MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

A NEW METHOD OF DETERMINING THE MIXED-MODE STRESS INTENSITY FACTORS FROM ISOCHROMATIC FRINGE LOOPS

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ABSTRACT

The presence of cracks in any structure or structural component may result in the total failur of the structure. The stress intensity factor (SIF) provide a single parameter representation for the stress condition at the crack tip.

This paper descripes a new evaluation method of the isochromatic fringe pattern for the determination of mixed mode stress intensity factor. The method bypasses the error-prone measurements on the isochromatic pattern near to the crack tip and uses data from the near of the crack tip as well as extended stress field. The four parameter method of determining the opening mode stress itensity factor (K_I), developed by the authors [1] was extended to incorporate the more general case of an arbitrarily oriented crack problem, wher the stress field is governed by two mixed mode factors K_I and K_{II}.

The method presented here is easy and the use of the full-field data parmits a significant improvement in the accuracy of determining the mixed mode stress intensity factors. Experimental evidence corroborated the results obtained by the new mehtod.

INTRODUCTION

By employing fracture mechanics concepts it is possible to characterize the stress singularity at the crack tip by means of stress intensity factors K_I and K_{II}. These stress intensity factors are associated with a opening mode K_I and shearing mode K_{II}.

Most of the photoelastic studies deal with mode-I stress intensity factor, and except for a few isolated studies and little attention has been paid to mixed mode stress intensity factors K_I and K_{II}.

Cheng [2] in a brief note determined the mode-I and mode-II SIFs by making photoelastic measurements on sixteen points near the crack tip. Based on the idea of Irwin, an approach was developed by Smith and Smith [3] for determination of mixed mode K_I and K_{II}. Gdoutos and Theocaris [4] have developed a method for the determination of mixed mode stress intensity factors and extrapolation laws were given for transfeering the far from crack

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tip data to the near crack tip region. Sanford and Dally [5] using computer generated theoretical fringe loops and computed numerical values for stress intensity factors K_I and K_{II}. Rossmanith [6] developed a general relationship between N, r, and ϕ and the general stress field expressed in terms of several parameters. These parameters are varied and isochromatic crack tip fringe loops are constructed for a set of clasified combination for infinite plates.

STATIC CRACK ANALYSIS

The stress field in an infinite plate with a central crack of length 2a subjected to axial tension σ may be approximated by selecting Westergaard stress functions of the form

$$Z' = \frac{K_j}{\sqrt{2\pi z}} \cdot \frac{1}{2} \left[-\frac{1}{z} + (\beta/a) \right] \qquad (j=1,2) \qquad (1)$$

where the subscript (j) refer to the diferent crack opening modes.

The real and imaginary parts of the Westergaard stress function are

$$R_e\, Z_j = \frac{K_j}{\sqrt{2\pi r}} \cos \phi/2 \; (1 + \beta(r/a))$$

$$I_m\, Z_j = \frac{K_j}{\sqrt{2\pi r}} \sin \phi/2 \; (1 - \beta(r/a))$$

$$R_e\, Z_j' = \frac{K_j}{\sqrt{2\pi r}} \frac{1}{2r} (-\cos 3\phi/2 + \beta(r/a) \cos \phi/2)$$

$$I_m\, Z_j' = \frac{K_j}{\sqrt{2\pi r}} \frac{1}{2r} (\sin 3\phi/2 - \beta(r/a) \sin \phi/2) \qquad (2)$$

Upon substituting expressions (2) into the mixed mode stress components σ_x, σ_y and τ_{xy} the stress field in the neighbourhood of the crack tip is given as

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \Big[\cos \phi/2 \; (1 + \beta(r/a)) - \frac{1}{2} \sin \phi \; (\sin 3\phi/2$$

$$- \beta(r/a) \sin \phi/2) \Big] - \frac{K_{II}}{\sqrt{2\pi r}} \Big[2 \sin \phi/2 \; (1 - \beta(r/a))$$

$$+ \frac{1}{2} \sin \phi \; (\cos 3\phi/2 - \beta(r/a) \cos \phi/2) \Big] + \sigma_{ox} \qquad (3)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \Big[\cos \phi/2 \; (1 + \beta(r/a)) + \frac{1}{2} \sin \phi \; (\sin 3\phi/2$$

$$- \beta(r/a) \sin \phi/2 \,)\Big] - \frac{K_{II}}{\sqrt{2\pi r}} \Big[\frac{1}{2} \sin \phi \,(\cos 3\phi/2$$

$$- \beta(r/a) \cos \phi/2 \,)\Big] \qquad (4)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \Big[\frac{1}{2} \sin \phi \,(\cos 3\phi/2 - \beta(r/a) \cos \phi/2 \Big]$$

$$+ \frac{K_{II}}{\sqrt{2\pi r}} \Big[\cos \phi/2 \,(1 + \beta(r/a)) - \frac{1}{2} \sin \phi \,(\sin 3\phi/2$$

$$- \beta(r/a) \sin \phi/2 \Big] \qquad (5)$$

The maximum in-plane shear stress τ_{max} is related to cartesian components of stress by

$$2\tau_{max} = (\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2 \qquad (6)$$

The photoelastic method of stress analysis enables the direct isochromatic fringe pattern determination of the difference of the principal stresses $(\sigma_1 - \sigma_2)$ given by

$$(\sigma_1 - \sigma_2) = 2\tau_{max} = \frac{N f_\sigma}{h} \qquad (7)$$

where f_σ is the material fringe value, h is the thicness of the plate and N is the order of the isochromatic fringe.

The analytical expression for the mixed mode isochromatic in the vicinity of the crack tip is obtained by substituting Eqs.(3-5). into Eq.(6) and takes the form

$$(2\tau_{max})^2 = \Big(\frac{K_I}{\sqrt{2\pi r}}\Big)^2 \Big\{ \sin \phi \,(\sin 3\phi/2 - \beta(r/a) \sin \phi/2)$$

$$+ \Big(\frac{K_{II}}{K_I}\Big) \Big[\sin \phi \,(\cos 3\phi/2 - \beta(r/a) \cos \phi/2 \,) + 2 \sin \phi/2$$

$$(1 - \beta(r/a))\Big] - \alpha\sqrt{r/a} \Big\}^2 + \Big(\frac{K_I}{\sqrt{2\pi r}}\Big)^2 \Big\{ \sin \phi \,(\cos 3\phi/2$$

$$- \beta(r/a) \cos \phi/2 \,) + \Big(\frac{K_{II}}{K_I}\Big) \Big[2 \cos \phi/2 \,(1 + \beta(r/a))$$

$$- \sin \phi \,(\sin 3\phi/2 - \beta(r/a) \sin \phi/2 \,)\Big]\Big\}^2 \qquad (8)$$

substituting Eq.(7) into Eq.(8) one gets

$$K_I = \left(\frac{N f_\sigma}{h}\right)\sqrt{2\pi r_m}\ \Big[\Big\{\sin\phi_m \sin 3\phi_m/2 - \beta(r_m/a)\sin\phi_m/2 + M\Big[\sin\phi_m(\cos 3\phi_m/2 - \beta(r_m/a)\cos\phi_m/2) + 2\sin\phi_m/2\,(1-\beta(r_m/a)) - \alpha\sqrt{r_m/a}\Big]^2 + \sin\phi_m(\cos 3\phi_m/2 - \beta(r_m/a)\cos\phi_m/2) + M\Big[2\cos\phi_m/2\,(1+\beta(r_m/a)) - \sin\phi_m(\sin 3\phi_m/2 - \beta(r_m/a)\sin\phi_m/2)\Big]\Big\}^2\Big]^{-1/2} \qquad (9)$$

where $M = K_{II}/K_I$

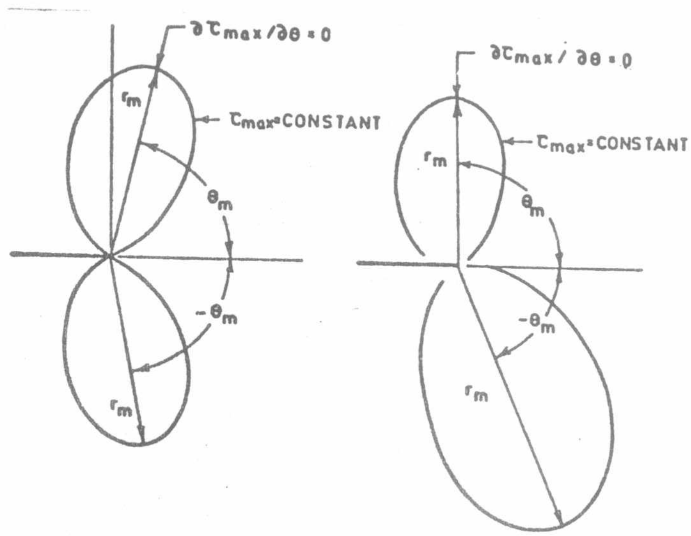
For solving Eq.(9), it is necessary to introduce a new equation for (K_{II}/K_I) relation for crack problems in finite plates to take care of finite dimension (2a/W) as well as crack tilt angle Φ. An expression is derived based on the analytical solution andthe graphical plots given by Wilson [7] as follows

$$(K_{II}/K_I)\tan\Phi = 1 - 0.146(\lambda\cos\Phi/2) - 0.169(\lambda\cos\Phi/2)^2 - 0.07(\lambda\cos\Phi/2)^3 \qquad (10)$$

Following Fattah [1] for the evaluation of β and α values, the following equation can be written for data collected from two neighbouring isochromatic fringe loops around the crack tip stress field as follows

$$0 = (1/2b_{mi})\left(\frac{\sigma h F_p}{N_i f_\sigma}\right)^2 \Big[\Big\{\sin\phi_{mi}(\sin 3\phi_{mi}/2 - \beta b_{mi}\sin\phi_{mi}/2) + M\Big[\sin\phi_{mi}(\cos 3\phi_{mi}/2 - \beta b_{mi}\cos\phi_{mi}/2) + 2\sin\phi_{mi}/2\,(1-\beta b_{mi})\Big] - \alpha\sqrt{b_{mi}}\Big\}^2 + \Big\{\sin\phi_{mi}(\cos 3\phi_{mi}/2 - \beta b_{mi}\cos\phi_{mi}/2) + M\Big[2\cos\phi_{mi}/2\,(1+\beta b_{mi}) - \sin\phi_{mi}(\sin 3\phi_{mi}/2 - \beta b_{mi}\sin\phi_{mi}/2)\Big]\Big\}^2\Big] - 1 \qquad (11)$$

where the subscript (i) refers to the order of isochromatic fringe, F_p is the fourth parameter introduced in this work, and $b_m = r_m/a$.



(A) Opening mode (B) Mixed mode

Fig.1. Isochromatic fringe loops at a crack tip

The fourth parameter F_p can be computed for the mixed mode crack problems as follows

$$F_p = 1 + \frac{(2r_m/a)\left[\cos \phi\right]^{3r_m/a} \lambda \; f(n) \; \cos^6 \Phi}{\left[\sin \phi_m/2 \; \cos \phi_m/2 \; \cos 3\phi_m/2\right]} \qquad (12)$$

where $f(n) = (N_2/N_1)^{2(1-N_1/N_2)} \, (N_3/N_2)^{2(1-N_2/N_3)} \cdots\cdots$

$$\cdots\cdots (N_{n+1}/N_n)^{2(1-N_n/N_{n+1})}$$

N_n and N_{n+1} represent the fringe order of the N^{th} and $(N_{n+1})^{th}$ fringe loop and $N_{n+1} > N_n$.

Φ is the crack tilt angle and λ is the relative crack length (crack length to the plate width ratio, 2a/W).

Thus, the photoelastic determination of mixed mode SIF's K_I and K_{II} by the method developed in this investigation, consists of the following steps.

a) Measurements of the isochromatic coordinate fringe loop parameters (r_{m1} , ϕ_{m1}) and (r_{m2} , ϕ_{m2}) for any two neighbouring fringe loops Fig.1.

b) Use of computer program (EGYPT-II) Fig.2. with the system of two equations, geting from Eq.11. by using data information from two neighbouring isochromatic fringe loops available around the crack tip,

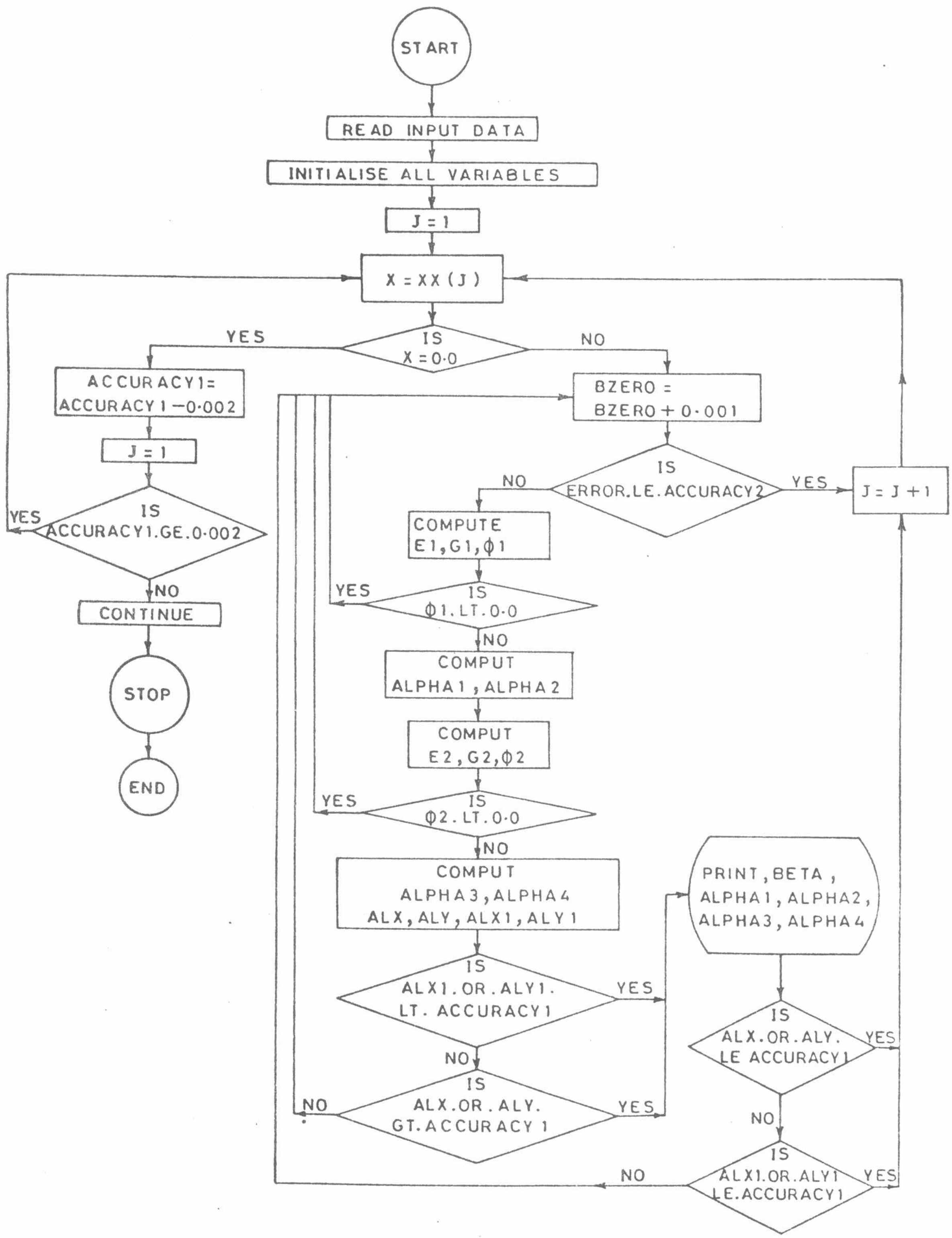


Fig.2. Flow chart for the (EGYPT-II) program

applying Eq.10. of (K_{II}/ K_I) and determining the fourth parameter F_p Eq.12. and evaluation of β and α accurately.

c) Determination of K_I and K_{II} accurately by substituting the values β and α, and $M = (K_{II}/ K_I)$ into Eq.9.

EXPERIMENTAL PROCEDURE

The above described method for the photoelastic determination of mixed mode stress intensity factors, by using data information from near the crack tip as well as extended stress field, was applied to a simple examle in order to be tested.

A Homolite-100 rectangular specimen having a centrally located inclined crack with a tilt angle Φ =30 deg. and λ = 0.143, was subjected to a uniform tensile stress. Four tests were run for the same λ value by changing the load levels. Isochromatic fringe patterns corresponding to the different values of applied stress were photographed Fig.3. shows a typical fringe loops . The r_m, ϕ_m values for two neighbouring fringe loops were measured and recorded after suitable enlargement.

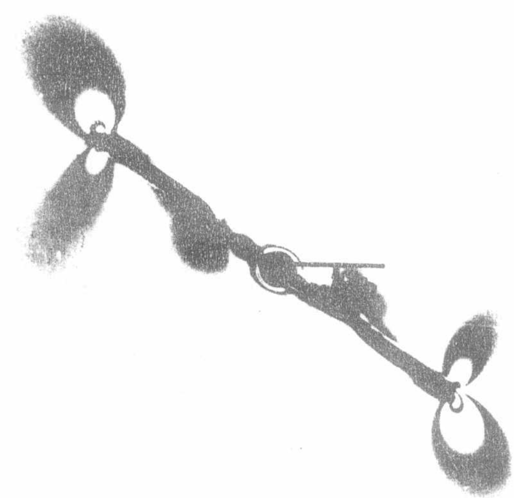


Fig.3. Isochromatic fringe loops for a finite plate (λ=0.143) having central crack of iclination Φ=30 deg.

RESULTS AND DISCUSSION

Comparison of the results of mixed mode SIF's from two parameter Smith and Smith [3] method and the new four prameter method with the analytical results of K_I and K_{II} SIF values are tabulated in Table.1.

Examination of the results indicate that, by using Smith and Smith approach and their equation for (K_{II}/ K_I) relation, the values of K_{II} are highly inaccurate.

To overcome this problem, Eq.10 for (K_{II}/ K_I) reltion developed herein has been used along with the four parameter method for solving mixed mode in finite plates. The K_I and K_{II} SIF's values are both in closed agreement with the analytical results.

Table.1. SIF values and error using Smith and Smith and Four parameter methods, for a specimen having inclined crack.

So. No.		Smith and Smith method				Four parameter mehtod				Analytical	
	kg/cm^2	$kg\ cm^{-3/2}$		Error %		$kg\ cm^{-3/2}$		Error %		$kg\ cm^{-3/2}$	
	σ	K_I	K_{II}	K_I	K_{II}	K_I	K_{II}	K_I	K_{II}	K_I	K_{II}
1	20	53.2	37.6	71	106	33.5	19.7	8	8	30.9	18.2
2	30	49.2	38.5	6	41	48.4	28.5	4	4	46.5	27.4
3	35	60.9	46.3	12	45	58.1	34.2	7	7	54.2	31.9
4	40	70.6	55.7	14	53	66.2	38.9	7	7	62.0	36.6

CONCLUSIONS

A new four prameter method of analysis was presented for the accurate determination of mixed mode SIF's, in finite CCT, plates. A new equation for (K_{II}/K_I) relationship has been developed which has to be used in conjunction with the four parameter method. The method permits the use of fringe loop information from near the crack tip ($0.02 < r_m/a < 0.2$) as well as the extended stress field ($0.02 < r_m/a < 0.5$). A comparison of the values of K_I and K_{II} using the present four parameter method of analysis with the analytical results have clearly shown good agreement. This indicates the accuracy of the method.

REFERENCES

1. Fattah, A.A., Rajaiah, K. and Bose, M.S.C.'' A new method of determining the stress intensity factor from isochromatic fringe loops'',Proc. A.M.E., 1, 1-9 (1984).
2. Cheng, Y.F. '' Simultaneous determination of th first and second mode stress intensity factors'',Int. J Fracture Mech., 7, 119-121 (1971).
3. Smith, D.G. and Smith, C.W. '' Photoelastic determination of mixed mode stress intensity factors'', Engg. Fract. Mech., 4, 337- 366 (1972).
4. Gdoutos, E.E. and Theocaris, P.S. '' A photoelastic determination of mixed mode stress intensity factors'', Exp. Mech., 87-96 (1978).
5. Sanford, R.T. and Dally, J.W. '' A general method for determining mixed mode stress intensity factors from isochromatic fringe pattern'',Engg. Fract. Mech., 11, 621-633 (1979).
6. Rossmanith, H.P. '' Analysis of mixed-mode isochromatic crack tip fringe patterns'', Ac ta Mechanica, 34, 1-38 (1979).
7. Wilson, W.K. '' Numerical method for determining stress intensity factors of an interior crack in finite plates'', J. Basic Engineering, 12, 685-690 (1971).