INTRODUCING HEURISTIC METHODS TO SOLVE SCHEDULING OF MULTISTAGE FLOWSHOP WITH IDENTICAL PARALLEL MACHINES

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ABSTRACT

The objective of this study is to develop an algorithm which can solve the scheduling problem of multistage flowshop with identical parallel machines in every stage. This was achieved heuristically and an algorithm was developed to solve this problem. This heuristic algorithm was applied to schedule the production of special purpose servicing equipment capital repair workshop. As an indication of algorithm efficiency, the absolute maximum production volume was determined using a linear programming model and compared to that realized by the proposed algorithm.

INTRODUCTION

An important function of industrial management is the co-ordination and control of complex activities, including optimum resource allocation fulfilling the objectives. However, detailed production planning should take into consideration resource allocation and sequencing of activities performed. This type of problem is known as scheduling problem.

It is important, from the begining, to distinguish between two complementary aspects: sequencing and scheduling. Erschler, Roubellat and Vernhes |1| report that sequencing deals with the order in which the jobs are carried out on the machines, while the scheduling deals with the determination of the starting times of these jobs.

In recent years, a nnmber of quantitative approaches to several types of scheduling problems have been proposed. It is worth to note that not all scheduling problems can be efficiently solved, and in several cases, heuristic techniques that yield non-optimal but relatively favourable solutions, are employed |2|.

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Garey, Graham and Johnson |3| in their study on performance guarantees for scheduling algorithms, reported that though there are problems for which finding even a feasible schedule seems computationally hopeless, for most schedule optimization problems there do exist simple heuristic algorithms that find feasible schedules quickly.

Wassenhove and Gelders |4| investigated the reliability of four heuristic methods when applied to more realistic and complex single-machine problem. Many other approaches for solving single-machine scheduling problems have been done, reffering to works of Weeda |5|, Lenstra |6| and Bansal |7|. These approaches are differed according to thetechniques used, their simplicity and the efficiency of solutions generated.

The most frequently cited paper in the field of scheduling is probably Johnson's solution to the two machine flowshop problem |8|. Johnson gave an algorithm for sequencing n jobs, all simultaneously available in a two-machine flowshop so as to minimize the maximum flow time. Apart from the Johnson two-machine and special three-machine cases, several attempts have been made to formulate algorithms which sequence jobs onto flowshops in an optimal way. Palmer's slope index heuristic method |9| is based on the idea that jobs placed early in the sequence should have processing times that tend to increase from machine to machine as the jobs progress through the technological ordering.

Bonney and Gundry |10| developed a method for obtaining an equivalent algorithm which represents the Johnson's two-machine algorithm.

Campbell, Dudek and Smith |11| proposed a procedure which is a heuristic generalization of the Johnson's three-machine algorithm. This method generates a set of m-1 , artificial two-machine problems from the original m-machine problem, each of which is then solved using Johnson's two-machine algorithm. The best of m-1 solutions becomes the heuristic optimal solution to the m-machine problem.

Dannenbring |12| proposed a rapid access procedure. Its purpose is to provide a good solution as quickly and easily as possible. A single, two-machine problem is formed where the processing times are determined from the weighing scheme and the problem is solved using Johnson's two-machine algorithm.

Giglo and Wagner |13| have applied several computational methods for solving the classic three machine scheduling model. They came to the conclusion that the integer programming approach has the obvious advantage that when it succeeds, an optimal solution to the problem can be obtained.

DESCRIPTION OF THE CASE UNDER STUDY

The case under study consists of a workshop that performs a capital repair process on special purpose surv icing equipment There are 10 kinds of equipment which are denoted by symbols $X_1, X_2, \ldots, X_{10}$.

When these equipment are regarded as trucks, they are mainly of two groups:

1. Diesel engine motorized equipment, denoted as diesel equipment, these are equipment X_1, X_2, X_6 and X_8.
2. Gasoline engine motorized equipment, denoted as gasoline equipment and these are: X_3, X_4, X_5, X_7, X_9 and X_{10}.

There are 23 professions contributing in the capital repair of all equipment. These professions can be classified into two main groups:

1. Truck group: these are professions that contribute in the repair of all kinds of equipment when regarded as trucks. These professions are denoted $Y_1, Y_2, Y_8, Y_{12}, Y_{13}, Y_{14}, Y_{15}, Y_{16}, Y_{17}, Y_{18}, Y_{19}, Y_{20}, Y_{21}, Y_{22}$ and Y_{23}.
2. Superstructure group: each profession in this group contributes repair of loaded superstructure (pumps,generators, compressors ...etc.) of one kind, or more, of the equipment. These are denoted by symbols $Y_3, Y_4, Y_5, Y_6, Y_7, Y_9, Y_{10}$ and Y_{11}.

   Due to certain considerations, equipments are of different weight of importance. When planning for repair, these weights must be taken into consideration. Equipment relative weights of importance are given in table(1). The larger the relative weight of importance, the more important is the equipment.

Table (1)

Equipment Symbol	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
Relative weight of importance	4	1	4	4	3	2	2	5	4	4

CONSTRUCTING THE LINEAR PROGRAMING MODEL

The objective function, or the management target, is to maximize the output expressed in total equivalent production, using the existing resources available for capital repair within the production period (one month).

To obtain the maximum product-mix, the objective function is usually expressed as follows:

$$X_1 + X_2 + X_3 + \ldots\ldots + X_n = \max$$

Such that X_i = amount of product i to be produced in the production period (i = 1, 2, ...n).

In the case under study, there is relative importance for different products. The previously mentioned form of the objective function does not reflect this relative importance. Therefore, the following form for the objective function may be used:

$$W_1X_1 + W_2X_2 + \ldots + W_nX_n = \max.$$

Where: X_i = volume of production of product i
and W_i = relative weight of importance.
As a result, the objective function will be:
Maximize $Z = 4X_1 + X_2 + 4X_3 + 4X_4 + 3X_5 + 2X_6 + 2X_7 + 5X_8 + 4X_9 + 4X_{10}$
Where Z is the value of the objective function.

The resource constraints can be expressed as follows:

$$P_{11}X_1 + P_{12}X_2 + \ldots + P_{1n}X_n \leqslant Y_1$$
$$P_{21}X_1 + P_{22}X_2 + \ldots + P_{2n}X_n \leqslant Y_2$$
$$\ldots\ldots$$
$$P_{m1}X_1 + P_{m2}X_2 + \ldots + P_{mn}X_n \leqslant Y_m$$
$$X_1, X_2, \ldots, X_n \geqslant 0$$

Where:

P_{ij} = the processing time needed by product i from resource j.
$i = 1,2, \ldots, n$
$j = 1,2, \ldots, m$
Y_j = the net man. hr available for resource j.

To achieve minimum operational requirements, management imposes a minimum production level of each product. This represents additional constraints. These constraints can be expressed in terms of a set of linear inequalities as follows:

$$X_1 \geqslant 4,\ X_2 \geqslant 3,\ X_3 \geqslant 4,\ X_4 \geqslant 2,\ X_5 \geqslant 2,\ X_6 \geqslant 3,\ X_7 \geqslant 1,$$
$$X_8 \geqslant 1,\ X_9 \geqslant 1 \text{ and } X_{10} \geqslant 1$$

As the elements of the linear programming model are defined and according to available data of the case under study, each of them can be expressed in a linear form.

This model gives a clear idea about the size of the problem which can be described as a problem of 10 variables and 33 constraints.

The only efficient optimization method for solving such a problem is the simplex method. This method is a collection rules applied in a relatively mechanical manner to obtain sequentially improved solutions to a problem with linear relationships.

A computer program was prepared for this case. The optimal solution has been attained after 20 iterative steps.

The optimum production mix that maximize the objective function which was attained after truncating the integer part of values obtained is as follows:

$$X_1 = 5,\ X_2 = 4,\ X_3 = 10,\ X_4 = 2,\ X_5 = 2,$$
$$X_6 = 3,\ X_7 = 1,\ X_8 = 4,\ X_9 = 1 \text{ and } X_{10} = 1$$

The value of the objective function Z in this case will be 114.

This solution will give some resedue of available man. hr resource. When studying this available man.hr in order to examine the possibility of repairing extra equipment, it was found that it was possible to obtain one extra equipment of each of the products X_2, X_5 and X_6. Thus, the possible near-optimal solution will be:

$$X_1 = 5,\ X_2 = 5,\ X_3 = 10,\ X_4 = 2,\ X_5 = 3,$$
$$X_6 = 4,\ X_7 = 1,\ X_8 = 4,\ X_9 = 1 \text{ and } X_{10} = 1$$

The new value of the objective function will hence be 120.

SCHEDULING PROBLEM OF CASE UNDER STUDY

In order to obtain a feasible production schedule, the problem under study is to be considered as a sequencing problem, having a target of reaching the production volume previously determined by linear programming model. This can be described as follows:

1. It is required to schedule 36 equipment (optimum product-mix of linear programming model), of 10 different relative weight of importance.
2. The pattern of flow is unidirectional. Then it is a flow shop scheduling problem.
3. It is a static problem in which the equipment arrive simultaneously into the shop to the crew that is free and immediately available for work.
4. The first stage (main workshop) is composed of two independent lines, one for the gasoline equipment and the other for the diesel equipment. The second stage is a complementary workshop, while the third stage is a finishing workshop.
5. For every stage there is more than one crew (machine) available. These crews are similar and working independantly parallel, i.e. it is a special case of scheduling parallel machines.
6. The scheduling is required so as to maximize the excutable number of equipment within the production period. This implies minimization of mean flow time.

All researches that developed heuristic methods for solving n job-3 machine flowshop sequencing problem did not treat a problem of such a structure which contains a set of parallel machines (crews) of the same type in every stage beside the complex nature of the first stage.

Now, it is possible to consider each stage separately such that the output of the first stage is the input to the second stage and so on. This will change the nature of the second and third stages to the dynamic nature.

The first stage is composed of two separate independent lines; each to be scheduled separately, but the input to second stage will be the total output of the two lines of the first stage.

In order to minimize the mean flow time, mean completion time and mean lateness, one would use the shortest processing time rule. This rule arranges the n jobs, to be sequenced, in an acsending order of processing time:

$$P_1 \leqslant P_2 \leqslant P_3 \leqslant \cdots\cdots \leqslant P_n$$

The flow time of a job in the k^{th} position of an arbitrary sequence is simply:

$$F_{[k]} = \sum_{i=1}^{k} P_{[i]}$$

The mean flow-times of the n jobs is given by

$$\bar{F} = \frac{\sum_{k=1}^{n} F_{[k]}}{n} = \frac{\sum_{k=1}^{n} \sum_{i=1}^{k} P_{[i]}}{n} = \frac{\sum_{i=1}^{n} (n-i+1) P_{[i]}}{n}$$

Now, to schedule a set of n jobs which have relative weights of importance W_i (the larger W_i, the more important the job) so as to minimize the mean weighted flow time we can use the following formula:

$$\bar{F}_w = \frac{\sum_{i=1}^{n} W_i \bar{F}_i}{n}$$

This may be accomplished by sequencing the jobs so that:

$$\frac{P_{[1]}}{W_{[1]}} \leqslant \frac{P_{[2]}}{W_{[2]}} \leqslant \cdots\cdots \leqslant \frac{P_{[n]}}{W_{[n]}}$$

On the other hand, the two lines of the first stage can be regarded as multiple classes of processing priorities. These classes are defined as follows:

- Class (A) : includes all equipment that are restricted to minimum volume of production, dictated by the management, after linear programming model solution.

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- Class (B): includes equipment having superstructure and not included in class (A) - As superstructure labour is a limiting resource.
- Class (C): includes equipment having no supersturcture and not included in class (A)-

This implies the following sequence between classes:

Sequence class (A), then class (B), then class (C).

SCHEDULING ALGORITHMS

Fig. (1) shows the flow chart of the computer programe which was developed in accordance with the preceded analysis for the first stage, while Fig. (2) and Fig. (3) show the flow charts of the computer programes developed for stage 2 and stage 3 respectively.

RESULTS AND DISCUSSION

Using previously mentioned algorithms, all products obtained by solving the linear programming model were scheduled within the production period (25 days).

The following are the obtained results:

1. First stage:

a- Diesel line:

Optimal sequence obtained for diesel line is as follows:

Crew No.	Sequence
1	$X_{81} - X_{83} - X_{23}$
2	$X_{82} - X_{84} - X_{24}$
3	$X_{11} - X_{13} - X_{15}$
4	$X_{12} - X_{14} - X_{22}$
5	$X_{61} - X_{62} - X_{63} - X_{64} - X_{21} - X_{25}$

N.B. X_{ab} is an equipment of type a(a=1,2,...., 10) and b is identification number for this equipment (b=1,2, ..., n) where n is the number of scheduled equipment of type a.

The mean flow time for diesel line $\bar{F}$=

$$\frac{\sum_{i=1}^{18} P_i}{18} + \frac{\sum_{i=1}^{18} W_i}{18} = 8.23 + 9.50 = 17.73 \text{ days.}$$

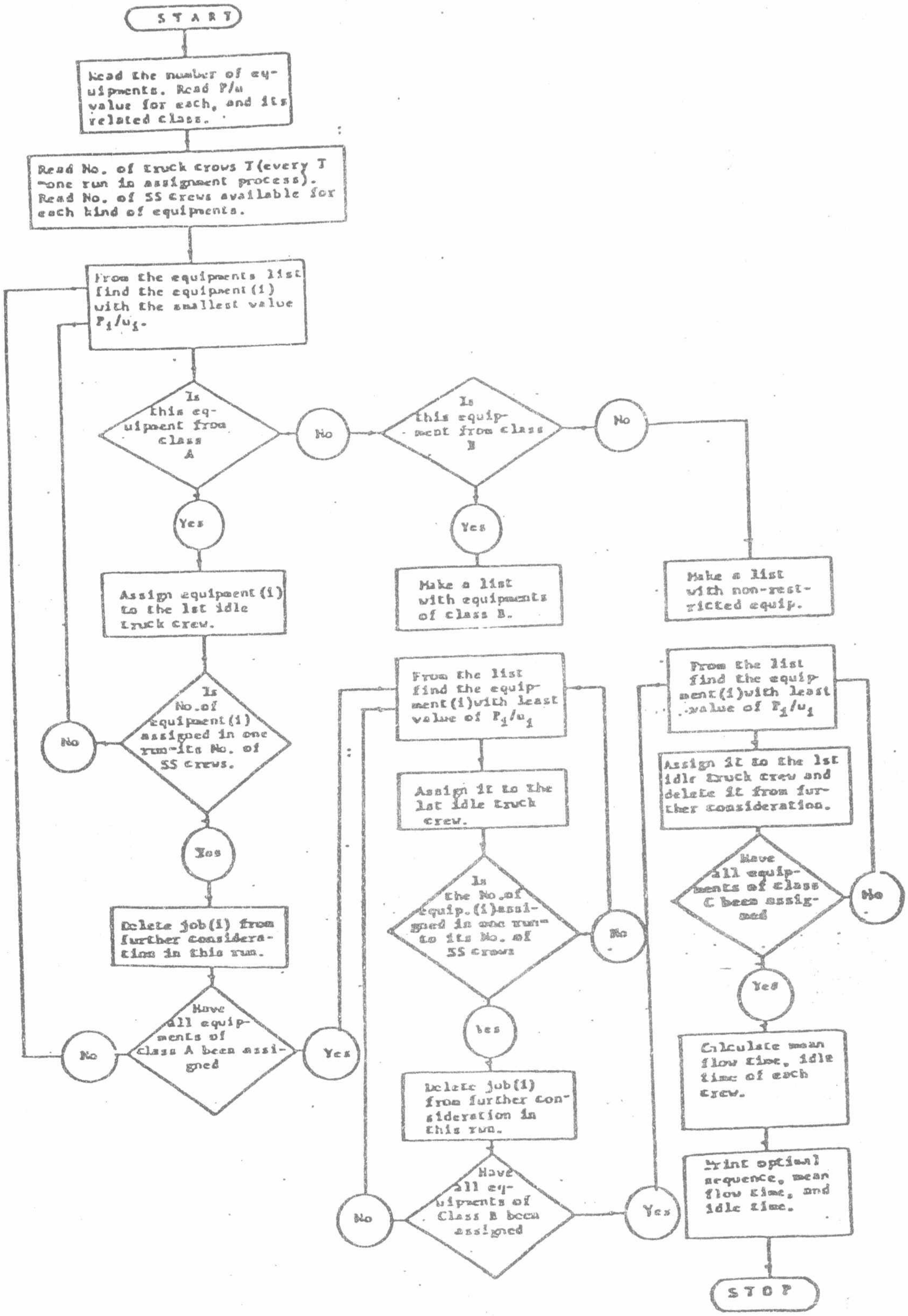

Fig.(1) First stage algofithm.

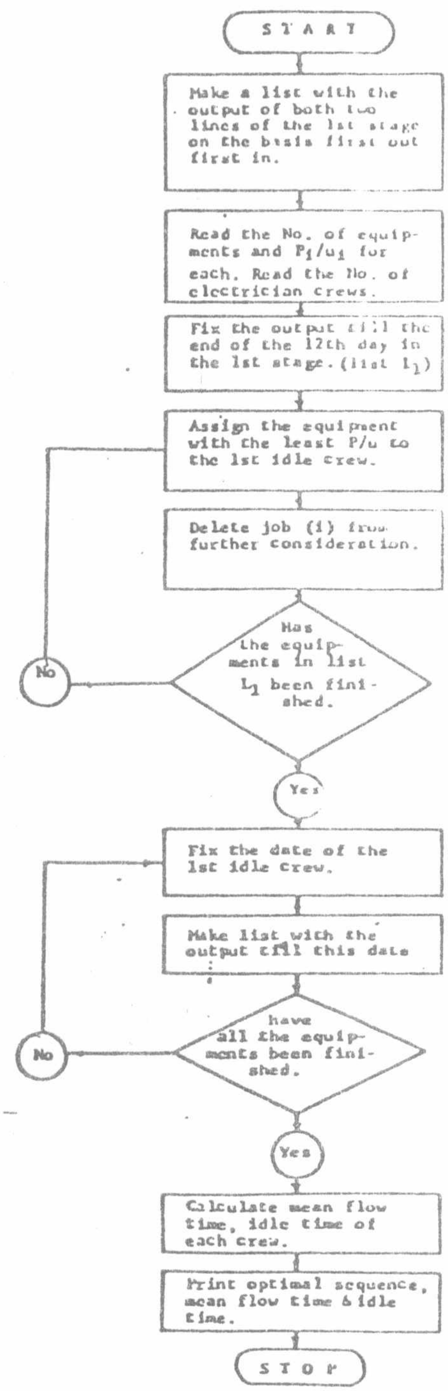

Fig. (2) Second stage algorithm.

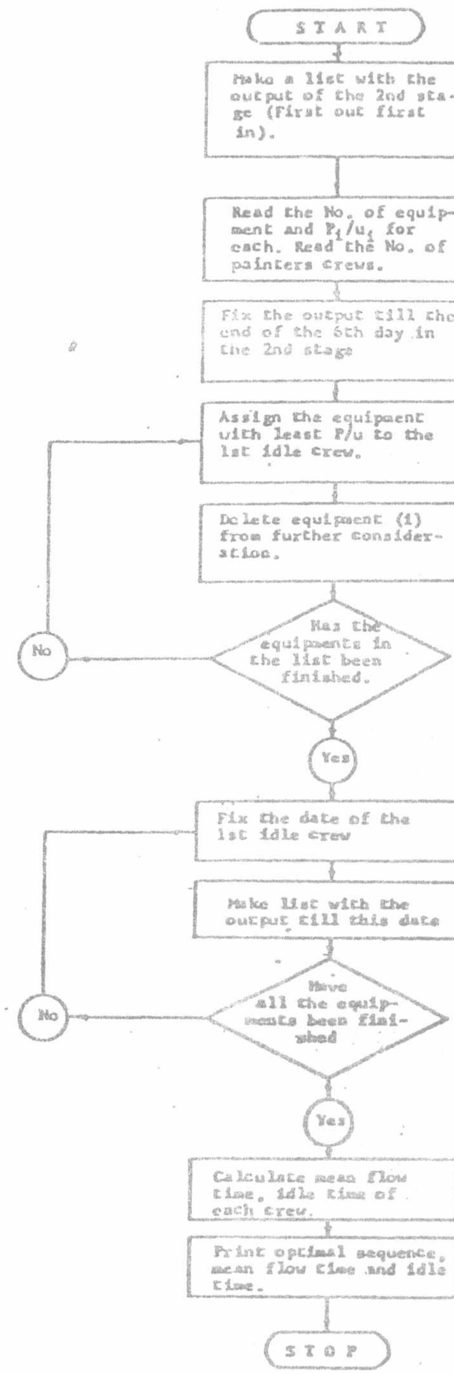

Fig. (3) Third stage algorithm.

b- Gasoline line:

Optimal sequence obtained for gasoline line is as follows:

Crew No.	Sequence
1	$X_{41} - X_{38} - X_{42}$
2	$X_{9} - X_{7} - X_{39}$
3	$X_{10} - X_{34} - X_{310}$
4	$X_{31} - X_{35} - X_{51}$
5	$X_{32} - X_{36} - X_{52}$
6	$X_{33} - X_{37} - X_{53}$

Mean flow time for gasoline line = 17.9 days.

2. Second stage:
The optimal sequence obtained according to the second stage algorithm is as follows:

Crew No.	Sequence
1	$X_{61} - X_{12} - X_{34} - X_{14} - X_{21} - X_{310} - X_{23}$
2	$X_{10} - X_{81} - X_{7} - X_{83} - X_{52}$
3	$X_{31} - X_{82} - X_{36} - X_{64} - X_{15} - X_{24}$
4	$X_{32} - X_{41} - X_{13} - X_{84} - X_{25}$
5	$X_{33} - X_{9} - X_{35} - X_{22} - X_{51} - X_{42}$
6	$X_{11} - X_{62} - X_{63} - X_{37} - X_{39} - X_{53}$

The mean flow for second stage = 7.1 days.

3. Third stage:
Optimal sequence will be as follows:

Crew No.	Sequence
1	$X_{61} - X_{32} - X_{12} - X_{38} - X_{9} - X_{7} - X_{35} - X_{64}$
	$X_{83} - X_{51} - X_{23} - X_{24} - X_{42}$
2	$X_{10} - X_{33} - X_{62} - X_{34} - X_{82} - X_{13} - X_{36}$
	$X_{21} - X_{39} - X_{15} - X_{25} - X_{53}$
3	$X_{31} - X_{11} - X_{41} - X_{81} - X_{63} - X_{14} - X_{37}$
	$X_{22} - X_{310} - X_{84} - X_{52}$

The mean flow time for the third stage = 3.6 days.

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CONCLUSION

The results of applying the developed algorithms on the case under study revealed that all the production volume realized by the linear programming model were scheduled. This indicates that all the available potentials were optimally utilized.

It was possible to prepare a product schedule to produce 36 pieces of equipment per month while this workshop had an average volume of production of 18.58 pieces per month.

The proposed algorithm can be applied to any multi-stage flow job-shop problem with identical parallel machines in each stage, as it does not impose any special constraints for its application.

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