A NEW METHOD OF DETERMINING THE STRESS INTENSITY FACTOR FROM ISOCHROMATIC FRINGE LOOPS.

A.A. Fattah*, K. Rajaiah** & M.S.C.Bose***

ABSTRACT

This paper describes a new four parameter method of analysis for determining of the stress intensity factor (SIF). The method bypasses the error-prone measurements on the isochromatic pattern near to the crack tip and uses data from the near of the crack tip as well as extended stress field. Suitable expression was developed for the determination of the fourth parameter based on isochromatic fringe loop information. Results were obtained by the computer program (EGYPT)for different values of the parameters β and α to give the best accuracy of determining the stress intensity factor (K). The results obtained for Dobeckote-505 show good agreement with the analytical results.

INTRODUCTION

One of the most effective methods of experimentally determining the stress intensity factor for a body containing a crack is to analyse the isochromatic pattern obtained from a photoelastic model.

Wells and Post [1] and Post [2] were the first to study the stress distribution for a static as well as for a dynamic crack. Irwin [3] in a discussion to Ref.[2] showed that the stress intensity factor could be determined from a single isochromatic fringe loop. Bradley and Kobayashi [4] and Schrodel and Smith [5] have modified Irwin's method. Bradley and Kobayashi introduced a differencing technique, involving measurements of r and θ on two different fringe loops, in an attempt to avoid the difficults associated with singular terms in Irwin's method. Schrodel and Smith pointed out that the isochromatic loops are easily observed along a line perpendicular to the crack tip at $\theta = \pi/2$, along which several data points are available, therefore, they simplify the

* Lecturar, Dept. of Design and Production Engg.,Faculty of Engineering Mansoura University, EGYPT, ** Professor, Dept. of Aeronautical Engg., I.I.T., Bombay, INDIA. *** Assistant Prof. Dept. of Mechanical Engg., I.I.T., Bombay, INDIA.

method by measuring r at $\theta = \pi/2$ and omitting one term in one of Irowin's equations. However, these methods are all based on a two-parameters (K_I; σ_{ox}) analysis. A critical review of these two parameter methods was written by Etheridge and Dally [6] , where these methods are compared and the errors are committed in the evaluation of the stress intensity factors are analyzed.

Etheridge and Dally [7] introduced a third parameter into the analysis by modiffying the Westergaard stress function to more closely account for stress field variations near the crack tip.

Etheridge, Dally and Kobayashi [8] improved upon the three parameter method, and presented a four parameter method for determining the dynamic stress intensity factor. This method of analysis involves four parameters which include crack length (a) as a fictitious parameter.

The method presented here is more easy and the use of the full-field (near field as well as extended stress field)data parmits a significant improvement in the accuracy of determining the stress intensity factors.

The method of analysis discussed herein involves four parameters which include: the stress intensity factor K, a normal stress σ_{ox}, a parameter β in a term added to the stress function, and a parameter F_p which introduced here to take care of far field and accurate determination of the stress intensity factor (K_I).

STATIC CRACK ANALYSIS

Following Irwin [3] and factoring out $K/\sqrt{2\pi Z}$ the stress field in the vicinity of the crack tip may be approximated by selecting Westergaard stress function.

$$Z_{(z)} = \frac{K}{\sqrt{2\pi z}} \left[1 + \beta(z/a)\right] \qquad (1)$$

the stress field can be obtained from the stress function Z by the relation

$$\sigma_x = R_e Z - y\, I_m Z' \qquad (2)$$

$$\sigma_y = R_e Z + y\, I_m Z' \qquad (3)$$

$$\tau_{xy} = -y\, R_e Z' \qquad (4)$$

from equation (1), it is evident that

$$R_e Z = \frac{K}{\sqrt{2\pi r}} \left[1 + \beta(r/a)\right] \cos\theta/2 \qquad (5)$$

$$I_m Z = \frac{-K}{\sqrt{2\pi r}} \left[1 - \beta(r/a)\right] \sin\theta/2 \qquad (6)$$

$$R_e Z' = \frac{K}{2\sqrt{2\pi r}} \left[-\frac{1}{r}\cos\frac{3\theta}{2} + \frac{\beta}{a}\cos\theta/2\right] \qquad (7)$$

$$I_m Z' = \frac{K}{2\sqrt{2\pi r}} \left[\frac{1}{r}\sin\frac{3\theta}{2} - \frac{\beta}{a}\sin\theta/2\right]$$

Substituting eq.(5, (7) and (8) into equs. (2-4) gives

$$\sigma_x = \frac{K}{\sqrt{2\pi r}} \left[\cos\frac{\theta}{2}(1 - \sin\theta/2 \sin 3\theta/2) + \cos\theta/2(1 + \sin^2\theta/2)\beta(r/a) + \alpha\sqrt{\frac{r}{a}}\right] \qquad (9)$$

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \left[\cos\frac{\theta}{2}(1 + \sin\theta/2 \sin 3\theta/2) + \cos\theta/2(1 - \sin^2\theta/2)\beta(r/a)\right] \qquad (10)$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}} \sin\theta/2 \,.\, \cos\theta/2 \left[\cos 3\theta/2 - \beta(r/a)\cos\theta/2\right] \qquad (11)$$

The maximum shear stress can be computed from

$$(2\tau_{max})^2 = (\sigma_y - \sigma_x)^2 + (2\tau_{xy})^2 \qquad (12)$$

The fundamental equation for the determination of the form of the isochromatic fringe loop follows upon substituting equ. (9)-(11) into eque. (12) and takes the form:

$$\tau^2_{max} = \frac{K_I^2}{8\pi r_m} \left[\sin^2\theta_m(1 - 2\beta(r_m/a)\cos\theta_m + \beta^2(r_m/a)^2 - 2\alpha\sqrt{r_m/a}\,\sin\theta_m(\sin 3\theta_m/2 - \beta(r_m/a)\sin\theta_m/2 + \alpha^2(r_m/a)\right] \qquad (13)$$

The stress optic law states that

$$\tau_{max} = \frac{Nf_\sigma}{2h} \qquad (14)$$

where: N is the isochromatic fringe order, f_σ is the material fringe values, h is the model thickness.

By equating equs. (13) and (14), one gets,

$$K_I = \left(\frac{Nf_\sigma}{h}\right)\sqrt{2\pi r_m}\left[\sin^2\theta_m(1 - 2\beta b_m\cos\theta_m + \beta^2 b_m^2) - 2\alpha\sqrt{b_m}\sin\theta_m(\sin 3\theta_m/2 - \beta b_m \sin\theta_m/2) + \alpha^2 b_m\right]^{-1/2} \qquad (15)$$

where $b_m = r_m/a$.

INTRODUCTION OF THE FOURTH PARAMETER

Let τ_{max} and τ^*_{max} represent respectively the maximum shear stress arising from approximate and exact solutions for infinite plates. Theocaris and Gdoutos [9] have shown that, the error in τ_{max} values can be corrected by estimating the errors in the τ_{max}/τ^*_{max} relationship and then using that information for correction. The introduction of the fourth parameter will follow on a similar basis for finite plates.

Let τ^*_{max} and K^*_I correspond the exact, maximum shear and SIF values respectively for finite plate. One can then write:

$$K^*_I = \sigma \sqrt{a \pi}\ Q \qquad (16)$$

where Q is an unknown constant to be determined later and

$$K_I = \sigma \sqrt{a \pi}$$

K_I being exact SIF value for infinite plates.

Substituting for τ_{max} and K_I by τ^*_{max} and K^*_I respectively in equation (13), one gets.

$$0 = \left(\frac{\sigma}{\sqrt{2\ b_m}}\right)^2 \left(\frac{b\ F_p}{N\ f_\sigma}\right)^2 \left[\sin^2 \theta_m (1-2\beta\, b_m \cos\theta_m + \beta^2 b_m^2) - 2\alpha\sqrt{b_m}\sin\theta_m(\sin 3\theta_m/2 - \beta\, b_m \sin\theta_m/2) + \alpha^2 b_m\right] - 1 \qquad (17)$$

where $F_q = \tau_{max}/\tau^*_{max}$

$F_p = Q\ F_q$

$F_p = 1$, For infinite plate solution.

$F_p \neq 1$, For finite plate solution.

COMPUTATION OF ACCURATE VALUES OF β AND α

Using the data collected from two neighbouring fringe loops Fig. 1 arround the crack tip (N_1, r_{m1}, θ_{m1} and N_2, r_{m2}, θ_{m2}, two equations can be written from equation (17)

$$0 = \left(\frac{\sigma}{\sqrt{2\ b_{m1}}}\right)^2 \left(\frac{h\ F_p}{N_1\ f_\sigma}\right)^2 \left[\sin^2 \theta_{m1} (1-2\beta\, b_{m1} \cos\theta_{m1} + \beta^2 b_{m1}^2) - 2\alpha\sqrt{b_{m1}}\ \sin\theta_{m1}\left(\sin\frac{3\theta_{m1}}{2} - \beta\, b_{m1} \sin\frac{\theta_{m1}}{2}\right) + \alpha^2 b_{m1}\right] - 1 \qquad (18)$$

6

$$0 = \left(\frac{\sigma}{\sqrt{2 b_{m2}}}\right)^2 \left(\frac{h F_p}{N_2 f_\sigma}\right)^2 \left[\sin^2\theta_{m2}\,(1-2\beta b_{m2}\cos\theta_{m2}+\beta^2 b_{m2}^2) - 2\alpha\sqrt{b_{m2}}\,\sin\theta_{m2}\left(\sin\frac{3\theta_{m2}}{2} - \beta b_{m2}\sin\frac{\theta_{m2}}{2}\right) + \alpha^2 b_{m2}\right] - 1 \qquad (19)$$

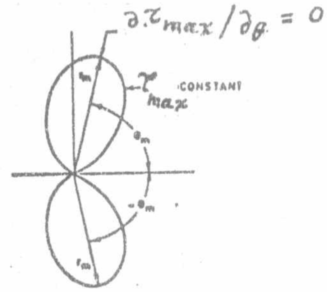

Fig. 1. Characteristic geometry of a pair of isochromatic fringe loops at the crack tip.

In the previous section, a technique for the determination of β and α was given, with the assumption that F_p will be known at that stage.

Here, the evaluation of F_p is introduced in form of general expression for center crack, single edge and double edge crack mode-I and mixed mode problems in plates.

$$F_p = C + \frac{(2\, r_m/a)\left[\cos\theta_m\right]^{(3\, r_m/a)}\, \lambda\, D\, f(N)\, \cos^6\phi}{\left[\sin\frac{\theta_m}{2}\,.\,\cos\frac{\theta_m}{2}\quad\cos\frac{3\theta_m}{2}\right]} \qquad (20)$$

where

$$f(N) = \left(\frac{N_2}{N_1}\right)^{2\left[1-\left(\frac{N_1}{N_2}\right)\right]} \times \left(\frac{N_3}{N_2}\right)^{2\left[1-\left(\frac{N_2}{N_3}\right)\right]} \ldots\ldots\ldots\ldots\ldots \left(\frac{N_{n+1}}{N_n}\right)^{2\left[1-\left(\frac{N_n}{N_{n+1}}\right)\right]}$$

$\left[\cos\theta_m\right]$ = modulus of $\cos\theta_m$

λ = 2a/W (2a crack length, W width of the plate)
D = 1, C = 1, for center cracked plated
D = 0.5, C = 10 for single edge cracks
D = 2, C = 2, for double edge cracks
F_p = 1, for an infinite plate with a central crack.

The computer program (EGYPT) can be used for the determination of β & α from equs. (18) and (19) together, and thus, one can gets accurate values of β and α.
Chart for the computer program presented in Fig. 2.

Using the accurate values of β and α, the stress intensity factor K_I can be precisely determined from equation (15).

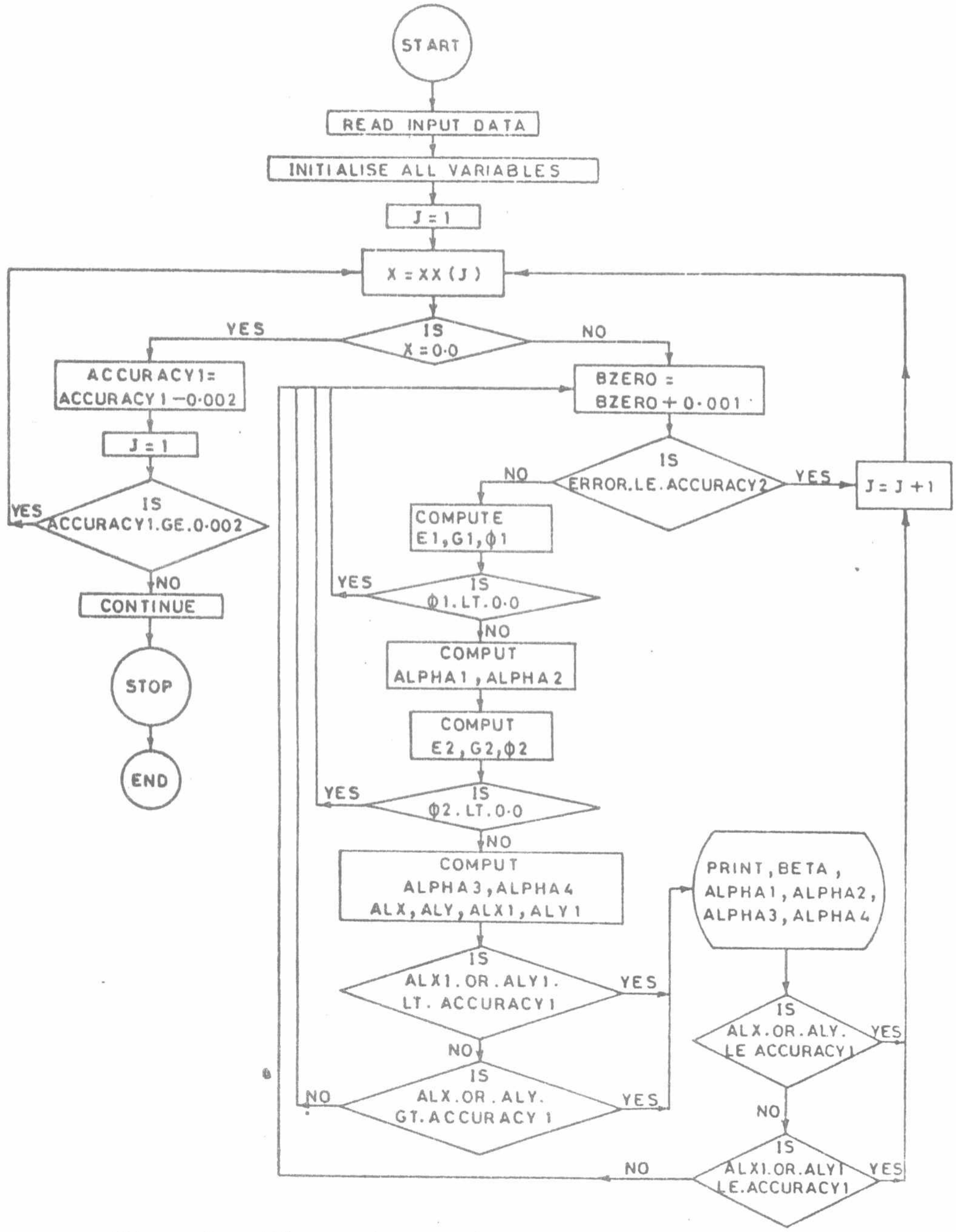

Fig. 2. Flow chart for the (EGYPT) Program.

EXPERIMENTAL PROCEDURE

In order to apply the above described new method for the determination of (SIF), experimental work was carried out on finite plate made of Dobeckot -505, having (CCT) central crack, subjected to uniform axial tensile stress using transmission technique. The mechanical and optical properties of the material used are summarized in Table 1.

Table 1. Mechanical and Optical Properties of Dobeckot-505

Material	f_σ Kg/cm^2.F.O	E Kg/cm^2 x 10^3	σ_{yp} Kg/cm^2	ϵ_y x 10^{-3}
Dobeckot-505	11.884	24.887	418.8	16.8

Ten tests were run for different crack length to the width ratio λ (=2a/w) varied in the range of 0.1 to 0.7. The uniform applied stress σ was maintained constant for all λ values. The isochromatic patterns shown in Fig. 3, was further enlarged so that the magnification of the order of 30 times was achieved from the real size.

Measurements of N, r_m and θ_m for two different neighbouring fringe loops were executed and recorded in the region $0.02 < (r_m/a) < 0.5$.

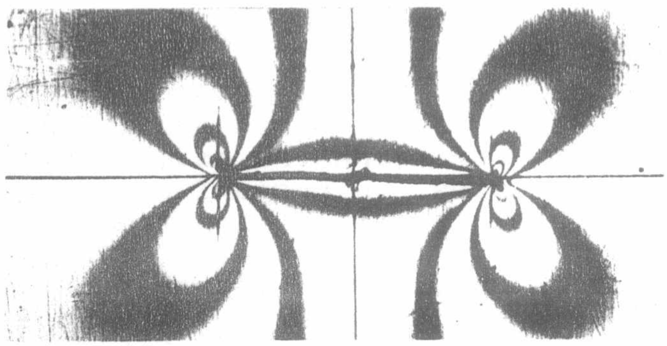

Fig. 3. Isochromatic fringe loops for a finite plate (λ = 0.5) having central crack.

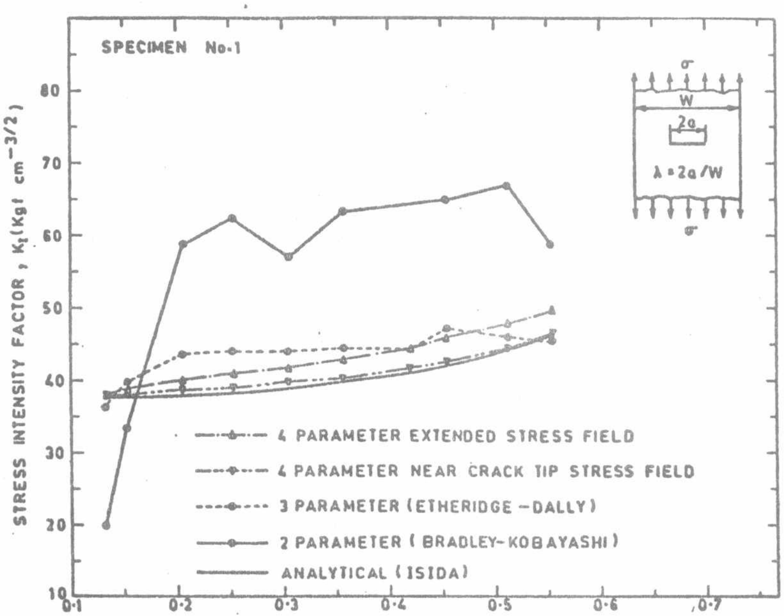

Fig. 4. Comparison of SIF values from Two, Three and Four Parameter Methods with Analytical Results.

RESULTS AND DISCUSSION

Fig. 4 show a comparison between the SIF values determined using two parameter (Bradley and Kobayashi), three parameter (Etheridge and Dally) and four parameter (developed in this work) methods with the analytical results for various values of (λ =2a/W), applied to finite plate with CCT crack problem.

Near field as well as extended stress field (0.02 r_m/a 0.5) data is used to determine SIF values by the four parameter method and the two results are compared. Fig. 4 show that four parameter method gives very good agreement with the analytical results in the range of λ (=2a/W) varying from 0.1 to 0.6. Errors in SIF values by two and three parameter methods when compared with the analytical results are in the range of 3.2 of 45.9 percent, while the error in SIF values using four parameter method, is in the range of -0.08 to 3.6 percent only.

CONCLUSIONS

A new four parameter method of analysis was presented and used to accurately determine the mode-I (K_I) stress intensity factor in CCT plates. A comparison of the values of SIF calculated from near the crack tip stress field data using Bradly and Kobayashi's two parameter, Etheridge and Dally's three parameter and four parameter methods with the analtical results have clearly shown that the superiority of the present four parameter method. The SIF results are found to be reasonable with the present method even when extended stress field data ($0.02 < r_m/a < 0.5$) is used.

NOTE

A further series of experiments were also carried out for finite plates having SEN and DEN for mode-I as well as mixed mode crack problems on finite plates as well as in cylindrical shells. The results to be published successively.

REFERENCES

1. Wells, A.A. and Post, D. "The dynamic stress distribution surrounding a runing crack-A photoelastic analysis" Proc. SESA, 16, 69-92 (1958).
2. Post, D. "Potoelastic analysis for an edge crack in tensile field" Proc. SESA, 12, 99-116 (1954).
3. Irwin, G.R. "Discussion of Ref. 1" Proc. SESA, 16, 93-96 (1957).
4. Bradley, W.B. and Kobayashi, A.S. "An investigation of propagation on crack by dynamic photoelasticity" Exp. Mech., 10, 106-113 (1970).
5. Schroedle, M.A. and Smith, C.W. "Local stress near deep surface flaws under cylindrical bending fields" Progress in flaw growth and fracture toughness

6

testing, ASTM, STP, 536, 45-63 (1973).

6. Etheridge, J.M. and Dally, J.M. "A critical review of methods for determining stress intensity factors from isochromatic fringes" Exp. Mech., 17, 248-254 (1977).
7. Etheridge, J.M. and Dally, J.M. "A three-parameter method for determining stress intensity factors from isochromatic fringe loops" J. Strain, Analysis, 13, 91-94 (1979).
8. Etheridge, J.M., Dally, J.W. and Kobayashi, T. "A new method of determining the stress intensity factor K from isochromatic fringe loops" Engg. Fract. Mech., 10, 81-93 (1978).
9. Theocaris, P.S. and Gdoutos, E.E. "A photoelastic determination of K_I stress intensity factor" Engg. Fract. Mech., 7, 331-339 (1975).