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THEORETICAL INVESTIGATION OF TRANSMISSION LOSS FOR DIFFERENT IN-PARALLEL TUBING CONFIGURATIONS

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ABSTRACT

A generalized formula predicting the attenuation of a main pipe branched into two in-parallel pipes joined together at their inlet and outlet in terms of the system dimensions has been investigated. The obtained formula reduces to that of known shapes and configurations through simple adjustments in the model. Examples of the expansion chamber and the side branch transmission loss equations have been achieved. A computer model for the system has been developed and proven applicable.

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INTRODUCTION:

When a main pipe is branched into two in-parallel pipes joined together at their inlet and outlet, those pipes become acoustically coupled causing attenuation characteristics that depend mainly on the configuration dimensions. In order to account for wave motion in the axial direction of such parallel coupled system, it is necessary to relate the acoustic pressure and volume velocities at the inlet and outlet of the main pipe. Configurations of this type are of major intrest for applications in the design and analysis of mufflers and exhaust systems of internal combustion engines.

THE MODEL:

The configuration of the model is shown in Fig.(1). The continuity of acoustic pressure and volume velocity at the inlet and outlet cross sections-shown in Fig.(2) is applied using the four-pole-parameters technique[2]. Each transmission matrix formed by four parameters corresponds to a pipe branch, while the overall transmission matrix is formed by the four pole parameters representing the whole system. This is demonstrated in Fig.(3). The model is adopted considering that the folowing conditions hold:

(1) Plane wave propagation.

(2) Conservative system (no losses).
   (a) Viscosity effects are neglected.
   (b) The pipes walls neither conduct nor transmit sound energy.

(3) The propagation of sound waves is one-dimensional in the axial direction of the pipes. The cross sectional dimensions are assumed to be small compared to the wave length. Limiting frequency values are to be calculated for practical applications. Analysis must be restricted by an upper frequency bound above which the effect of the neglected higher order modes of propagation should be accounted for.

It has been successfully proven by different authors[6,7,8] that a complex in-series system can be modeled as a four-pole net work that as long as the pipe system fulfills the assumption of linear plane wave theory. Using this theory the output pressure and volume velocity are dependent on the input pressure and volume velocity and constants to be obtained by experiment or theory. For branches (a) and (b) of Fig.(3), the acoustic pressure and volume velocities are related as follows:

$$P_{1a} = A_1 P_{2a} + A_2 U_{2a} \quad \text{............... (1)}$$

$$U_{1a} = A_3 P_{2a} + A_4 U_{2a} \quad \text{.............. (2)}$$

$$P_{1b} = B_1 P_{2b} + B_2 U_{2b} \quad \text{.............. (3)}$$

$$U_{1b} = B_3 P_{2b} + B_4 U_{2b} \quad \text{............. (4)}$$

It has been shown by reciprocity theorem that:

$$A_1 A_4 - A_2 A_3 = 1$$

$$B_1 B_4 - B_2 B_3 = 1$$

For the main pipe, this relation could be written as:

$$P_1 = C_1 P_2 + C_2 U_2 \quad \text{............. (5)}$$

$$U_1 = C_3 P_2 + C_4 U_2 \quad \text{............. (6)}$$

where (C_1, C_2, C_3, C_4) are the four-pole-parameters of the overal transmission matrix of the main pipe. They are to be expressed in terms of (A_1, A_2, A_3, A_4) and (B_1, B_2, B_3,B_4) subject to the following boundary conditions:

$$P_1 = P_{1a} = P_{1b} \quad \text{............. (7)}$$

$$P_2 = P_{2a} = P_{2b} \quad \text{.............. (8)}$$

$$U_1 = U_{1a} + U_{1b} \quad \text{............. (9)}$$

$$U_2 = U_{2a} + U_{2b} \qquad \text{.......... (10)}$$

The four pole parameters (C_1, C_2, C_3, C_4) appearing in equations (5) and (6) expressed in terms of (A_1, A_2, A_3, A_4) and (B_1, B_2, B_3, B_4) are*

$$C_1 = \frac{A_1 B_2 + A_2 B_1}{A_2 + B_2} \qquad \text{.......... (11)}$$

$$C_2 = \frac{A_2 B_2}{A_2 + B_2} \qquad \text{.......... (12)}$$

$$C_3 = \frac{A_2 B_3 + A_3 B_2 + A_1 B_4 + A_4 B_1 - 2}{A_2 + B_2} \qquad \text{.......... (13)}$$

$$C_4 = \frac{A_2 B_4 + A_4 B_2}{A_2 + B_2} \qquad \text{.......... (14)}$$

Those parameters satisfy the condition:

$$C_1 C_4 - C_2 C_3 = 1$$

Transmission loss calculations:

By definition , the transmissioin loss in decibels due to the combination of acoustic element between the two sections is:

$$TL = 20 \log_{10} \left[\frac{\text{Average incident sound power}}{\text{Average transmitted sound power}} \right]^{1/2}$$

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* See appendix for detailed calculations.

It is understood that the transmission loss could also be found by taking the ratio of pressure levels at both ends of the main pipe:

$$TL = 20 \, Log_{10} \left[\frac{P_2}{P_1} \right]$$

In terms of the four-pole-parameters (C_1, C_2, C_3, C_4) this transaction loss is given by[2]:

$$TL = 20 \, log_{10} \frac{1}{2} \left| (C_1 + \frac{S}{\rho c} \; C_2 + \frac{\rho c}{s} \; C_3 + C_4) \right| \quad \ldots\ldots\ldots\ldots\ldots (15)$$

using the drived values of the parameters; the transmission loss equation becomes:

$$TL = 20 \, Log_{10} \left[\frac{(A_1B_2+A_2B_1) + \frac{S}{\rho C}(A_2B_2) + \frac{\rho C}{S}(A_2B_3+A_3B_2+A_1B_4+A_4B_1-2) + (A_2B_4+A_4B_2)}{2(A_2 + B_2)} \right] \quad \ldots\ldots\ldots\ldots (16)$$

For the case under study where the two branches are simple pipes of cross sectional areas (s_a), (S_b), and lengths (ℓ_a), (ℓ_b)

the four-pole parameters of the branches are:

$$A_1 = \cos k\ell_a \qquad B_1 = \cos k\ell_b$$

$$A_2 = i \frac{\rho c}{S_a} \sin k\ell_a \qquad B_2 = i \frac{\rho c}{S_b} \sin k\ell_b$$

$$A_3 = i \frac{S_a}{\rho c} \sin k\ell_a \qquad B_3 = i \frac{S_b}{\rho c} \sin k\ell_b$$

$$A_4 = Cos \, k\ell_a \qquad B_4 = Cos \, k\ell_b$$

The corresponding four-pole-parameters forming the overall transmisson matrix are:

$$C_1 = \frac{S_a Cos \, k\ell_a Sin \, k\ell_b + S_b Sin \, k\ell_a Cos \, k\ell_b}{S_b Sin \, k\ell_a + S_a Sin \, k\ell_b} \quad \ldots\ldots\ldots (17)$$

$$C_2 = (j\rho c)\frac{\mathrm{Sin}\,k\ell_a \,\mathrm{Sin}\,k\ell_b}{S_b \mathrm{Sin}\,k\ell_a + S_a \mathrm{Sin}\,k\ell_b} \quad \ldots\ldots\ldots\ldots (18)$$

$$C_3 = \left(\frac{i}{\rho c}\right)\frac{(S_a^2 + S_b^2)\mathrm{Sin}\,k\ell_a \mathrm{Sin}\,k\ell_b + 2S_aS_b - 2S_aS_b \mathrm{Cos}\,k\ell_a \mathrm{Cos}\,k\ell_b}{S_b \mathrm{Sin}\,k\ell_a + S_a \mathrm{Sin}\,k\ell_b} \quad (19)$$

$$C_4 = \frac{S_a \,\mathrm{Cos}\,k\ell_a \mathrm{Sin}\,k\ell_b + S_b \mathrm{Sin}\,k\ell_a \mathrm{Cos}\,k\ell_b}{S_b \mathrm{Sin}k\ell_a + S_a \mathrm{Sin}\,k\ell_b} \quad \ldots\ldots\ldots (20)$$

And the general formula of transmission loss for such ocnfiguration becomes:

$$T.L = 10\log_{10}\left\{\left[(S_a \mathrm{Cos}\,k\ell_a \mathrm{Sin}\,k\ell_b + S_b \mathrm{Sin}\,k\ell_a \mathrm{Cos}\,k\ell_b)^2 + \left(\frac{S_a^2 + S_b^2 + S^2}{2S}\,\mathrm{Sin}\,k\ell_a \mathrm{Sin}\,k\ell_b + \left(\frac{S_aS_b}{S}\right)(1 - \mathrm{Cos}\,k\ell_a \mathrm{Cos}\,k\ell_b)\right)^2\right] / (S_a \mathrm{Sin}\,k\ell_a + S_b \mathrm{Sin}\,k\ell_a)\right\} \quad \ldots\ldots\ldots (21)$$

INVESTIGATION OF TRANSMISSION LOSS FOR PIPE CONFIGURATIONS OF KNOWN RESULTS:

The transmission loss formula (21) is assumed applocable to any inversion of the in-parallel-branched pipe configuration. A computer program to verify the model has been developed. To verify the model, two test cases (the expansion chamber and the tuned side branch) have been considered:

(a) Expansion Chamber:

Considering the assumption stated for the model shown in Fig. (1), this model will converge to an expansion chamber by setting $\ell_a = \ell_b = \ell$ as shown in Fig.(4) Transmission loss curve for an expansion chamber and the frequencies of maximum and minimum attenuation are shown in Fig.(5).

Let $\dfrac{S_a + S_b}{S} = m$, and substituting these values in equation (21):

$$\therefore \text{T.L.} = 10 \log_{10}\left[1 + \frac{1}{4}\ (m - 1/m)^2 \sin^2 k\ell\right] \quad \ldots\ldots\ldots (22)$$

The general formula of transmission loss for the model under study (21) reduces to that of an expansion chamber . Figures (6) and (7) show the transmission loss predicted by the developed model for two selected values of (m).

(b) Tuned side branch:

If one of the side braches' length is set to (zero) with the length of the other branch set to (ℓ), the model reduces to that of a side branch with a length ($\ell/2$) and a cross sectional area of ($2S_a$) as in Fig.(8). A typical transmission loss curve for a turned side branch and the frequencies of maximum and minimum attenuation is shown in Fig. (9).

Set $m = \dfrac{2\ S_a}{S}$, and substituting those values in

equation (21):

$$\therefore \text{T.L} = 10 \log_{10}\left[1 + \frac{1}{4}\ \left(\frac{m}{\text{Cot}\ k\ell/2}\right)^2\right] \quad \ldots\ldots\ldots (23)$$

The general formula of transmission loss for the model under study (21) reduces to that of a tuned side branch . Figures (10), and (11) show the transmission loss predicted by the developed model for selected values of (m).

4 CONCLUSIONS AND RECOMMENDATIONS:

The use of the four-pole-parameters technique to relate acoustic pressure and volume velocities between the inlet and outlet of a main pipe branched into two in-parallel pipes has proven to be useful to calculate the attenuation in the configuration. The driven generalized formula is flexible and applicable to many inversions of the model under study. This formula should be of major interest to those involved in the design and analysis of mufflers and exhaust systems of internal combustion engines where numerous shapes and configurations ofthis sort are used.

It is recommended that the developed formula not only be applied to in-parallel branched pipes, but also be used to investigate and predict acoustic characteristics of more complicated shapes through rearranging the configuration under study to fit the model.

Nomenclature:

A_i = Four-pole-parameters of the first branch, i = 1,2,3,4.

B_j = Four-pole-parameters of the second branch, j = 1,2,3,4.

C_k = Overall four-pole-parameters of the main pipe, k= 1,2,3,4.

c = Velocity of sound.

f = Frequency.

k = Wave number.

ℓ = length of pipe (or branch).

m = Expansion ratio.

p = Acoustic pressure.

s = Cross sectional area.

u = Volume velocity.

ρ = Average density of sound-conducting medium.

ρc = Specific acoustic impedance.

References:

(1) Roushdy, E.H. "one dimensional model for the muffler reversing section", Herrick laboratories internal report 78-9-21 Purdue University, 1978.

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(3) Sullivan, J.W. "Theory and methods for modeling acoustcally long unpartitioned cavity resonators for engine exhaust systems", Ph.D. thesis, Purdue University, Dec. 1974.

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(5) Davis Jr. D. , Stokes G., Moore D., and Stevens Jr. G., "Theoretical and experimental investigation of mufflers with comments on engine exhaust muffler design", Langley Aero- nautical Laboratory, Report 1192.

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(7) Rschekin, S.N. "The theory of sound", Pergamon Press, Handan, 1963.

(8) Morse P.M., Ingard K.U., "Theoretical Acoustics", McGraw Hill, New York, 1978.

(9) Kinsler L.E., Frey A.R., "Fundamentals of Acoustics", John Wiley and Sons, New York, 1962.

Appendix

(a) Development of the general model:

Equations (1) and (3) give:

$$(P_{1a} + P_{1b}) = (A_1P_{2a} + B_1P_{2b}) + (A_2U_{2a} + B_2U_{2b}) \quad \ldots.(A\text{-}1)$$

$$2P_1 = (A_1 + B_1)\ P_2 + A_2(U_2 - U_{2b}) + B_2U_{2b}$$

$$2P_1 = (A_1 + B_1)\ P_2 + A_2U_2 + (B_2-A_2)U_{2b} \quad \ldots.. (A\text{-}2)$$

Equations (2) and (4) give:

$$(U_{1a} + U_{1b}) = (A_3P_{2a} + B_3P_{2b}) + (A_4U_{2a} + B_4U_{2b})$$

$$U_1 = (A_3 + B_3) + A_4(U_2 - U_{2b}) + B_4U_{2b}$$

$$U_1 = (A_3 + B_3) + A_4U_2 + (B_4 - A_4)U_{2b} \quad \ldots.. (A\text{-}3)$$

Subtracting equation (3) from (1) gives:

$$(P_{1a} - P_{1b}) = (A_1 - B_1)P_2 + (A_2U_{2a} - B_2U_{2b})$$

$$0 = (A_1 - B_1)P_2 + A_2(U_2 - U_{2b}) - B_2U_{2b}$$

$$0 = (A_1 - B_1)P_2 + A_2U_2 - (A_2 + B_2)\ U_{2b}$$

$$U_{2b} = \left[\frac{A_1 - B_1}{A_2 + B_2}\right]P_2 + \left[\frac{A_2}{A_2 + B_2}\right]U_2 \quad \ldots\ldots\ldots..(A\text{-}4)$$

Substituting for U_{2b} as expressed by equation (A-4):

i. In equation (A-2)

$$P_1 = \left[\frac{A_1 + B_1}{2}\right] P_2 + \left[\frac{A_2}{2}\right] U_2 - \left[\frac{(A_2 - B_2)(A_1 - B_1)}{2(A_2 + B_2)}\right]P_2 + \left[\frac{A_2(B_2 - A_2)}{2(A_2 + B_2)}\right]U_2$$

$$P_1 = \left[\frac{A_1 B_2 + A_2 B_1}{A_2 + B_2}\right] P_2 + \left[\frac{A_2 B_2}{A_2 + B_2}\right] U_2 \qquad \text{.......... (A-5)}$$

ii. In equation (8)

$$U_1 = (A_3 + B_3)\, P_2 + A_4 U_2 - \left[\frac{(A_1 - B_1)(A_4 - B_4)}{(A_2 + B_2)}\right] P_2 - \left[\frac{A_2 (A_4 - B_4)}{(A_2 + B_2)}\right] U_2$$

$$U_1 = \left[\frac{A_2 B_3 + A_3 B_2 + A_1 B_4 + A_4 B_1 - 2}{A_2 + B_2}\right] P_2 + \left[\frac{A_2 B_4 + A_4 B_2}{A_2 + B_2}\right] U_4 \qquad \text{...... (A-6)}$$

The four pole parameters (C_1, C_2, C_3, C_4) appearing in equations (5) and (6) and expressed in terms of (A ,A ,A ,A) and ($_1$B ,$_2$B ,$_3$B ,$_4$B ,) are:

$$C_1 = \frac{A_1 B_2 + A_2 B_1}{A_2 + B_2} \qquad \text{........... (A-7)}$$

$$C_2 = \frac{A_2 \; B_2}{A_2 + B_2} \qquad \text{........... (A-8)}$$

$$C_3 = \frac{A_2 B_3 + A_3 B_2 + A_1 B_4 + A_4 B_1 - 2}{A_2 + B_2} \qquad \text{........... (A-9)}$$

$$C_4 = \frac{A_2 B_4 + A_4 B_2}{A_2 + B_2} \qquad \text{.......... (A-10)}$$

The parameters satisfy the condition

$$C_1 C_4 - C_2 C_3 = 1$$

(b) Transmission loss expression for an expansion chamber:

$$\therefore \text{T.L.} = 10 \log_{10}\left[\cos^2 k\ell + \left(\frac{\sin^2 k\ell((S_a + S_b)^2 + S^2)}{2S}\right)^2 / (S_a + S_b)^2 \sin^2 k\ell\right]$$

$$= 10 \log_{10}\left[\, 1 - \sin^2 k\ell + \sin^2 k\ell \left(\frac{1}{4}\left(\frac{S_a + S_b}{S}\right)^2\right.\right.$$

$$+ \frac{1}{4} \left(\frac{S}{S_a + S_b}\right)^2 + \frac{1}{2}\Big)\Big]$$

$$= 10 \log_{10}\left[1 + \frac{1}{4} (m^2 - 1 + 1/m^2) \sin^2 k\ell\right]$$

$$= 10 \log_{10}\left[1 + \frac{1}{4} (m - 1/m)^2 \sin^2 k\ell\right]$$

(c) Transmission loss expression for a tuned side branch

$$T.L. = 10 \log_{10}\left[1 + \left(\frac{S_a}{S}\right)^2 \left(\frac{1 - \cos k\ell}{\sin k\ell}\right)^2\right]$$

$$= 10 \log_{10}\left[1 + \frac{1}{4}\; m \left(\frac{(1-\cos^2 k\ell/2) + \sin^2 k\ell/2}{2 \sin k\ell/2 \cos k\ell/2}\right)^2\right]$$

$$T.L = 10 \log_{10}\left[1 + \frac{1}{4}\left(\frac{m}{\cot k\ell/2}\right)^2\right] \qquad \text{.......... (A-1)}$$

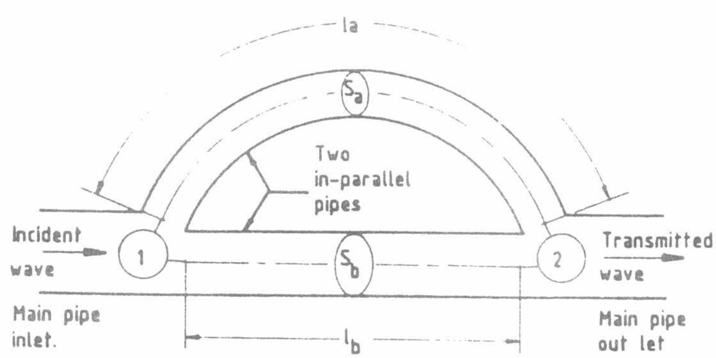

FIG. (1) Configuration of the model under study

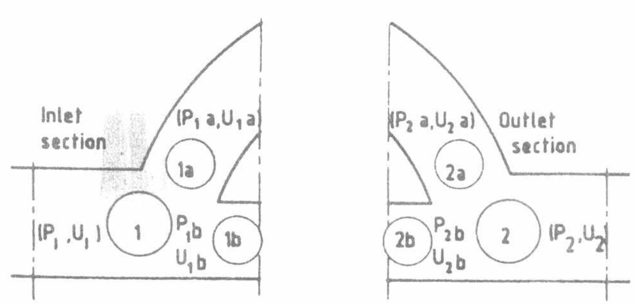

FIG.(2) Acoustic pressure and volume velocity at the inlet and outlet sections

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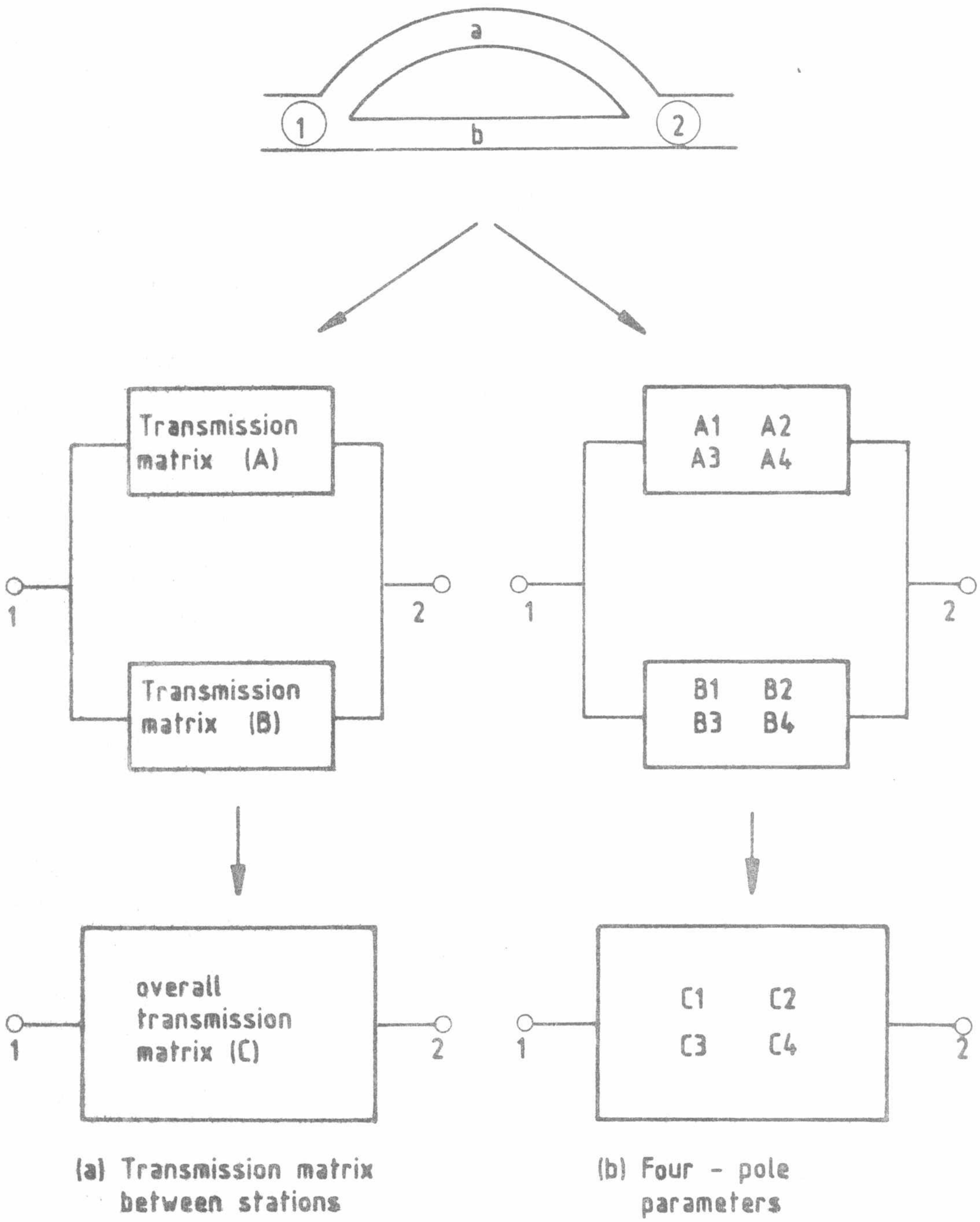

FIG. (3) Modeling of the system

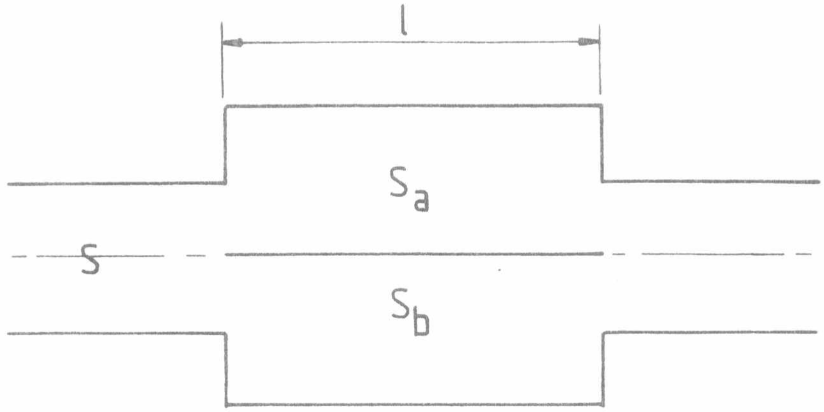

FIG. (4) Two-in-parallel pipes of equal lengths (Expansion chamber).

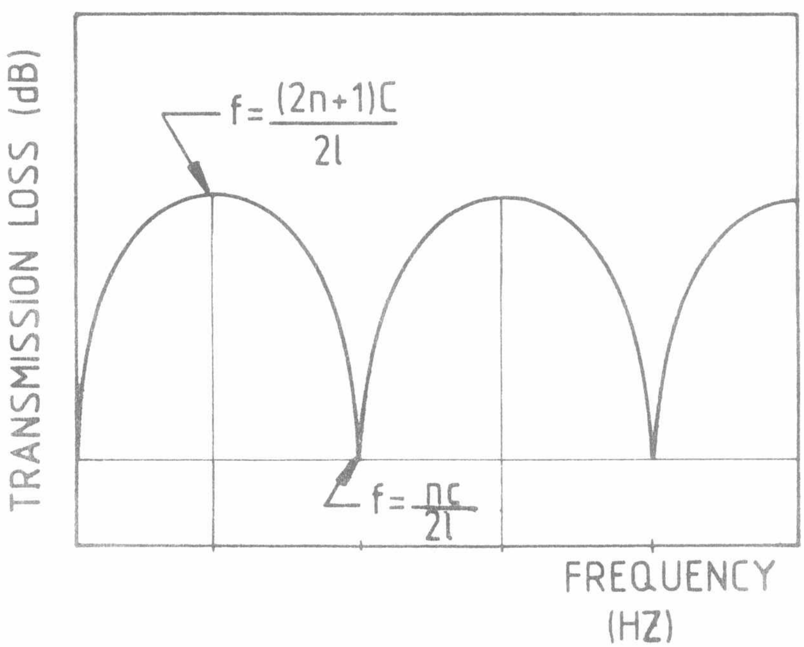

FIG. (5) Shape of transmission loss characteristics of the expansion chamber.

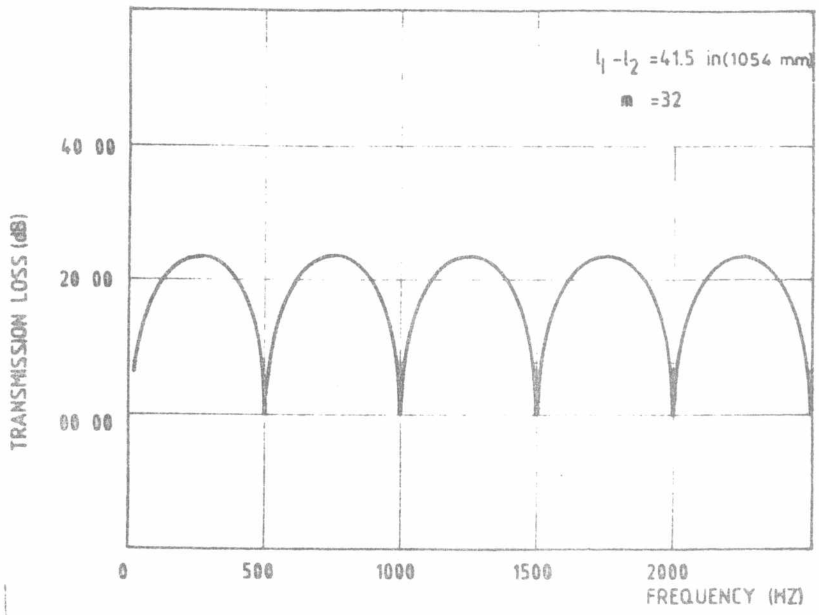

FIG. (6) Predicted transmission loss charateristics of an expansion chamber using the developed model.

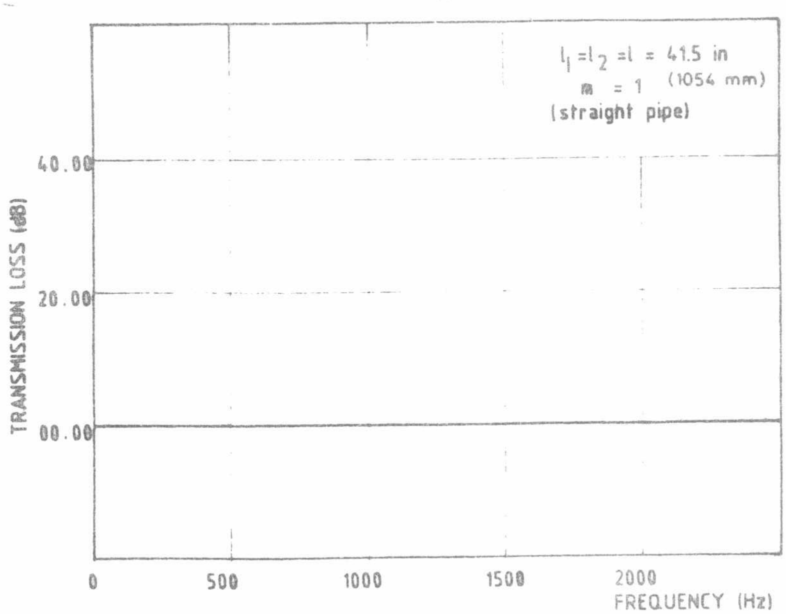

FIG. (7) Predicted transmission loss characteristics of a straight pipe (a special case of the expansion chamber) using the developed model.

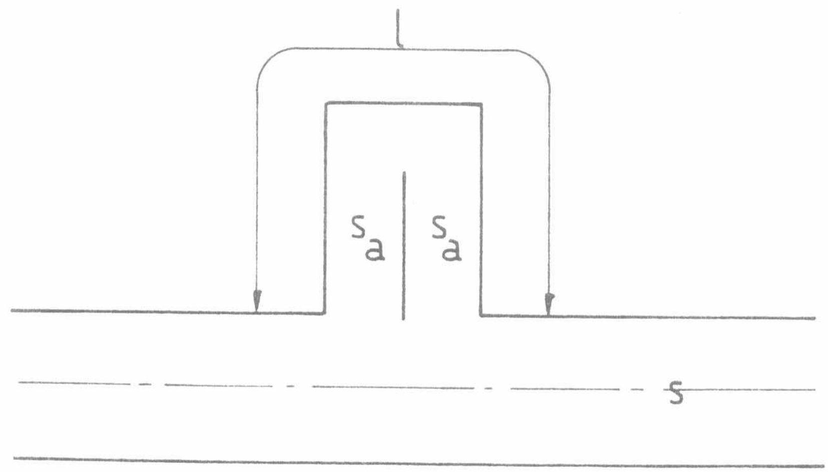

FIG. (8) Two-in-parallel pipes with one length set to zero (tuned side branch)

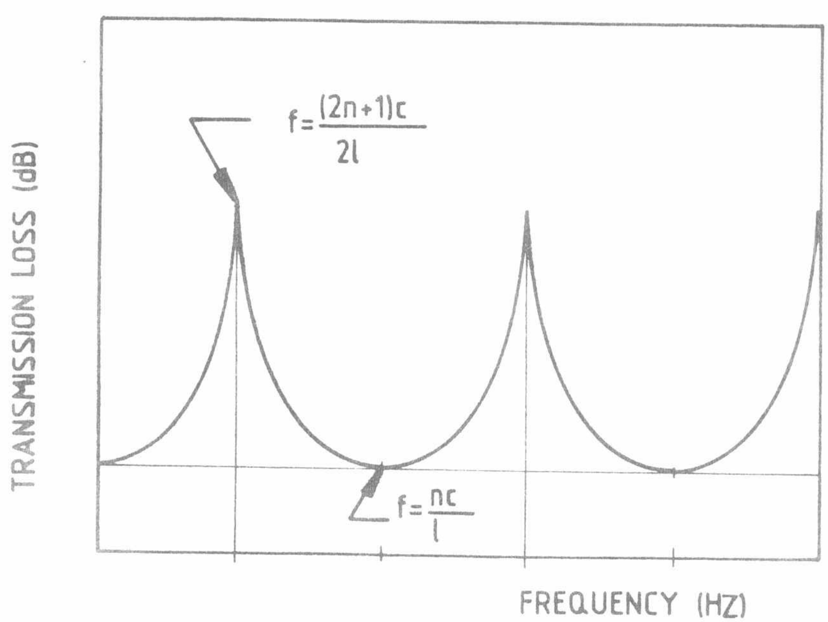

FIG. (9) Shape of transmission loss characteristics of the tuned side branch.

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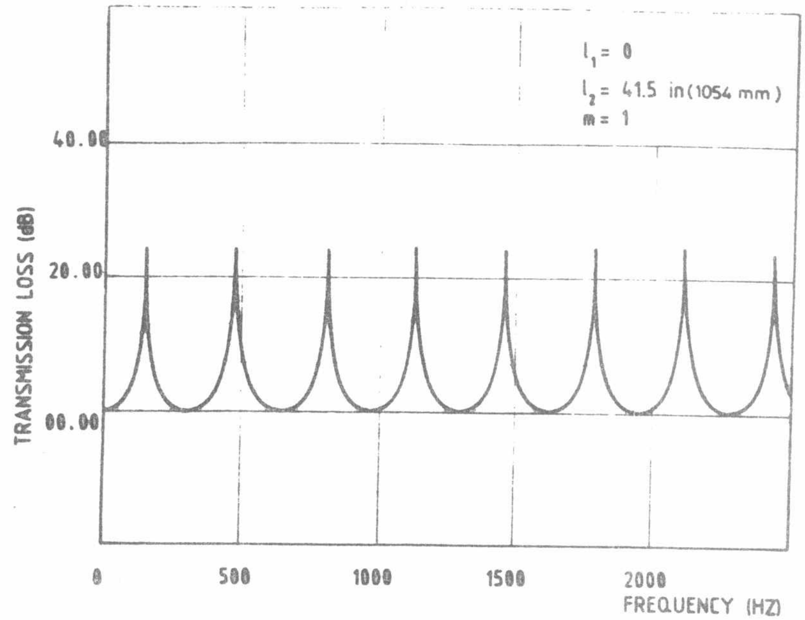

FIG. (9) Predicted transmission loss characteristics of a tuned side branch using the developed model.

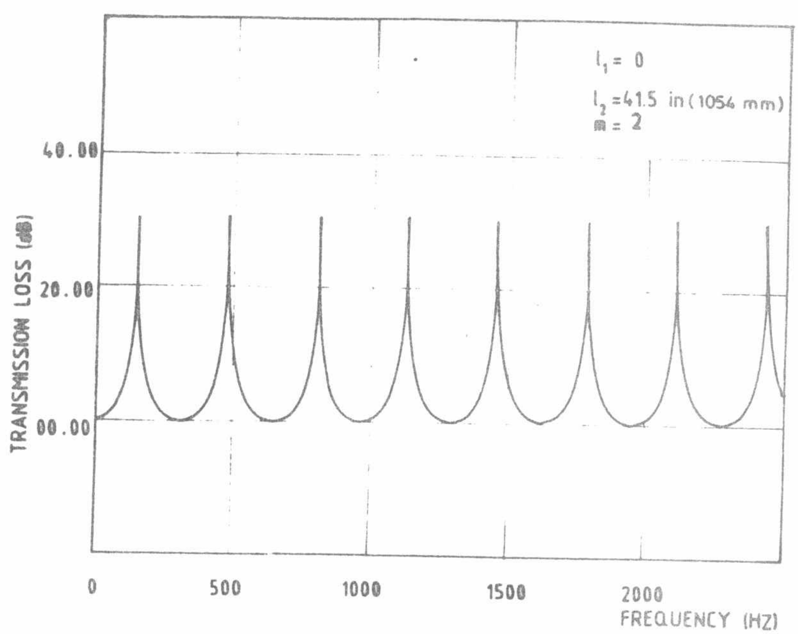

FIG. (10) Predicted transmission loss characteristics of a tuned side branch using the developed model.