MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

1

2

A GENERAL PROCEDURE FOR INVESTIGATING THE INSTANTANEOUS KINEMATICS OF MULTI-LOOP MECHANISMS

3

MAHER GABER MOHAMED*

4

ABSTRACT

The screw theory is applied to the study of the instantaneous kinematics of multi-loop mechanisms. A general procedure for investigating the instantaneous kinematics of multi-loop mechanisms is introduced using only better interpretation and implementation of the two fundamental laws of instantaneous kinematics; series and parallel laws. The procedure is applicable to any mechanism specially to that type of practical mechanisms with pre-specified actuated joints. It neither requires special cases to be identified nor requires any more laws to be introduced. Numerical example is presented.

INTRODUCTION

The study of instantaneous kinematics of kinematic chains was pioneered by Waldron [1]. He introduced two fundamental laws, best known as the series and parallel laws. Later Davies and Primrose in [2] and Baker in [3] pointed out that these two laws were inadequate for determining the relative freedom between two bodies, or links, divided by cross-jointing within the context of Fig. 1. Therefore, a third law was introduced. Moreover, Davies and Primrose make no effort for computing the magnitudes of the twists involved which allows them to use a projective space in their exposition. Baker in [3] used the motor notation of screw system algebra in computing the magnitudes of the twists. However, the procedure introduced in [3] does require special cases to be identified.

Davies [4] had to adapt the graph theoretic formulation of Kirchhoff's circulation law to create the constraint equations for multi-loop kinematic chains with mobility M using screw motor notation. The instantaneous kinematics of the chain is then described by solving these constraint equations in terms of a certain set of M velocity scalars. These M velocity scalars had then to be associated with the input actuated joints. However in practice, the location of the input actuators is

*Lecturer, Mechanical Engineering Department, Faculty of Engineering and Technology, El-Minia University, El-Minia, Egypt.

usually prespecified in advance and not to be chosen after and according to the analysis. Furthermore, in this procedure one has to solve the constraint equations and compute the magnitude of the scalars associated with all joints in order to compute the required twists. The author introduced in [5,6] a direct procedure for determining and describing the instantaneous kinematics of fully-parallel multi-loop mechanisms with prespecified actuated joints that specially oriented for robotic uses using expanded algebra of screws.

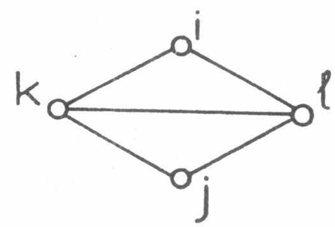

Fig. 1 Coupling graph of a kinematic chain with cross-jointing.

With only a better understanding of the two fundamental laws; series and parallel laws and without introducing any new laws, a general procedure is here introduced for the investigation of the instantaneous kinematics of multi-loop mechanisms with prespecified actuated joints.

SCREW ALGEBRA

A screw is defined by Ball [7] as a straight line in space with which a definite scalar called the pitch, h, is associated. The straight line is called the screw axis. A screw can be uniquely specified by five independent quantities (coordinates), four for the axis and one for the pitch. If (L,M,N ; P,Q,R) are the six plücker coordinates of the screw axis, then the screw coordinates are (L,M,N ; P+hL, Q+hM, R+hN).

The general instantaneous motion of body i relative to body j is a combination of an instantaneous rotation $\underline{\omega}_{ij}$ about an instantaneous screw axis $\underline{S}_{ij}$ (ISA) and an instantaneous translation $\underline{V}_{ij}$ along the axis (Fig. 2). Such instantaneous motion can be expressed by the dual vector,

$$\hat{\$}_{ij} = \omega_{ij}\, \$_{ij} = \omega_{ij}[\underline{S}_{ij} ; \underline{S}^{\circ}_{ij}]$$

where

$$\underline{S}^{\circ}_{ij} = (\underline{r}_{ij} \times \underline{S}_{ij}) + h_{ij}\, \underline{S}_{ij} .$$

Ball called this motion a twist $\hat{\$}_{ij}$ of amplitude $\underline{\omega}_{ij}$ about a unit screw $\$_{ij}$. Generally, the instantaneous relative motion of two bodies connected by a joint of connectivity n may take place and can be expressed as a twist about any ISA which is a member of the screw system, of order n, of that joint. A screw system of order n is defined as a vector space formed by all possible linear combinations of n independent instantaneous screw axes. The order of the screw system of a joint is the connectivity of that joint.

If $\underline{\omega}_{ij} \neq o$ and $h_{ij} = o$, i.e., the joint connecting bodies i and j is a revolute joint, the motion reduces to an instantaneous rotation which can be expressed as a scalar multiple of a unit line vector, and

$$\underline{\omega}_{ij}\, \$_{ij} = \omega_{ij} [\underline{S}_{ij} ; \underline{r}_{ij} \times \underline{S}_{ij}] .$$

Also if $\omega_{ij} = o$ and $h_{ij} = \infty$, i.e., the joint is a prismatic joint, the motion reduces to an instantaneous translation along the direction $\underline{S}_{ij}$

with the linear velocity $\underline{V}_{ij}$. This motion is a scalar multiple of a free vector,

$$V_{ij}\,[\,\underline{o}\ ;\ \underline{S}_{ij}\,].$$

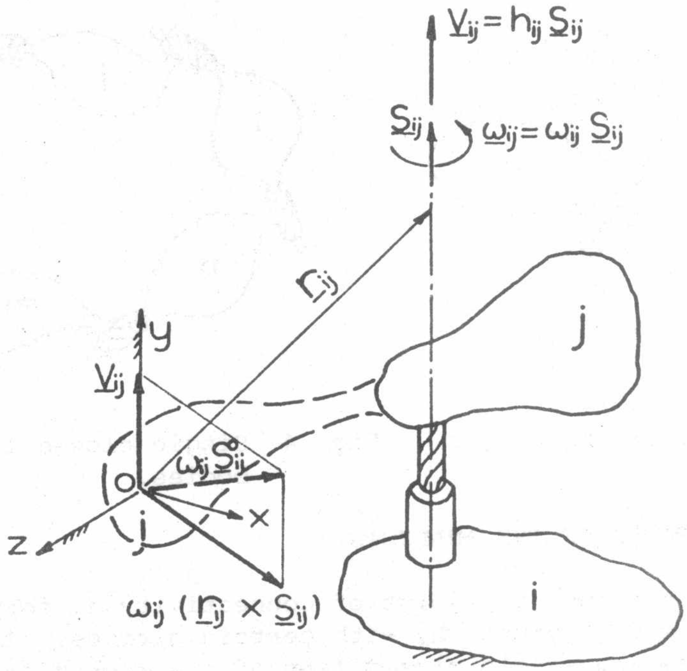

Fig. 2 General relative motion of bodies i and j.

SERIES AND PARALLEL LAWS OF INSTANTANEOUS KINEMATICS

The instantaneous motion of body k relative to body i as shown in Fig. 3 is determined by the screw system of the complex joint between them. This resultant Instantaneous Screw System (ISS) is formed by screws of the two screw systems of both simple joints simultaneously, and can be expressed as,

$$S_{ik} = S_{ij} + S_{jk}$$

where S_{ij} is the ISS of joint ij connecting bodies i and j, and the "+" sign means the sum or jointing of the two ISS's. Furthermore, in a closed loop of rigid bodies successively connected as shown in Fig. 4, the concept of the series law can be expressed as follows,

$$S_{ij} + S_{jk} + \cdot\cdot\cdot + S_{mn} + S_{ni} = o$$

which will be called the "loop equation" version of the series law.

If a coupling chain C is the union of couplings C_1, C_2, ... that have only bodies i and j in common as illustrated in Fig. 5, the relative instantaneous motion of i and j is subjected to the constraints of both complex joints between them simultaneously. Motions can now only occur about the ISA's which are members of the screw systems of both joints, and

$$S^{c}_{ij} = S^{c1}_{ij} \cap S^{c2}_{ij} \cap \cdot\cdot\cdot$$

where "$\cap$" means the intersection of the ISS's,

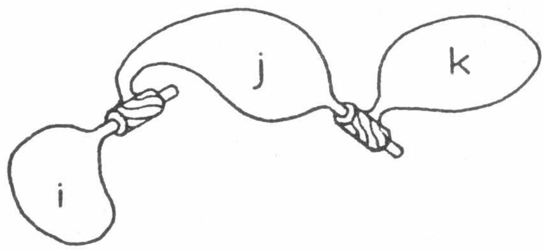

Fig. 3 The concept of the series law.

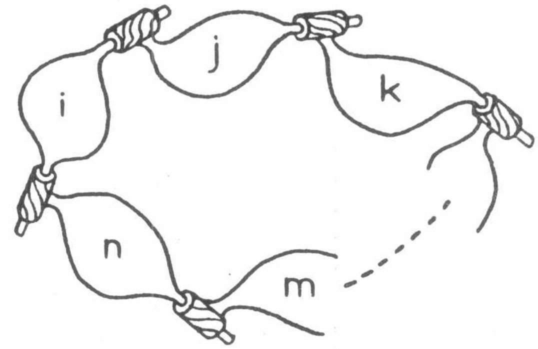

Fig. 4 Single closed loop of rigid bodies.

THE PROCEDURE OF THE ANALYSIS

It is important to consider any joint of connectivity n, for $n \geqslant 1$, as a combination of n screw joints, H, with certain pitches. It is now necessary to designate a single closed loop of the mechanism which contains the frame as loop 1 (see Fig. 6). Start the analysis with that loop as follows.

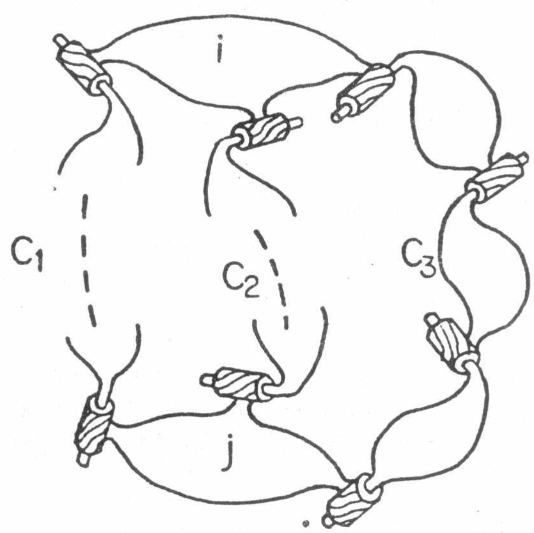

Fig. 5 The concept of the parallel law.

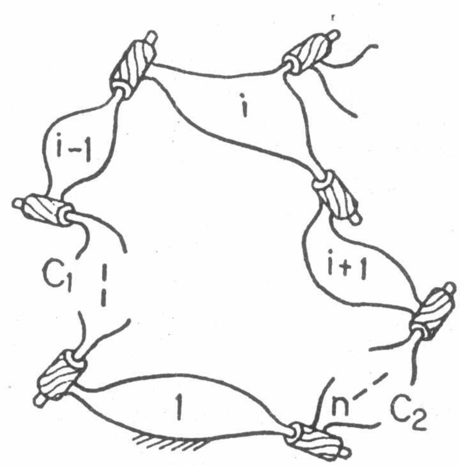

Fig. 6 Loop 1, contains the frame.

Using only the series and parallel laws, as discussed earlier, the screw system representing all possible motion of a body (or link) i in loop 1 that is connected to other loop(s) of the given multi-loop mechanism is determined. This is to be done as follows.

6

a- The concept of the loop equation can now be applied on loop 1. Writing the loop equation in a matrix form, the unknown ω's can be solved in terms of the known -prespecified- input velocity scalars and some dummy ω's if needed.

b- Since the two couplings C_1 and C_2 have only bodies i and 1 in common, then applying the concept of the parallel law, the ISS, S_{1i}, that represents the possible motion of link i relative to the frame, link 1, must lie in each of the ISS's S_1 and S_2 where,

$$S_1 = \text{span} \langle \underline{\$}_{12}, \underline{\$}_{23}, \cdot \cdot \cdot, \underline{\$}_{(i-1)i} \rangle$$

and

$$S_2 = \text{span} \langle \underline{\$}_{1n}, \underline{\$}_{n(n-1)}, \cdot \cdot \cdot, \underline{\$}_{(i+1)i} \rangle .$$

Then all possible motion of link i can be represented by the ISS,

$$S_{1i} = S_1 \cap S_2 .$$

There are many comutational methods to find a linearly independent set of screws that spans the above intersection subsystem S_{1i}. One of which is the well-known Gram-Schmidt process. The unknown scalars associated with these linearly independent screws can now be expressed in terms of the known input scalars ω's and the dummy ω's using the concept of the loop equation. Therefore, the ISS, S_{1i}, representing the possible motion of link i relative to the frame in loop 1 can be determined.

The analysis can then be repeated for any other single closed loop which contains the frame of the mechanism. Finally, the multi-loop mechanism is reduced to a single loop, that may or may not contain the frame, acted upon by the ISS's representing the constraints caused by the previous single loops. Using the concept of loop equation, the instantaneous screw system representing all possible instantaneous relative motion of any two bodies of the mechanism can be determined.

In the interest of applying the preceding analysis and to make the task easier, the following example is chosen.

EXAMPLE

The two- Dof multi-loop mechanism illustrated in Fig. 7 is chosen from both [3] and [4], so we could check and compare results. The impotance of this example is that it carries the main idea of [3], i.e., having cross-jointing, which the authors in [2] and [3] had to introduce a third law in order to perform the analysis. Also in this example we have to use dummy ω's. The objective is to determine the ISS representing all possible relative motion of bodies 4 and 7.

It is clear from the configuration of Fig. 7 that,

$$\underline{\$}_{21} = (o,o,1 \; ; \; o,o,o)$$

$$\underline{\$}_{32} = (o,o,1 \; ; \; 1.5,o,o)$$

$$\underline{\$}_{43} = (o,o,1 \; ; \; 3,2,o)$$

$$\underline{\$}_{54} = (o,o,1 \; ; \; 2,3,o)$$

$$\underline{\$}_{15} = (o,o,1 \; ; \; o,1,o)$$

$$\underline{\$}_{63} = (0,0,1 \; ; \; 3,-1,0)$$
$$\underline{\$}_{76} = (0,0,1 \; ; \; 4,-2,0)$$
$$\underline{\$}_{17} = (0,0,1 \; ; \; 2.5,-3,0)$$

and joints 15 and 17 are the two prespecified actuated joints. It is evident that only three components are required for each screw as the result of the mechanism being planar. A convenient basis could have the form $\{\underline{k} \; ; \; \underline{i},\underline{j}\}$.

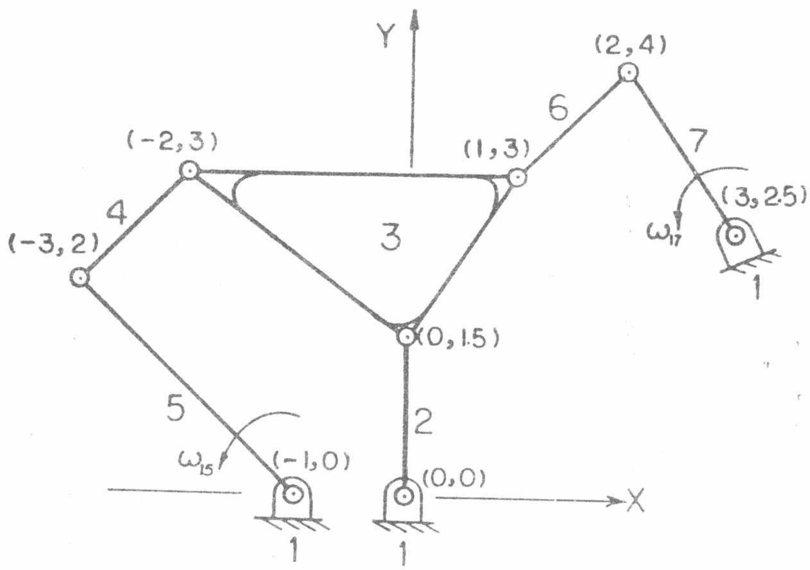

Fig. 7 Example 1.

a- Let the single closed loop 12345 be chosen as loop 1. Using the loop equation,

$$\omega_{15}\underline{\$}_{15} + \omega_{54}\underline{\$}_{54} + \omega_{43}\underline{\$}_{43} + \omega_{32}\underline{\$}_{32} + \omega_{21}\underline{\$}_{21} = 0 \tag{1}$$

where $\hat{\underline{\$}}_{15} = \omega_{15}\underline{\$}_{15}$ is the twist representing the motion of link 5 relative to the frame 1. Because the loop is indeed a pentagon, then we could carry both the known input velocity scalars ω_{15} and one of the unknowns, say ω_{21}, as a dummy ω. Then,

$$\omega_{54}\underline{\$}_{54} + \omega_{43}\underline{\$}_{43} + \omega_{32}\underline{\$}_{32} = -\omega_{15}\underline{\$}_{15} - \omega_{21}\underline{\$}_{21}$$

and

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1,5 \\ 3 & 2 & 0 \end{bmatrix} \begin{Bmatrix} \omega_{54} \\ \omega_{43} \\ \omega_{32} \end{Bmatrix} = \begin{Bmatrix} -\omega_{15} - \omega_{21} \\ 0 \\ -\omega_{15} \end{Bmatrix} .$$

Then,

$$\omega_{54} = -\frac{9}{7}\omega_{15} - \frac{6}{7}\omega_{21}$$

$$\omega_{43} = \frac{10}{7}\omega_{15} + \frac{9}{7}\omega_{21}$$

$$\omega_{32} = -\frac{8}{7}\omega_{15} - \frac{10}{7}\omega_{21}$$

Let $S_1 = \text{span}\langle \underline{\$}_{32}, \underline{\$}_{21} \rangle$ and $S_2 = \text{span}\langle \underline{\$}_{43}, \underline{\$}_{54}, \underline{\$}_{15} \rangle$, then using the concept of parallel law,

$$S_{13} = S_1 \cap S_2 \ .$$

Using Gram-Schmidt process to find two linearly independent screws,

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} , \text{ and } \begin{bmatrix} 0 \\ -1.5 \\ 0 \end{bmatrix}$$

that span the above intersection subspace S_{13}. Then the twist representing all possible instantaneous motion of link 3 relative to the frame 1,

$$\hat{\underline{\$}}_{13} = \lambda_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 \\ -1.5 \\ 0 \end{bmatrix} \qquad (3)$$

where λ_1 and λ_2 are unknown scalars. However, since

$$\hat{\underline{\$}}_{13} = \omega_{43}\underline{\$}_{43} + \omega_{54}\underline{\$}_{54} + \omega_{15}\underline{\$}_{15}$$

$$= -(\omega_{32}\underline{\$}_{32} + \omega_{21}\underline{\$}_{21})$$

then,

$$\hat{\underline{\$}}_{13} = \lambda_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 \\ -1.5 \\ 0 \end{bmatrix} = -(\omega_{32}\underline{\$}_{32} + \omega_{21}\underline{\$}_{21}) .$$

Substitude the expression for ω_{32} in eq. (2), then

$$\hat{\underline{\$}}_{13} = \left(\frac{8}{7}\,\omega_{15} + \frac{3}{7}\,\omega_{21} \right) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \left(-\frac{8}{7}\,\omega_{15} - \frac{10}{7}\,\omega_{21} \right) \begin{bmatrix} 0 \\ -1.5 \\ 0 \end{bmatrix} .$$

The multi-loop mechanism shown in Fig. 7 is then reduced to the single loop illustrated in Fig. 8. The constraints acted upon by loop 1 is then represented by the twist $\underline{\$}_{13}$. Its loop equation then becomes,

$$\omega_{17}\underline{\$}_{17} + \omega_{76}\underline{\$}_{76} + \omega_{63}\underline{\$}_{63} - \hat{\underline{\$}}_{13} = 0.$$

Solving for the unknown velocity scalars ω_{76}, ω_{63}, and ω_{21} in terms of the known input velocity scalars ω_{15} and ω_{17},

$$\begin{bmatrix} 1 & 1 & -\frac{3}{7} \\ 4 & 3 & -\frac{15}{7} \\ -2 & -1 & 0 \end{bmatrix} \begin{Bmatrix} \omega_{76} \\ \omega_{63} \\ \omega_{21} \end{Bmatrix} = \begin{Bmatrix} \frac{8}{7}\,\omega_{15} - \omega_{17} \\ \frac{12}{7}\,\omega_{15} - \frac{5}{2}\,\omega_{17} \\ 3\,\omega_{17} \end{Bmatrix} .$$

Then,

$$\omega_{76} = -\frac{4}{3}\,\omega_{15} - \frac{7}{6}\,\omega_{17}$$

$$\omega_{63} = \frac{8}{3}\,\omega_{15} - \frac{2}{3}\,\omega_{17}$$

$$\omega_{21} = \frac{4}{9}\,\omega_{15} - \frac{35}{18}\,\omega_{17} \qquad (4)$$

All possible motion of link 4 relative to link 7 can then be represented by the twist,

$$\hat{\underline{\$}}_{74} = \omega_{76}\underline{\$}_{76} + \omega_{63}\underline{\$}_{63} - \omega_{43}\underline{\$}_{43}$$

where from eqs. (2) and (4),

$$\omega_{43} = 2\,\omega_{15} - \frac{5}{2}\,\omega_{17}.$$

Then,

$$\hat{\underline{\$}}_{74} = -\frac{2}{3}\,\omega_{15}\begin{bmatrix}1\\5\\6\end{bmatrix} + \frac{2}{3}\,\omega_{17}\begin{bmatrix}1\\5/4\\12\end{bmatrix}. \tag{5}$$

which is a two-system consisting of all screws of zero pitch, normal to the plane of motion and lie on a line through points (-6,5) and (-12,5/4). These results are identical to those results given in both [3] and [4].

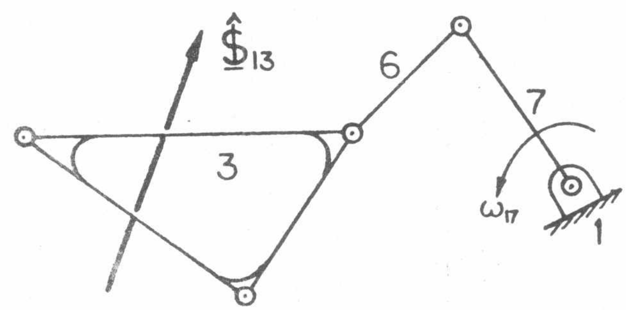

Fig. 8 The remaining loop.

CONCLUSIONS

A logical analysis procedure for investigating the instantaneous kinematics of multi-loop mechanisms requires neither special cases to be identified nor any new laws to be introduced. It just requires better interpretation and implementation of the two fundamental laws; series and parallel laws. This analysis can be applied to the study of the instantaneous kinematics of any multi-loop mechanisms.

REFERENCES

1 Waldron,K.J., "The Constraint Analysis of Mechanisms," Journal of Mechanisms, Vol. 2, 1966, pp. 1o1-114.

2 Davies, T.H., and Primrose,E.J.F., "An Algebra for the Screw Systems of a Pair of Bodies in a Kinematic Chain," Proceedings of the Third World Congress on the Theory of Machines and Mechanisms, Kupari, Yugoslavia, Paper D-14, 1971, pp. 199-212.

3 Baker, J.E., "On Relative Freedom Between Links in Kinematic Chains with Cross-Jointing," Mechanism and Machine Theory, Vol. 15, 198o, pp. 397-413.

4 Davies, T.H., "Kirchhoff's Circulation Law Applied to Multi-loop Kinematic Chains," Mechanism and Machine Theory, Vol. 16, 1981, pp. 171-183.

5 Mohamed, M.G., Instantaneous Kinematics and Joint Displacement Analysis of Fully-Parallel Robotic Devies, Ph.D. Dissertation, University of Florida, Gainesville, FL, 1983.

6 Mohamed, M.G., and Duffy, J., "A Direct Determination of the Instantaneous Kinematics of Fully-Parallel Manipulators," Journal of Mechanisms, Transmission, and Automation in Design, Vol. 1o7, June 1985, pp. 226-229.

7 Ball, R.S., A Treatise on the Theory of Screws, Cambridge University Press, Cambridge, 19oo.