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STEADY STATE FORCE ANALYSIS OF FOUR WAY UNDERLAP HYDRAULIC SPOOL SERVOVALVES

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ABSTRACT

The analysis of one of the common four way underlap hydraulic spool valve construction is presented. The flow force in the valve is deduced and used in building a steady state model for the valve based on the continuity and momentum equations.

The flow damping coefficient is illustrated for differnt damping length ratios. The steady state flow force is discussed for different flow parameters. The effect of the spool valve parameters, the flow stiffness and the positioner spring rate on the valve performance are illustrated.

1. INTRODUCTION

Evaluation of the different forces encountered in the underlap spool servovalves during their use is of vital importance. Such analysis helps in accurate characterization of the valves under different operating conditions. Also, it allows successful selection of the type of spool actuation depending on the level of the required driving force at different pressures and flow rates through the valve.

The study illustrates the importance of the design and operating conditions of such hydraulic valves from point of view of stability since the underlap valve may be unstable under certain conditions [1] .

The four way, four land underlap spool servovalve shown in Fig.1 is chosen for the analysis.

2. ASSUMPTIONS

The analysis involves the following assumptions:
(a) The supply flow rate q_3 enters region b (see Fig.1) and immediately establishes an average velocities between the supply port and the two centre lands. Similarly for the two drain flow rates q_4 and q_d

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(b) The stream lines inside the control volume are assumed parallel, uniform and directed along the axis of the spool
(c) The effect of any fluid velocity between the sealing lands and drain ports of regions a and c is neglected
(d) The entering and leaving jet angle at the ports are assumed constant and equal to 69° [2,3]
(e) The velocity profile in the vena contracta of the entering or leaving jet is assumed uniform
(f) Neglecting pressure drop in the control volumes,temperature effect and coulomb friction
(g) Flow density is constant

3. CONTINUITY EQUATION OF THE VALVE

Using the continuity equation, the relationships of flow through orifices and gauge pressures provides the various flow rates through the valve. They are given by [2,4] (see Fig.1):

$$q_s = C_d A_s \sqrt{2(p_s - p_1)/\rho} \qquad (1)$$

$$q_5 = C_d A_5 \sqrt{2(p_s - p_2)/\rho} \qquad (2)$$

$$q_4 = C_d A_4 \sqrt{2 p_1/\rho} \qquad (3)$$

$$q_d = C_d A \sqrt{2 p_2/\rho} \qquad (4)$$

$$q_1 = q_s - q_4 \qquad (5)$$

$$q_2 = q_d - q_5 \qquad (6)$$

$$q_3 = q_s + q_5 \qquad (7)$$

and

$$q_6 = q_d + q_4 \qquad (8)$$

In the vast majority of cases, the valve orifices are matched and symmetric [3]. Therefore,

$$A = A_s = B(u + x) \qquad (9)$$

and

$$A_4 = A_5 = A_d = B(u - x) \qquad (10)$$

where C_d is the discharge coefficient, A is the port area, p_s, p_1 and p_2 are the supply and load pressures respectively, u is the valve underlap and x is the spool displacement.

4. MOMENTUM EQUATION OF THE VALVE

The flow force on a control volume, f_f, is the sum of the force due to the rate of change of momentum through the control surface, the force due to the rate of increase of momentum inside the control volume, the force due to the control volume inertia and the force due to the compressibility effect. Therefore, f_f is given as [2,4] :

$$f_f = \int v\rho v \, dA + \frac{d}{dt}\int \rho v \, dV + \frac{d^2x}{dt^2}\int \rho \, dV + \frac{dp}{dt}\int \frac{\rho}{N} v \, dV \qquad (11)$$

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where V is the control volume, v is the flow velocity and N is the oil bulk modulus.

Applying Eq.(11) on the three control volumes shown in Fig.1 gives the component of the flow force in the x-direction, f_x which is the force exerted on the spool by the oil flow. That is:

$$f_x = \int \rho (q_d/C_cA)^2 \cos 69^\circ \, dA - \int \rho (q_5/C_cA_5)^2 \cos 69^\circ \, dA + \int \rho (q_s/C_cA_s)^2 \cos 69^\circ \, dA - \int \rho (q_4/C_cA_4)^2 \cos 69^\circ \, dA + \frac{d}{dt}\Big[-\int \rho q_d/A_c \, dV - \int \rho q_5/A_c \, dV + \int \rho q_s/A_c \, dV + \int \rho q_4/A_c \, dV \Big] + \frac{d^2x}{dt^2}\int \rho \, dV + \frac{dp_s}{dt}\Big[\int \rho q_s/NA_c \, dV - \int \rho q_5/NA_c \, dV\Big] + \frac{dp_d}{dt}\Big[\int \rho q_4/NA_c \, dV - \int \rho q_d/NA_c \, dV\Big] \quad (12)$$

where C_c is the jet contraction coefficient.

Let the switching functions S and S_1 be defined as:

$$S = \begin{cases} 0 & \text{for } A_s \leqslant 0 \\ 1 & \text{for } A_s > 0 \end{cases} \quad (13)$$

and

$$S_1 = \begin{cases} 0 & \text{for } A_d \leqslant 0 \\ 1 & \text{for } A_d > 0 \end{cases} \quad (14)$$

where A_s and A_d are the areas of supply and drain orifices respectively.

Combining Eqs.(1) through (6),(9),(10) and (12) through (14) gives f_x as:

$$f_x = 2C_d^2 B \cos 69^\circ / C_c \Big[S(u+x)(p_s - p_1 + p_2) - S_1(u-x)(p_s + p_1 - p_2)\Big] + C_d BL_s \sqrt{2\rho}\Big[S\sqrt{p_s - p_1} + S_1\sqrt{p_s - p_2} - LS_1\sqrt{p_1} - LS\sqrt{p_2}\Big] dx/dt + C_d BL_s \sqrt{2\rho}\Big[0.5S_1 L(u-x)/\sqrt{p_1} - 0.5S(u+x)/\sqrt{p_s - p_1}\Big] dp_1/dt + C_d BL_s \sqrt{2\rho}\Big[0.5S_1(u-x)/\sqrt{p_s - p_2} - 0.5SL(u+x)/\sqrt{p_2}\Big] dp_2/dt + m_{cv}\, d^2x/dt^2 \quad (15)$$

where L, L_s and m_{cv} are the ratio of the damping lengths, the supply port -centre land distance and the mass of the control volume respectively.

Define the load pressure $p_L = p_1 - p_2$ and for $q_1 = q_2 = q_L$ (load flow), we get $p_s = p_1 + p_2$. Then, the flow force component f_x and the load flow q_L will take the form:

$$f_x = 2C_d^2 B \cos 69^\circ / C_c \Big[S(u+x)(p_s - p_L) - S_1(u-x)(p_s + p_L)\Big] + C_d BL_s\sqrt{\rho}\,(1-L)\Big[S\sqrt{p_s - p_L} + S_1\sqrt{p_s + p_L}\Big] dx/dt + 0.5C_d BL_s\sqrt{\rho}\,(L-1)\Big[S(u+x)/\sqrt{p_s - p_L} + S_1(u-x)/\sqrt{p_s + p_L}\Big] dp_L/dt + m_{cv}\, d^2x/dt^2 \quad (16)$$

and

$$q_L = C_d B/\sqrt{\rho}\left[S(u+x)\sqrt{p_s-p_L} - S_1(u-x)\sqrt{p_s+p_L}\right] \qquad (17)$$

5. SPOOL EQUATION OF MOTION

Applying the Newton's second law of motion on the valve spool gives its equation of motion. That is [2,5] :

$$m_s\ddot{x} = f - kx - c\dot{x} - f_x \qquad (18)$$

where m_s is the spool mass, k is the positioner spring rate, c is the viscous damping coefficient, f_x is the axial component of the flow force and f is the input force.

From Eqs.(16) and (18), the input force to the spool f takes the form:

$$f = m\ddot{x} + c\dot{x} + kx + f_s(x,p_L) + g(p_L)x + h(x,p_L)p_L \qquad (19)$$

where

$$f_s(x,p_L) = 2C_d^2 B\cos 69^\circ/C_c\left[S(u+x)(p_s-p_L) - S_1(u-x)(p_s+p_L)\right] \qquad (20)$$

$$g(p_L) = C_d B L_s\sqrt{\rho}\,(1-L)\left[S\sqrt{p_s-p_L} + S_1\sqrt{p_s+p_L}\right] \qquad (21)$$

$$h(x,p_L) = C_d B L_s/\sqrt{\rho}\,(L-1)\left[S(u+x)/\sqrt{p_s-p_L} + S_1(u-x)/\sqrt{p_s+p_L}\right] \qquad (22)$$

and $m = m_s + m_{cv}$

The function $g(p_L)$ of Eq.(21) can be viewed as a flow damping coefficent. Normalizing $g(p_L)$ by dividing Eq.(21) by $(C_d B L_s\sqrt{\rho p_s})$ yields the dimensionless flow damping function $G(P_L)$ as:

$$G(P_L) = (1-L)\left[S\sqrt{1-P_L} + S_1\sqrt{1+P_L}\right] \qquad (23)$$

where $P_L = p_L/p_s$

Fig.2 illustrates the variation of $G(P_L)$ with load pressure and damping lengths ratio.

The function $f_s(x,p_L)$ given by Eq.(20) is the steady state flow force. Dividing Eq.(20) by $(2C_d^2 B\cos 69^\circ\, u p_s/C_c)$ yields the dimensionless form of that force $F(X,P_L)$ as:

$$F_s(X,P_L) = (1+X)(1-P_L)S - (1-X)(1+P_L)S_1 \qquad (24)$$

where $X = x/u$

The dimensionless load flow rate $Q_L(X,P_L)$ is obtained by dividing Eq.(17) by $(C_d Bu\sqrt{p_s/\rho})$. That is:

$$Q_L(X,P_L) = (1+X)\sqrt{1-P_L}\;S - (1-X)\sqrt{1+P_L}\;S_1 \qquad (25)$$

Fig.3 shows the variation of F_s with X, P_L and Q_L.

For static conditions, the valve force equation(Eq.(19)) reduces to the form:

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$$f = kx + f_s(x,p_L) \quad (26)$$

Define the spring rate of the flow at zero load flow rate, k_f as:

$$k_f = \partial f_s/\partial x \qquad (q_L = 0) \quad (27)$$

Combining Eqs.(20) and (27) gives k_f as:

$$k_f = 4C_d^2 \, B\cos 69^\circ \, p_s/C_c \quad (28)$$

Now, dividing Eq.(26) by $k_f u$ gives the dimensionless driving force F as:

$$F = KX + F_s(X) \qquad (q_L = 0) \quad (29)$$

Fig.4 shows the relation between the input force, spool displacement and the ratio of the positioner spring rate to the magnitude of the flow spring rate.

6. DISCUSSIONS

The flow inside the underlap spool servovalve is analyzed in details and the relationships of the flow force, the driving force, the flow spring rate and the flow damping coefficient were derived and studied.

The flow damping coefficient increases by increasing the port width, the supply port-centre land distance and the supply pressure and also by decreasing the ratio of the damping lengths. It may be negative, zero or positive depending on the value of the ratio of the damping lengths. The ratio of the damping lengths should be less than or equal to one for zero or positive damping in the valve.

The steady state flow force acting on the valve spool decreases by decreasing the port width, valve underlap and supply pressure. Also, it decreases by increasing the spool displacement and the load flow rate.

The input force required to actuate the valve spool increases by increasing the ratio of the positioner spring rate to the flow spring rate. It increases also by increasing the spool position if the ratio of the positioner to the flow spring rates is greater than or equal to one. The negative slope at X = 0 (see Fig.4) indicates that the origin has the potential of being an unstable equilibrium point if K is less than one. For the origin to be a stable position and the spool stays centred, the values of K must be greater than or at least equal to one.

The analysis presented in this paper is a part of a complete research work carried out at Cairo University to study in details the forces encountered in underlap, centred and overlap spool servovalves [6].

7. REFERENCES

1. Noton, G. and Turnbull,D., "Some Factors Influencing the Stability of Piston Type Control Valves", Proc. Instn. Mech. Engrs. , 172, 40, p.1065,(1958)
2. Blackburn,J., Reethof,G. and Shearer,J. "Fluid Power Control" , MIT Press and Wiley, (1960)

3. Merritt,H., "Hydraulic Control Systems", J. Wiley and Sons , (1967)
4. Streeter,V. and Wiley,E., "Fluid Mechanics", McGraw Hill Book Co., (1975)
5. Butler,R. "A Theoretical Analysis of the Response of a Loaded Hydraulic Relay", Proc. Instn. Mech. Engrs., 173, 16, p.429, (1959)
6. Farid,M.S., M.Sc. Thesis, Faculty of Engineering, Cairo University, (1984)

8. NOMENCLATURE

A_c, A_d, A_s = Area of control volume, drain and supply orifices, m^2
B = Port width, m
c = Viscous damping coefficient, Ns/m
C_c, C_d = Jet contraction and flow discharge coefficients
f, f_f = Input and flow forces, N
f_s, f_x = Steady state and axial component of flow force, N
F = Dimensionless steady state force
k, k_f = Positioner and flow spring rates, N/m
K = Ratio of the positioner to the flow spring rates
L = Ratio of damping lengths
L_d, L_s = Drain and supply ports-centre land distance, m
m_{cv}, m_s = Masses of control volume and spool, kg
N = Oil bulk modulus, N/m^2
p_d, p_L, p_s = Drain, load and supply pressures, N/m^2
P_L = Dimensionless load pressure
q_d, q_L, q_s = Drain, load and supply flow rates, m^3/s
S, S_1 = Switching functions
t = Time, s
u = Valve underlap, m
x = Spool displacement, m
X = Dimensionless spool displacement
ρ = Oil density, kg/m^3

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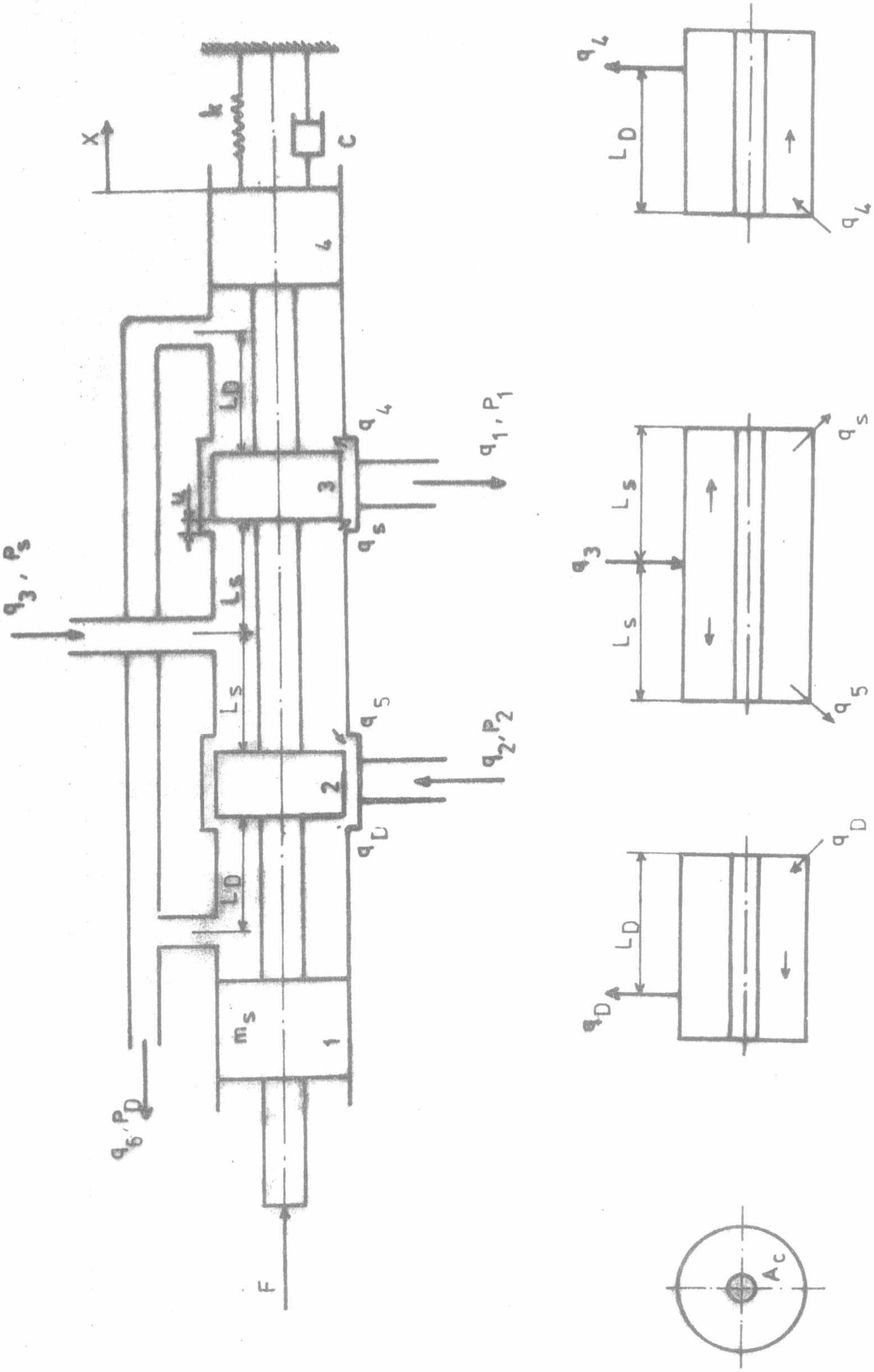

Fig.1 Underlap spool servovalve and control volumes

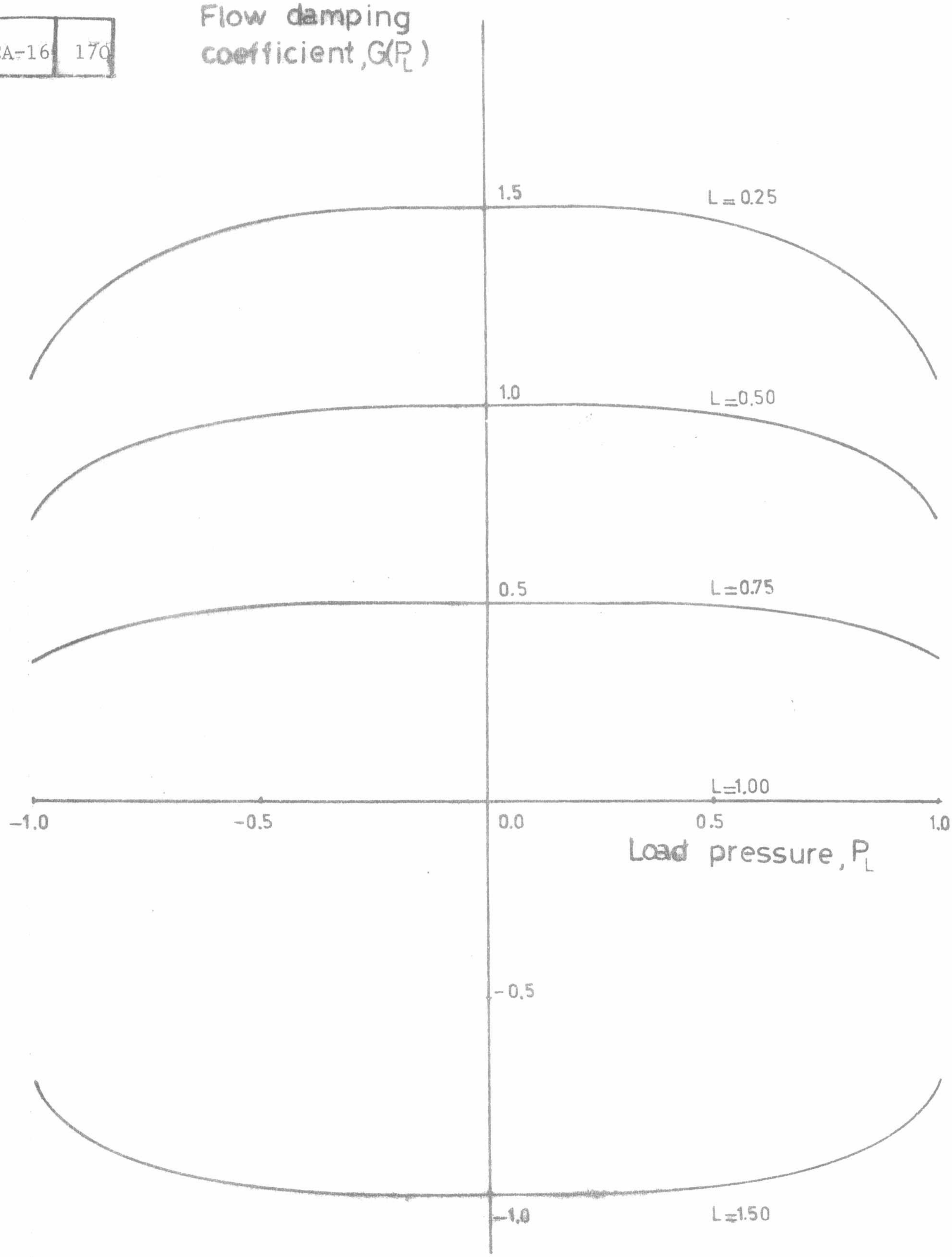

Fig. 2 Dimensionless flow damping coefficient in underlap region

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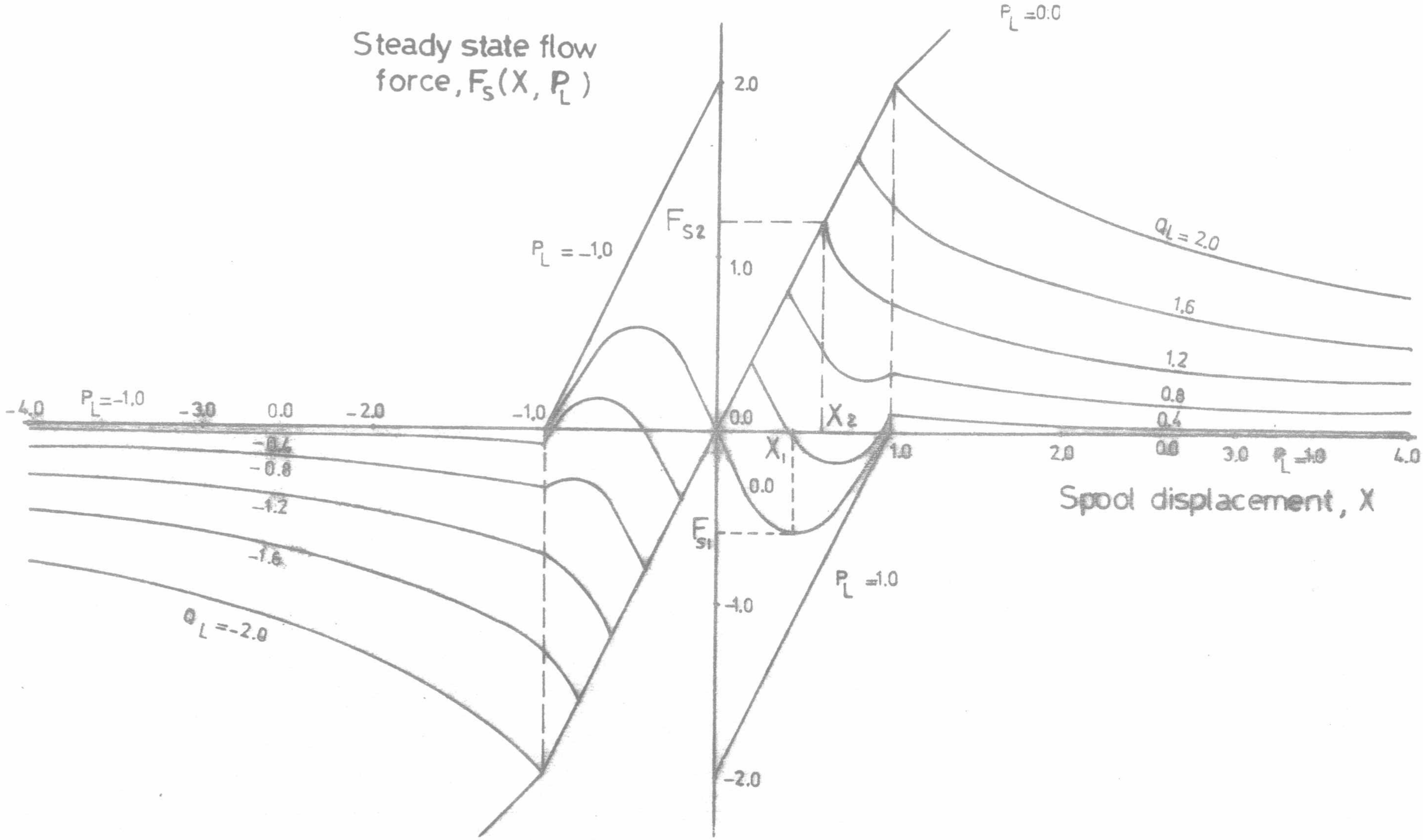

Fig. 3 Dimensionless static flow force for underlap servovalve

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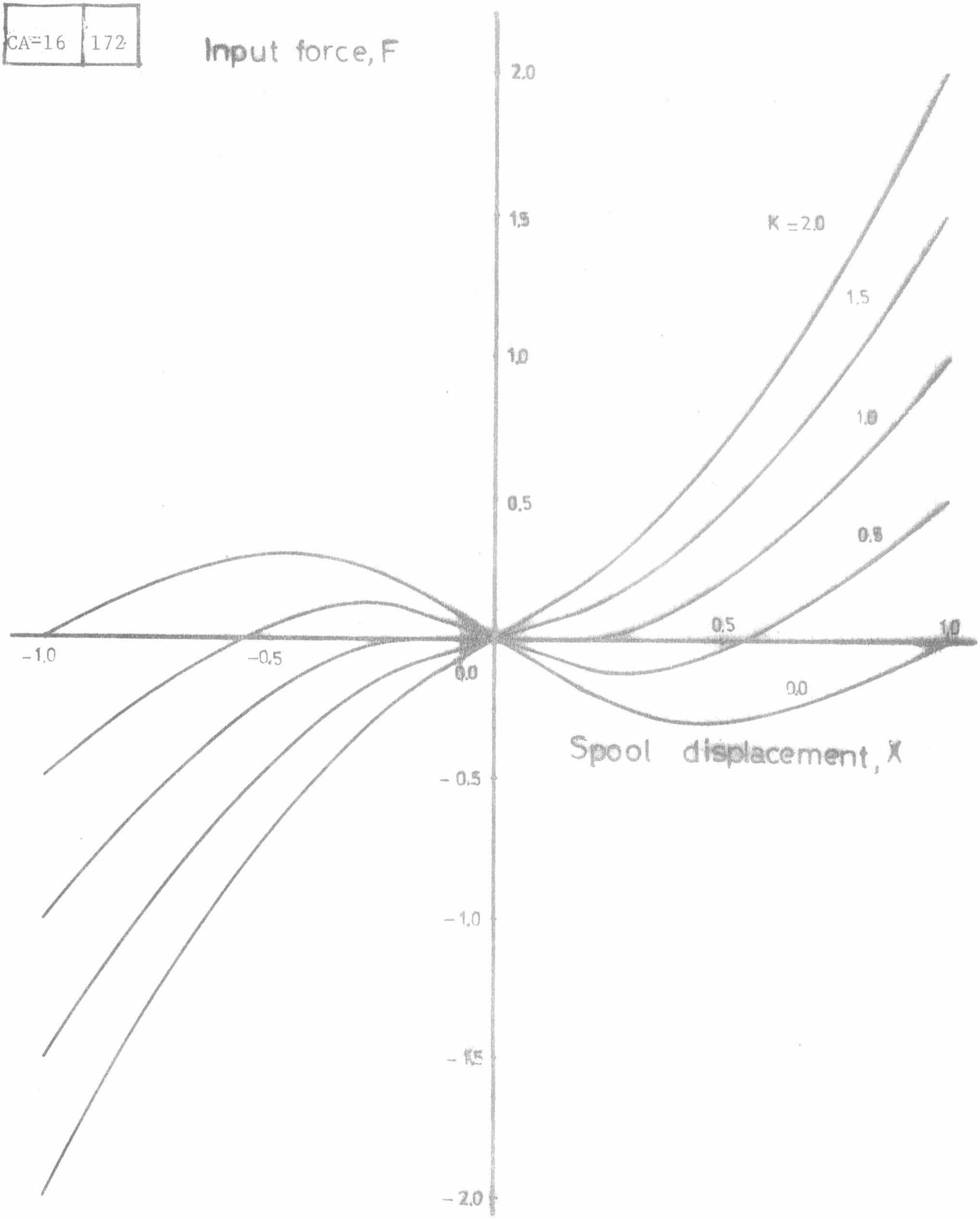

Fig.4 Dimensionless static positioner force for various positioner spring rates