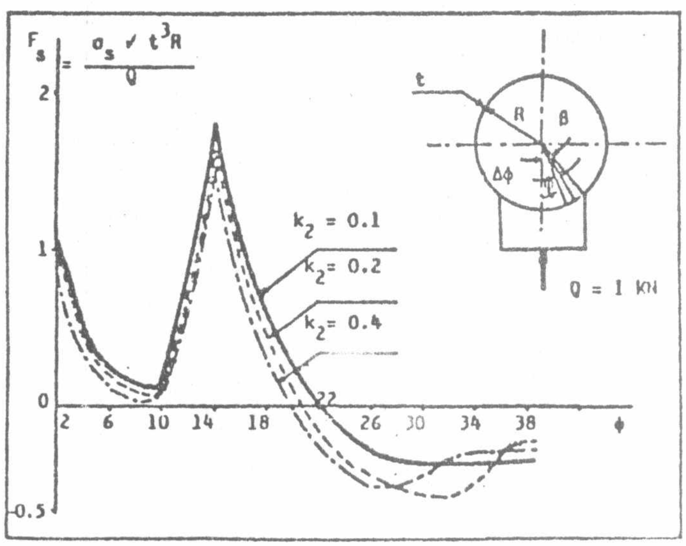

Fig. 8 Variation of stress coefficient F_s for ($k_1 = 0.0$[illegible] , $\delta = 0.036$, $\beta = 16°$, $\Delta\phi = 4°$) and different value of end factor k_2.

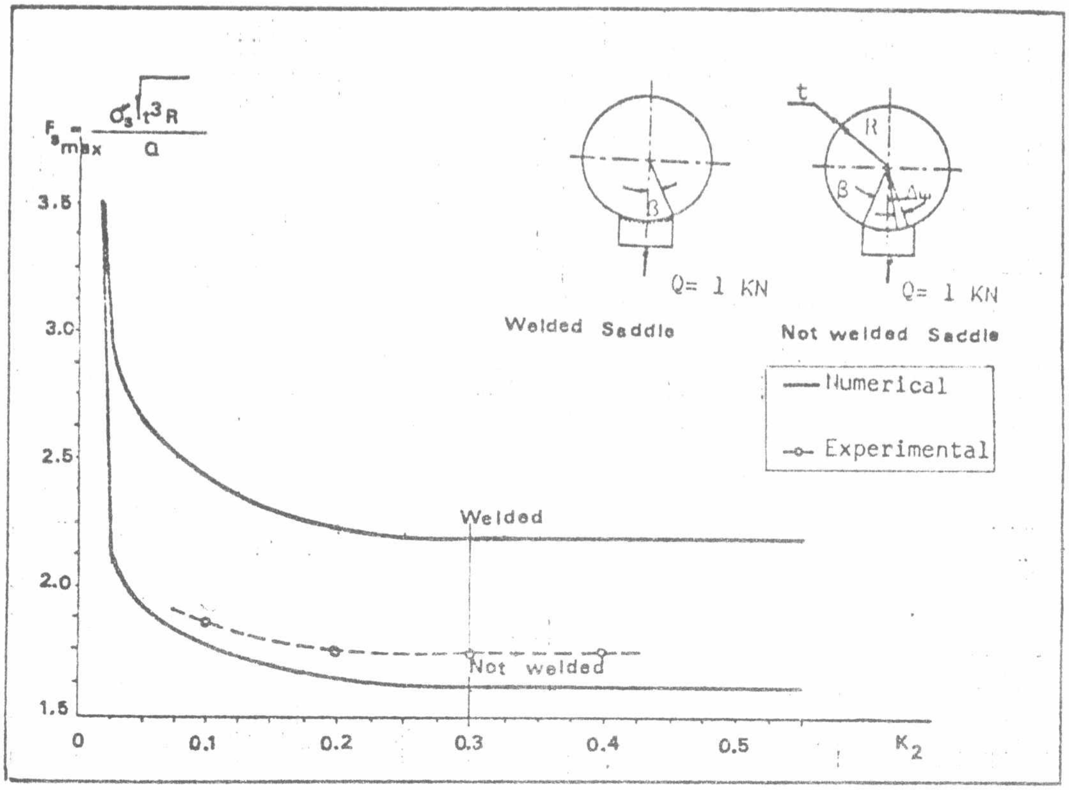

Fig.9 Variation of the max. stress coefficient at the saddle horn (ϕ =14°) $F_{smax.}$ with end factor k_2 for (β =16°, k_1=0.02372, δ = 0.036)

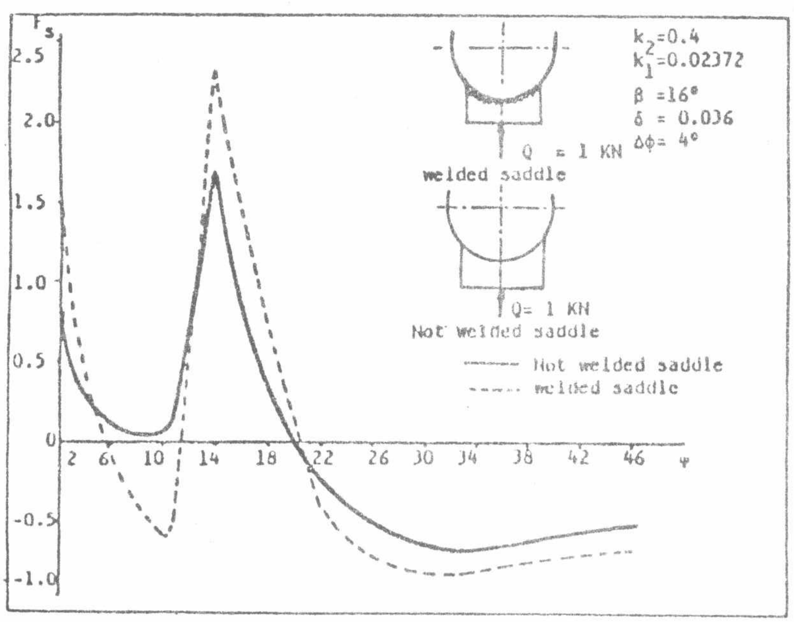

Fig. 10 Variation of circumferential stress coefficient F_s for welded and not welded saddle.

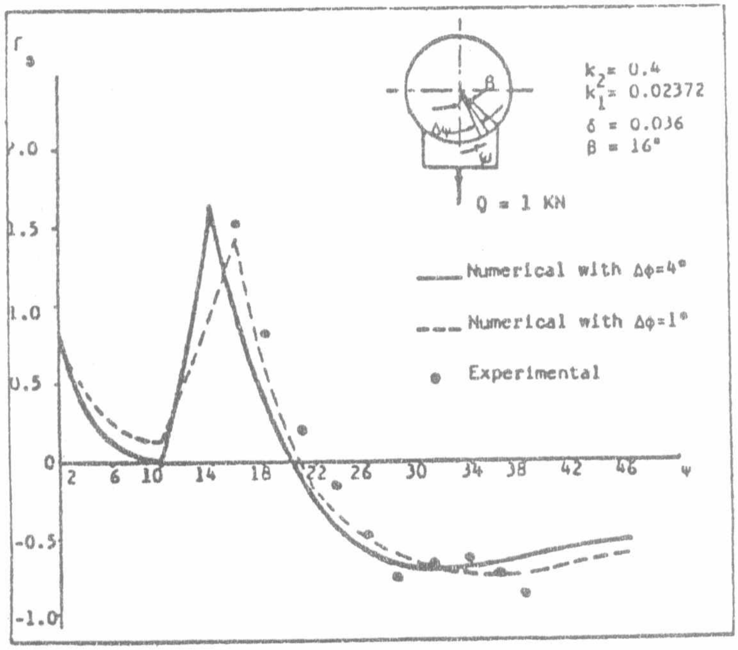

Fig. 11. Experimental and numerical stress coefficient F_s with ψ for not welded saddle.

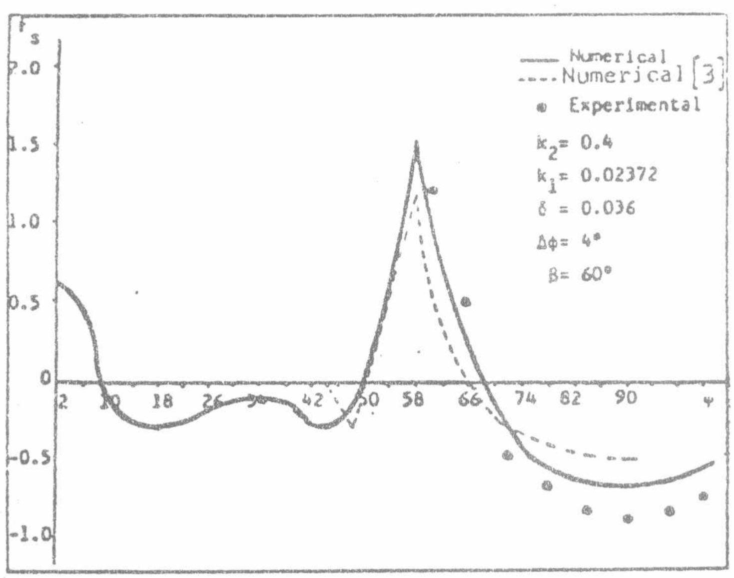

Fig. 12. Numerical and experimental stress coefficient F_s for not welded saddle and half saddle angle β=60°

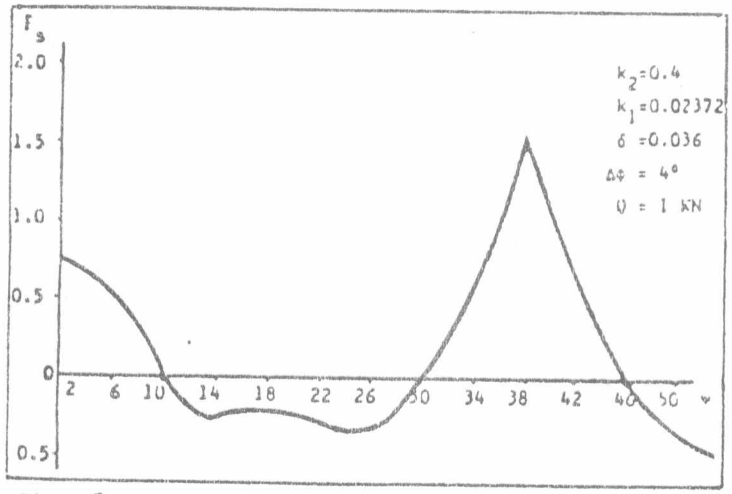

Fig. 13. Variation of stress coefficient F_s for half saddle angle β= 40° and not welded saddle.

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