



TOWARD THE DESIGN OF AN INTELLIGENT AEROSPACE
CONTROL SYSTEM

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ABSTRACT

The development of control technology has been directed toward the design of more accurate, more reliable, more adaptive and responsive control systems. To accomplish such goal some artificial intelligence features should be incorporated in the design. Information identification, recognition and sorting are the corner stones of artificial intelligence. Since the only intelligent creature is the human being, therefore we have to immitate his pattern of thinking to get a job done. The present article is an attempt to define the sequence of events carried out by a human brain to perform a certain task and to identify their mathematical methodology counterparts. It is believed that following the presented approaches an intelligent control system may be constructed.

Introduction

It has been recognized during the last decade that conventional control systems analysis by itself whether in the frequency domain, time domain or modal space will not be adequate to meet the demands of advanced and sophisticated aerospace systems. Due to the mathematical difficulties encountered when employing any approach of control methodologies, many idealistic and perfect conditions assumptions are to be made. These simplifying assumptions in some cases will alter considerably the desired performance of the controlled plant. Adaptive control techniques, robust controllers, self tuned controllers and the introduction of artificial intelligence logics in control schemes are significant steps forward toward the design of a reliable and accurate control system. An ideal control system should perform in an intelligent manner. It has to be firmly understood that intelligence is a feature given by GOD to humanbeings only. No man made system will ever match or duplicate human intelligence. All what we have to do is to try to represent human intelligence by a sequence of logic steps carried out by a normal humanbeing in order to accomplish a certain task, then we have to identify the

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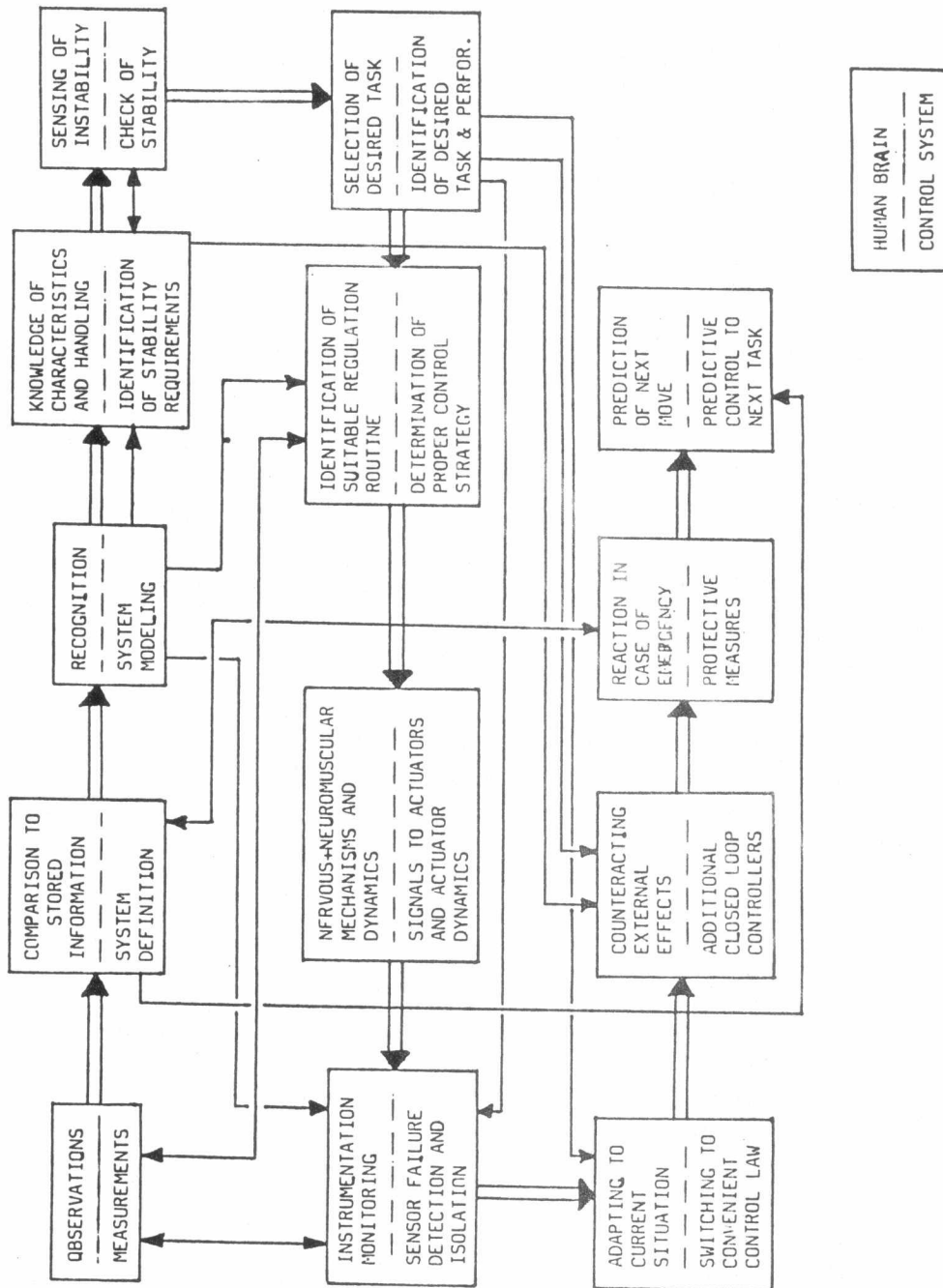


Fig. 1 Intelligent Sequence of Human Brain and Their Control Counterparts



suitable mathematical methodology that produces as close as possible the outcome of each human logic step. A control system incorporating such features is undoubtedly an intelligent control system. Fig. 1 shows the sequence of human logic steps and their mathematical methodologies counterparts. In the following sections the components of the proposed intelligent control system are introduced.

I. Measurements

The rigid body dynamics of aerospace vehicles are modeled by six degrees of freedom mathematical equations. Treatment of flexible effects requires additional mathematical representations. The basic measurement devices used are different types of gyroscopes, accelerometers, air data probes, star tracking probes, ..., etc.. For each sensor the functional relation between the measurements and the plant states together with the sensor noise is adequately represented by;

$$z = h(x) + v \quad (1)$$

In the above representation the stochastic characteristics of the noise vector should be described. If the noise is nonwhite gaussian a shaping filter should be constructed. Sensor location should be accounted for by making the proper transformations to the reference coordinate system used, Fig. 2. Sensor redundancy is usually applied (quad redundancy for most systems).

II. System Definition and Modeling

Accurate system definition and modeling is the corner stone for designing a reliable and effective control system. The full order state, control and parametric vectors should be carefully defined. Representation of the uncertainties by an additive noise vector whether colored or white gaussian does not yield in general true simulation of the plant stochastic characteristics. A better representation is achieved by considering the state vector to be composed of two parts, a deterministic part and a stochastic part, i.e.;

$$\dot{x} = f(x, p, u, t) \quad (2a)$$

$$x = x_d + x_s \quad (2b)$$

where x, p and u are the state, parametric and control vectors respectively, x_d is the deterministic part and x_s is the stochastic part of the state vector. Proper linearization of Eqn. (2) gives;

$$\dot{\bar{x}} = (F_d + F_s) \bar{x} + (G_d + G_s) \bar{u} \quad (3)$$

where \bar{x} and \bar{u} are the perturbed state and control vectors respectively, F_d and G_d are deterministic (nominal) coefficient matrices and F_s and G_s are stochastic coefficient matrices incorporating the stochastic characteristics of the uncertainties. Equations (2) and (3) are more realistic and accurate representation of the plant dynamics than the traditional additive noise representation.

For large as well as moderate scale systems, dividing the system into interconnected subsystems leads to more computational efficiency specially for regulating purposes. The overall system state and control vectors are composed of those of the different subsystems; i.e.

$$x^T = [x^{(1)T} \quad x^{(2)T} \quad \dots \quad x^{(N)T}] \quad (4a)$$

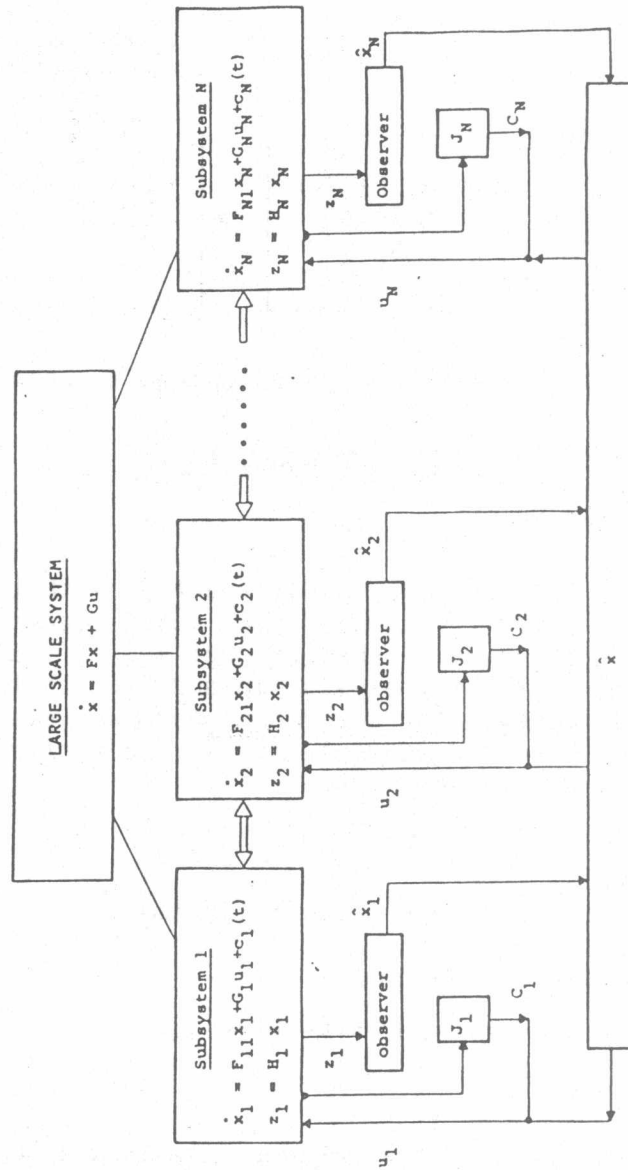


Fig. 3 The present concept for decentralized control of large scale system





coefficient matrices F and G. Due to uncertainties in system modeling and compensating such deficiency by random vectors with known characteristics, the system equations become stochastic. It is therefore necessary to estimate the mean of the state and the parametric vectors. This is done by means of Kalman filter for linear cases and Extended Kalman filter for nonlinear cases. The Kalman filter algorithm becomes an integrated part of the system mathematical model as shown in Fig. 4.

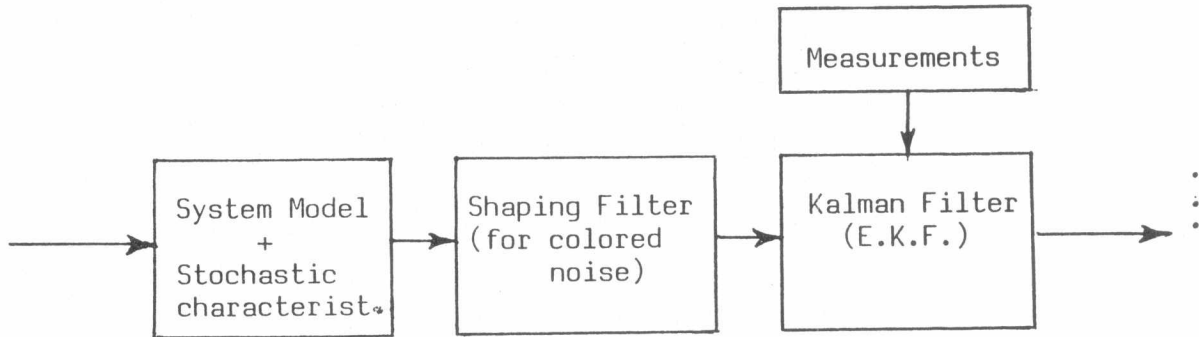


Fig. 4 The Integrated System Model for Stochastic Processes

III. Identification of Stability Requirements and Stability Check

Stability Analysis of nonlinear systems is different from that of linear systems. Also stability analysis for time varying coefficients is different from that of constant coefficients. In that respect identification of the system structure should precede the analysis. In addition to the above classification stability requirements for interconnected subsystems (decentralized control) are dealt with in a different manner. The system structures requiring different stability analysis are:-

- Nonlinear systems.
- Time invariant linear systems.
- Linear systems with time varying coefficients.
- Systems with time delays.
- Linear interconnected subsystems.

In the following each case will be discussed briefly;

Nonlinear Systems

A general nonlinear system may be represented by the vector differential equation;

$$\dot{x} = f(x, u, t) \quad ; \quad x_0 = x(t_0) \quad (7)$$

The stability of this nonlinear system may be checked indirectly by means of Lyapunov or Malkin methods. The powerful Lyapunov method starts with the choice of a positive Lyapunov function $V(t)$. Such choice should represent the stored energy of the system. The system will be stable if $\dot{V}(t) < 0$, critically stable for $\dot{V}(t) = 0$ and unstable for $\dot{V}(t) > 0$. In practice difficulties may arise due to unsuitable choice of the function $V(t)$ and also difficulties may be encountered at conditions where equilibrium points are



critical. Moreover, for some cases different choices of Lyapunov function $V(t)$ may result in different stability status (stable or unstable). A direct and computationally efficient method for checking the stability of nonlinear systems has been introduced by the author⁴. The nonlinear continuous system is first discretized by means of the fourth order Runge-Kutta numerical scheme. Sensitivity analysis for the state propagation is then performed. The resulting stability criterion is of a direct form. It states that a nonlinear autonomous system will be stable at the equilibrium point "r" if the eigen values of the matrix $(U_r + M_r)$ are confined within the central unit circle in the complex plane. The general element of the matrix U_r is given by;

$$u_{ij} = \exp[h(\partial f_i / \partial x_j)_r] - (1 - \delta_{ij}) \quad (8)$$

Matrix M_r is given by;

$$M_r = (h^2/6)E_r(N_r^{(1)} + N_r^{(2)} + N_r^{(3)}) \quad (9)$$

where E_r is an n-square matrix whose general element is;

$$\sum_{\ell=1}^n \partial^2 f_i / \partial x_\ell \partial x_j$$

Matrices $N_r^{(1)}$, $N_r^{(2)}$ and $N_r^{(3)}$ are n-dimensional diagonal matrices with diagonal elements respectively;

$$f_{ii}(x_r, t_r) \quad ; \quad f_{ii}(a_r, t_r + h/2) \quad ; \quad f_{ii}[x_r + f(a_r, t_r + h/2)/2; t_r + h/2]$$

where $a_r = x_r + hf(x_r, t_r)/2$

Time Varying Systems

There is no explicit solution for time varying systems. This type of systems has been investigated a great deal during the past decade. It has been shown that linear time varying systems can be unstable even if all of its eigen values are located on the left half of the complex plane^{5,6,7}. In the mean time a linear time varying system can be asymptotically stable even if some of its eigen values are located on the right half of the complex plane⁸. There exists an algebraic transformation such that any linear⁹ time varying system can be transformed into a linear time-invariant system. A linear time varying system represented by the equation;

$$\dot{x} = A(t)x + B(t)u \quad (10)$$

could be transformed to a linear time invariant system by the following transformation;

$$x = T\bar{x} \quad (11a) \quad ; \quad T = \Phi(t, t_0) \Psi^{-1}(t_0, t) \quad (11b)$$

where matrices Φ and Ψ are given by the equations;

$$\dot{\Phi} = A(t)\Phi \quad (12a) \quad ; \quad \dot{\Psi} = B(t)\Psi \quad (12b)$$



Systems with Time Delays

In practice time delays are encountered. Time delays may occur in state variables and /or in control variables. State variables time delays result from nonsynchronization of the different coupled dynamic modes of the system. In such cases the magnitude of time delays vary from one system to another. On the other hand control variables time delays result from control mechanisms and the time required by the processor (computer) to compute the proper control signals. In any case whether the time delays are attributed to state variables or to control variables or both, such delays should be accounted for when designing the control system and also when analyzing the system performance. Considerable changes in system dynamics and stability occur due to time delays. A stable nondelayed system may become unstable when time delays are included. The time delays become more profound for systems with fast response such as high performance aircraft. The Cooper-Harper pilot rating technique¹⁰ gives a very important indication and measure of the effect of time delays on the aircraft performance. The Cooper-Harper pilot rating scale is shown in Fig. 5.

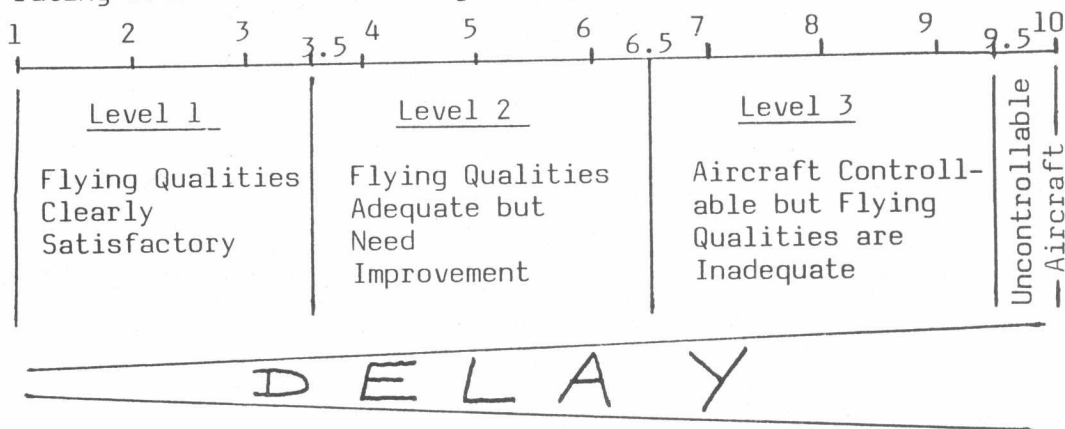


Fig. 5 Illustration for the Effect of Time Delay on the Cooper-Harper Pilot Rating Scale.

As seen the aircraft flying qualities change drastically as a function of time delays. Reference 11 presents an outstanding review of the different mathematical techniques dealing with time delays. It is found that for systems with state variables time delays good control quality may be achieved by using suboptimal control laws. However, for systems with control variables time delays the control quality is sensitive to the applied control law. A linear time invariant continuous system with multiple time delays in state and control variables is represented by;

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^n A_i x(t - \tau_{xi}) + B_0 u(t) + \sum_{i=1}^m B_i u(t - \tau_{ui}) \quad (13)$$

where τ_{xi} and τ_{ui} are delay times for the different state and control variables. When neglecting time delays, the above equation reduces to the nondelayed standard form. To show the effect of time delays on the system performance, the delayed state and control variables are expanded by Taylor's series as;

$$x(t - \tau_{xi}) = x(t) - \tau_{xi} \dot{x}(t) + (1/2!) \tau_{xi}^2 \ddot{x}(t) - \dots \quad (14a)$$



$$u(t-\tau_{ui}) = u(t) - \tau_{ui} \dot{u}(t) + (1/2!) \tau_{ui}^2 \ddot{u} - \dots \quad (14b)$$

In the above equations neglecting terms containing \ddot{x} , \dot{u} and \ddot{u} and substituting into Eqn.(13) yields;

$$\dot{x}(t) = [I + \sum_{i=1}^n \tau_{xi} A_i]^{-1} (\sum_{i=0}^n A_i) x(t) + [I + \sum_{i=1}^n \tau_{xi} A_i]^{-1} (\sum_{i=0}^m B_i) u(t) \quad \dots \dots \dots (15)$$

Equation (15) represents a linear time-invariant continuous system with modified state and control coefficient matrices. Such representation will hold whenever the inverse of the matrix appearing in the equation exists. Using any control methodology, it is seen from Eqn.(15) that the resulting control strategy when accounting for time delays is different from that when neglecting time delays.

Time Invariant Linear Systems

The stability of time-invariant system represented by the vector differential equation;

$$\dot{x} = Fx + Gu \quad (16)$$

is determined by the eigen values of the state coefficient matrix F . As well known the system will be stable if all the eigen values are located on the left half of the complex plane. On the other hand the stability of the discrete-time system represented by the equation;

$$x_{k+1} = \phi_k x_k + L_k u_k \quad (17)$$

is determined by the eigen values of the state transition matrix ϕ . As well known the system will be stable if the eigen values of the transition matrix ϕ are located inside a unit central circle in the complex plane.

For these type of systems the stability status as well as the stability margins are well defined.

Linear Interconnected Subsystems

For this type of system structure application of Lyapunov's concept yields a direct stability criterion for the interconnected subsystems^{1,2}. Based on the subsystems representation given by Eqns.(4)-(6) a Lyapunov function for the i th. subsystem is chosen as;

$$V(t) = x_i^T S x_i + \sum_{k=1}^{N-1} x_k^T \theta_k x_k \quad (18)$$

where matrices S and θ are semipositive definite symmetric matrices governed by matrix Ricatti equations derived for the decentralized control law. Substitution of the feed back decentralized control law into Eqn. (5) and lumping all subsystems other than the i th. together yields;

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} X^{(1)} & X^{(2)} \\ \Psi^{(1)} & \Psi^{(2)} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (19)$$



where $y^T = [x_1^T \quad x_2^T \quad \dots \quad x_{i-1}^T \quad x_{i+1}^T \quad \dots \quad x_N^T]$

The interconnected subsystems will be stable if the following matrices are negative definite;

$$(\chi^{(1)} + \chi^{(2)})_M \quad \text{and} \quad (\psi^{(1)})_{M+\psi^{(2)}}$$

where $M = \phi^{(4)}(t, t_0) \phi^{(2)-1}(t, t_0)$

$\phi^{(2)}$ and $\phi^{(4)}$ are submatrices of the transition matrix relevant to those appearing in Eqn.(19).

Stability criteria establish bounds for stable system operation. In aerospace systems nominal flight conditions are defined by solving the original non-linear differential equations governing the system trajectory or dynamics. Such solution can only be achieved by a numerical scheme. The continuous equations thus have to be discretized. State and parametric values at each discrete nodal point from the equilibrium condition for stability check at this particular time instant. Continuous stability check during system operation is therefore required to limit the response to the stable domain. The combined stability of both system dynamics and Kalman filter (for assumed stochastic process) should be checked. Kalman filter will be stable (convergent estimate) when all the eigen values of matrix $(F-KH)$ are located in the left half complex plane, where K is the Kalman gain matrix. The combined system-filter closed loop dynamics for a feed back control signal $u = -Cx$ is determined by the combined coefficient matrix;

$$\begin{pmatrix} F & -GC \\ 0 & (F-KH-GC) \end{pmatrix}$$

The combined system-filter will be stable when all the eigen values of the above matrix are located in the left half complex plane.

IV. Identification of Desired Task and Performance

The desired task identification means specifying the states (position, velocity, attitude, ..., etc.) of the aerospace vehicle to be reached after a fixed (or free) interval of time. Identification of the desired performance for the given task means that certain constraints such as acceleration limits, damping quality, stability margin, assigned trajectory, ..., etc. are to be satisfied. Such identification leads to the choice of the convenient control methodology and strategy. The importance of task and performance identification is dictated by the fact that there is no unifying control methodology capable of achieving the desired requirements. For tasks with specified constraints (equality or inequality) the optimal control approach (LQ, LQG, nonlinear) is most appropriate. For model or trajectory tracking, the model following techniques are more convenient. For stability and handling qualities requirements pole assignment and configuration control methodologies are to be employed. In all cases robustness analysis should be performed to yield a more reliable and accurate controller.

V. Determination of Control Strategies

As mentioned earlier the choice of the control methodology to be employed depends upon the task to be achieved. The different optimal control problems of Mayer, Bolza and Lagrange as well as the regulator problem (LQ and LQG)



the Hamiltonian of the system then enforcing the optimization necessary and sufficient conditions thus yielding the costate equations for the Lagrange multipliers and the optimal control law. In the presence of equality or inequality constraints and /or free terminal time, additional conditions have to be satisfied¹³. Though the obtained optimal control law justifies the optimality requirements, yet it can not provide the desired transient characteristics (stability margins and damping quality). In some cases the obtained optimal control gains could not be realized in practice. However, it has been shown that in cases where both state and control vectors have the same dimensions, the optimal control law may provide desired transient dynamics (closed loop eigen values)¹⁴. This may be achieved by partitioning the state vector into directly controllable state subvector and a non-directly controllable state subvector. This approach has proven its validity for control configured vehicles (CCV) such as the the Advanced Fighter Technology Integration F-16 aircraft (AFTI/F-16)¹⁵. The state vector and the state and control coefficient matrices of the linearized system are partitioned in the form;

$$x^T = [x_c^T \quad x_{nc}^T] ; \quad F = \begin{bmatrix} F^{(1)} & F^{(2)} \\ \text{-----} & \\ F^{(3)} & F^{(4)} \end{bmatrix} ; \quad G = \begin{bmatrix} G \\ \text{---} \\ 0 \end{bmatrix}$$

The linearized state equation may therefore be written in the decomposed form;

$$\dot{x}_c = F^{(1)} x_c + G_c u + c(t) \quad (20a)$$

$$\dot{x}_{nc} = F^{(4)} x_{nc} + d(t) \quad (20b)$$

where $c(t) = F^{(2)} x_{nc}$; $d(t) = F^{(3)} x_c$

and the quadratic performance index to be minimized is;

$$J = \frac{1}{2} \int_{t_0}^{t_f} (x_c^T A x_c + u^T B u) dt \quad (21)$$

The closed loop eigen values are not sensitive to the control weighting matrix B, however they are very sensitive to the state weighting matrix A. Matrix B will therefore be assigned and matrix A will be calculated to yield control gains with desired closed loop eigen values.

Pole placement techniques are more convenient for designing control systems dealing with the maneuverability, performance and handling qualities of the aircraft. During the past decade several pole assignment techniques have been published. A certain degree of mathematical and implementation difficulties are encountered in most approaches, specially for on-line computations. Severe limitations on eigen values shifts are placed particularly for coupled multi-input, multi-output systems. A powerful and computationally efficient methodology in that respect has been presented by the author¹⁶⁻¹⁷⁻¹⁸ which relaxes most constraints imposed on other techniques. The method is based on finding a direct relation between the eigen values shifts (difference between closed loop and open loop eigen values) and the corresponding changes in the system coefficient matrices (closed and open loop coefficient matrices). Shifts in all eigen values in any direction are admissible except for the case of repeated closed loop eigen values which in fact is a trivial one. In addition to conventional controllers the method



for nonconventional configuration control of the aerospace vehicles. The key equation for this methodology is;

$$\Delta f = -D^* [c^0 + \text{Re}(P_c^{-1} g_c)] \quad (22)$$

where D^* is the pseudo-inverse of matrix D ; i.e. $D^* = (D^T D)^{-1} D^T$, $\text{Re}(\cdot)$ indicates the real part of the expression between brackets. f is a vector whose elements are those of the open loop coefficient matrix which are allowed to vary. c^0 is a vector whose elements are the coefficients of the open loop characteristic equation. Matrices D and P_c and vector g_c are given by;

$$D = \begin{pmatrix} \frac{\partial c_{n-1}^0}{\partial f_{11}} & \frac{\partial c_{n-1}^0}{\partial f_{12}} & \dots & \frac{\partial c_{n-1}^0}{\partial f_{nn}} \\ \frac{\partial c_{n-2}^0}{\partial f_{11}} & \frac{\partial c_{n-2}^0}{\partial f_{12}} & \dots & \frac{\partial c_{n-2}^0}{\partial f_{nn}} \\ \vdots & \vdots & & \vdots \\ \frac{\partial c_0^0}{\partial f_{11}} & \frac{\partial c_0^0}{\partial f_{12}} & \dots & \frac{\partial c_0^0}{\partial f_{nn}} \end{pmatrix}$$

$$P_c = \begin{pmatrix} (e_1^c)^{n-1} & (e_1^c)^{n-2} & \dots & e_1^c & 1 \\ (e_2^c)^{n-1} & (e_2^c)^{n-2} & \dots & e_2^c & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ (e_n^c)^{n-1} & (e_n^c)^{n-2} & \dots & e_n^c & 1 \end{pmatrix}$$

$$g_c^T = [(e_1^c)^n \quad (e_2^c)^n \quad \dots \quad (e_n^c)^n]$$

where e_i^c ; $i=1, \dots, n$ are the desired closed loop eigen values.

This configuration control methodology is valid whenever D exists. Such condition can easily be realized in practice. It has to be mentioned here that this control method is most convenient for designing control systems for advanced CCV fighter aircraft.

One of the most needed concepts for guidance and control of aircraft and missiles is that of model tracking (model following; model matching). For the final landing approach of aircraft, for precise aircraft or missile navigation or for intercepting missiles a specified nominal trajectory and/or conditions are to be tracked (followed or matched) by the actual vehicle in order to achieve certain performance requirements. Many model following and matching techniques have been published in recent years. The linearized dynamics of the vehicle and the model are given by;

$$\text{Vehicle} \quad \dot{x} = Fx + Gu + w \quad ; \quad w \sim N(0, Q) \quad (22a)$$

$$z = Hx + v \quad ; \quad v \sim N(0, R) \quad (22b)$$

$$\text{Model} \quad \dot{x}_m = F_m x_m + G_m u_m + w_m \quad ; \quad w_m \sim N(0, Q_m) \quad (23a)$$



$$z_m \cong H_m x_m + v_m ; \quad v_m \sim N(0, R_m) \quad (23b)$$

An efficient model tracking method has been published¹⁹. It is based on forming the output dynamics of the vehicle and of the model. The dynamics of the output difference $\delta z = z - z_m$ is obtained as;

$$\dot{\delta z} = AN^{-1} \delta z + B \delta v + HGu + (AN^{-1} - A_m N_m^{-1}) z_m + (B - B_m) v_m - H_m G_m u_m + \mu \quad (24)$$

where in the above equation all matrices are defined, $\delta v = v - v_m$ where v and v_m are the nonobservable subvectors of state for the vehicle and for the model respectively. μ is a white gaussian noise vector. The state and control coefficient matrices for the vehicle and for the model are partitioned as;

$$F = \begin{bmatrix} \underbrace{F^{(1)}}_p & \underbrace{F^{(2)}}_{(n-p)} \\ \underbrace{F^{(3)}}_p & \underbrace{F^{(4)}}_{(n-p)} \end{bmatrix} ; \quad G = \begin{bmatrix} \underbrace{G^{(1)}}_m \\ \underbrace{G^{(2)}}_{(n-p)} \end{bmatrix}$$

where n is the dimension of the state vector and p is the dimension of the observation (output vector). Equation for the difference vector δv is then obtained as;

$$\delta \dot{v} = F^{(4)} \delta v + F^{(3)} N^{-1} \delta z + G^{(2)} u + (F^{(4)} - F_m^{(4)}) v_m + (F^{(3)} N^{-1} - F_m^{(3)} N_m^{-1}) z_m - G_m^{(2)} u_m + \mu^* \quad (25)$$

Forming the n -dimensional vector ξ such that;

$$\xi^T = [\delta z^T \quad \delta v^T]$$

The control strategy for optimal model tracking is obtained by minimizing the quadratic performance index;

$$J = \frac{1}{2} E \int_{t_0}^{t_f} (\xi^T Q^* \xi + u^T R^* u) dt$$

The resulting control law for optimal model tracking is;

$$u = -C_1 z - C_2 v + C_3 z_m + C_4 v_m + C_5 u_m \quad (26)$$

where C_1 to C_5 are defined gain matrices.

It is seen that this control law is a feedbackward with respect to vehicle information and a feedforward with respect to model information. The mean of the nonobservable subvector of state \hat{v} is computed by the reduced order estimator;

$$\hat{v} = F^{(4)} \hat{v} + F^{(3)} N^{-1} z + G^{(2)} u \quad (27)$$

The highlights of the main control strategies suitable for aerospace systems has been presented in this section. As stated earlier each control concept



has its merits and the applicability of each depends upon the desired outcome of the control system. In addition to the above mentioned control methodologies modal control approaches are also quite useful and they are used as supplementary techniques to the above main concepts. Reduced order control methods have received great attention in recent years specially when dealing with large scale systems. Predictive control also is a suitable technique for terrain following²⁰ and for missile guidance. To conclude, we emphasize the fact that control system analysis has just started to take shape and the field is still widely open for intuitions, innovations and development in conventional and nonconventional ways.

VI. Sensor Failure Detection and Isolation

Measurements are essential in the operation of any control system. They could be used directly for output feedback control or they could be used for state and parameter estimation in case of state feed back control. In all situations accurate measurements are needed to compute the required control signals. Sensors operational reliability must therefore be very high. In aerospace applications utilization of redundant sensors and processors is a common practice. All precautions should be taken to ensure receiving an observation signal as accurate as possible at all times. Several software as well as hardware techniques have been developed in that direction. The elements of a software sensor failure detection and isolation technique is described in this section²¹⁻²². The basic idea is to construct an ideal observation trajectory describing the measurements history of the given plant. The vehicle dynamics and its measurements are represented in discrete-time form as;

$$x_{k+1} = \Phi_k x_k + L_k u_k + w_k \quad ; \quad w_k \sim N(0, Q) \quad (28a)$$

$$z_k = H_k x_k + v_k \quad ; \quad v_k \sim N(0, R) \quad (28b)$$

An ideal observation trajectory is derived as;

$$z_{k+1} = \Psi_k^* z_k + \Gamma_k v_k + L_k^* u_k + \epsilon_k \quad (29)$$

where all matrices are defined, v is the unmeasured subvector of state and ϵ is the observation dynamics noise vector which is a combination of both the process and measurement noise vectors. Denoting the actual measurements by superscript "m" and the difference between the ideal and actual measurement vector by $\Delta z = z - z^m$, the following difference propagation equation may be obtained;

$$\Delta z_{k+1} = \Psi_k^* \Delta z_k + \mu_k + \epsilon_k \quad (30a)$$

$$\mu_k = \Gamma_k v_k + L_k^* u_k + \Psi_k^* z_k^m - z_{k+1}^m \quad (30b)$$

From the above equations it is seen that for ideal measurements $\mu_k + \epsilon_k = 0$. Therefore the measurements error (deviation between ideal and actual measurements) is generated by the sensors inaccuracy and the combined process and measurements noise vectors. The allowable deviations in measurements for sensors in the operational mode may be determined by the nonzero minimum of the input μ_k and the output Δz_k . The following quadratic performance index is therefore minimized;



$$J = \frac{1}{2} E \sum_{k=0}^{N-1} (\Delta z_k^T A_k \Delta z_k + \mu_k^T B_k \mu_k) \quad (31)$$

The propagation equation for the allowable measurement error covariance matrix is then obtained as;

$$\Pi_{k+1} = (I + B_k^{-1} S_{k+1})^{-1} \Psi_k^* \Pi_k \Psi_k^{*T} (I + B_k^{-1} S_{k+1})^{-1T} + R_k^* \quad (32)$$

Equation for the propagation of the actual measurement error covariance matrix Π^a is also obtained. The sensor failure detection criterion is therefore derived as;

$$\Delta \Pi_k = (\Pi_k + W_k) - \Pi_k^a \quad (33)$$

where W is a positive definite symmetric matrix that accounts for computations round off errors.

The i th. sensor will be considered in the failure mode if $\Delta \pi_{i,j} < 0$. The failed sensor/s isolation process is given in detail in references 24 and 22. The block diagram for sensor failure detection and isolation is shown in Fig. 6.

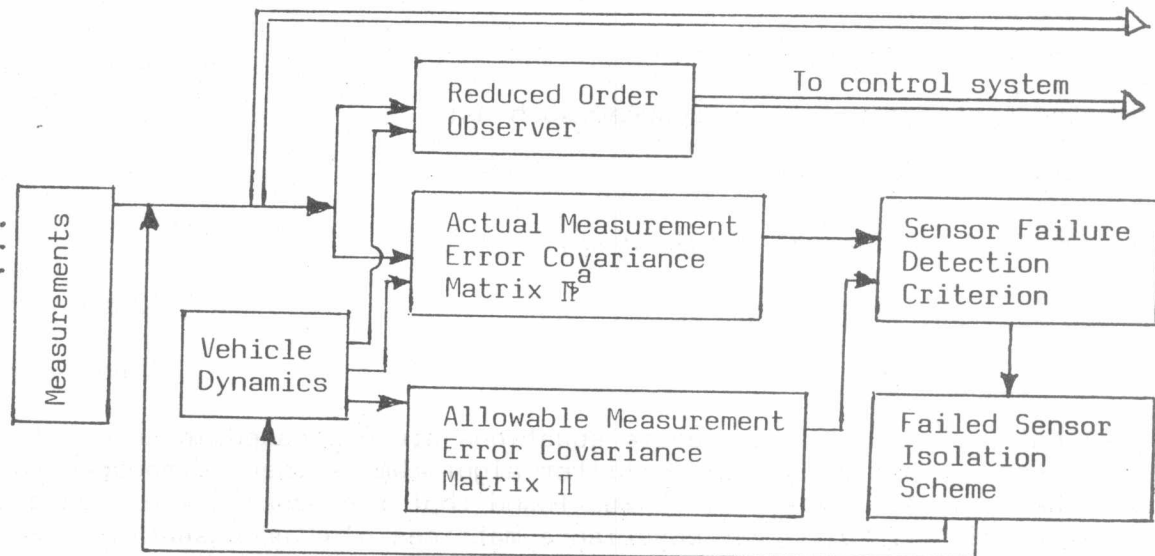


Fig. 6 Sensor Failure Detection and Isolation Block Diagram

VII. Adaptive Control and Robustness

Inaccuracy in system representation arises due to two factors, (i) simplified mathematical modeling and (ii) uncertainty in parameter values. Such inaccuracy in system representation results in inaccurate control signals. Restructuring the system model by measuring its response for a specified input is one way of improving the system representation. However, due to mathematical complexity such an approach could not be implemented on-line. On the other hand, uncertainties in parameters may be accounted for by employing an adaptive control technique. In such a technique, a parameter estimation algorithm is included. The parameters are estimated at each control interval and the updated values are used to compute the control signals. The mathematical basics for such an approach may be described by considering the nonlinear discrete-time



system given by²³;

$$x_k = (x_{k-1}, \xi, u_{k-1}) + w_k \quad ; \quad w_k \sim N(0, Q_k) \quad (34a)$$

$$z_k = h(x_k) + v_k \quad ; \quad v_k \sim N(0, R_k) \quad (34b)$$

ξ is an r -dimensional parametric vector whose nominal value ξ is known a priori. During system operation the parametric vector changes from its nominal value. It is required to estimate both the system state vector and the actual parametric vector at each time interval. The feed back control gain matrices depend on the updated value of the parametric vector. Denoting the estimate of the parametric vector at step $k-1$ by ξ_{k-1} , expansion and linearization of Eqns.(34) lead to;

$$s_k = \Phi_{k-2} s_{k-1} + (\Pi_{k-1} - \Pi_{k-2}) \gamma_{k-1} + U_{k-2} c_{k-1} + v_k \quad (35a)$$

$$y_k = H_{k-1} s_k + \mu_k \quad (35b)$$

where $s_k = x_k - x_{k-1}$; $\gamma_{k-1} = \xi - \xi_{k-1}$; $c_{k-1} = u_{k-1} - u_{k-2}$

$y_k = z_k - z_{k-1}$; $\Phi = \partial\phi/\partial x$; $\Pi = \partial\phi/\partial\xi$

$U = \partial\phi/\partial u$; $H = \partial h/\partial x$

An augmented state vector is introduced such as;

$$\eta^T = [s^T \quad \gamma^T]$$

and an augmented equation is obtained as;

$$\eta_k = A_{k-2} \eta_{k-1} + B_{k-2} c_{k-1} + L v_k \quad (36a)$$

$$y_k = D_{k-1} \eta_k + \mu_k \quad (36b)$$

where all matrices given in the above equations are expressed in terms of previously defined matrices. Kalman filter algorithm is then introduced to estimate the augmented vector η . It was found that the cross correlated state and parametric vectors error covariance matrices play an essential role in the updating process for the parametric vector estimation. The control system that incorporates a recursive parameter estimation algorithm is an adaptive control system since the control gain matrices are computed based on the updated value of the parametric vector.

Robust control system is that control system which is insensitive to parameter variations. The transient response of a linear system is defined by means of the eigen values and eigen vectors of the coefficient matrix. The eigen values and consequently the eigen vectors depend on the parametric vector. Any variation in the parametric vector will result in changes in eigen values and eigen vectors, i.e. result in different transient response (dynamics) of the system. The control system that will provide preassigned transient characteristics regardless of the changes in the parametric vector is known as robust control system. The design elements of a robust control system is given by considering the following²⁴.

$$\dot{x} = A(\alpha)x + B(\alpha)u \quad (37a)$$

$$y = C(\alpha)x \quad (37b)$$



where α is a scalar parameter with nominal value α_0 .
 An output feed back controller is described by the closed loop state equation

$$\dot{x} = A(\alpha) x \quad (38a)$$

$$A(\alpha) = A(\alpha) + B(\alpha)KC(\alpha) \quad (38b)$$

where K is the output feed back gain matrix.

The problem may be posed as, the determination of matrix K to achieve desired transient response. For distinct eigen values of matrix $A(\alpha_0)$ the following relation holds;

$$p_i^T A(\alpha_0) = \lambda_i p_i^T \quad (39)$$

where λ_i is the i th. eigen value and p_i is the corresponding i th. row eigen vectors of $A(\alpha_0)$. Perturbations in α cause perturbations in matrices A, B and C and hence perturbations in the eigen values and the eigen vectors of matrix A . Such perturbations are expressed by the Jacobi formula;

$$d\lambda_i = p_i^T dA(\alpha) v_i \quad ; \quad i=1, \dots, n \quad (40)$$

where v_i is the i th. eigen vector corresponding to λ_i . The variation of matrix $\bar{A}(\alpha)$ given by Eqn.(38b) is;

$$\begin{aligned} d\bar{A}(\alpha_0) &= [(dA/d\alpha)_{\alpha_0} + (dB/d\alpha)_{\alpha_0} KC(\alpha_0) + B(\alpha_0)K(dC/d\alpha)_{\alpha_0}] d\alpha \\ &= [\delta A + \delta BKC + BK\delta C] d\alpha \end{aligned} \quad (41)$$

Substitution of Eqn.(41) into Eqn.(40) shows that sufficient conditions for the eigen values and the corresponding eigen vectors of matrix $A(\alpha_0)$ to remain insensitive to small parameter variations is;

$$[\delta A + \delta BKC] v_i = 0 \quad (42a); \quad \delta C v_i = 0 \quad (42b)$$

These normality conditions are to be satisfied to ensure the controller robustness. It has to be noticed that further analysis is still needed to obtain applicable criteria for robust controllers.

VIII. Conclusion

An attempt is made in the present article to focus on the key issues to be considered for achieving the goal of designing an intelligent control system. Brief rather than detailed description has been given for the needed methodologies. The designer's task is to connect these information and the sequence of computations in a workable algorithm.

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