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AUTOMATIC NORMAL MODE TESTING

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Introduction

To provide data for the verification and improvement of analytical models of aircraft structures (once they have been designed), it is necessary to accurately determine the principal modes of the structure. This is carried out during what is normally called the ***Ground Vibration Test.***

Traditionally these tests have been based on the Normal Mode testing technique described by Lewis & Wrisley [1] in which the structure is excited at the resonant frequency by a set of monophase forces (0° or 180° out of phase with each other) distributed over the structure, and the amplitudes of the forces are adjusted ***manually*** until the velocities at all points on the structure are in phase with the forces. Using these forces which are called ***appropriated*** forces, each of the modes can be excited in turn and the modal parameters determined. To determine these forces, however, some form of iterative procedure for force appropriation is required to optimize the phase coherence of the responses. Manual iterative procedures are carried out whilst observing some form of phase display. Generally, use of Lissajous figures is quite convenient, as closing of the ellipses at all points implies not only proper force appropriation, but also excitation at the resonant frequency.

Difficulties, however, arise when a large number of shakers have to be adjusted to isolate modes in frequency ranges of high modal density. Therefore to automate the force appropriation task, different methods have been put forward. The one that will be described in this paper is due to Asher [2] who proposed a quantitative method, which detects the natural frequencies as well as provides the force ratios necessary for multiple-shaker tuning, using only experimental transfer admittance data as input.

Asher's Method

To illustrate the method suggested by Asher consider the following equation which describes the harmonic motion of an n degree of freedom linear, damped system i.e.

$$[m]\{\ddot{x}\} + [c(\omega)]\{\dot{x}\} + [k]\{x\} = \{F\}\sin\omega t \tag{1}$$

The damping is considered to be rather general, and is therefore represented by an arbitrarily frequency-dependent term

$$[c(\omega)] = [c] + 1/\omega\,[d] \tag{2}$$

where $[c(\omega)]$ encompasses any combination of the standard viscous and hysteretic types, given by $[c]$ and $[d]$ matrices respectively. Furthermore, in the general case, damping is non-proportional so that the damping matrix couples the otherwise uncoupled normal equations of motion.

The system is assumed to be excited by a set of mono-phase forces i.e. all the forces are either 0° or 180° out of phase with each other. Introducing complex notation (complex quantities are denoted by a bar) in eq. (1) yields

$$[m]\{\ddot{\bar{u}}\} + [c(\omega)]\{\dot{\bar{u}}\} + [k]\{\bar{u}\} = \{F\}e^{j\omega t} \tag{3}$$

where $$\{x\} = \mathrm{Im}\{\bar{u}\} \tag{4}$$

Substituting the trial solution

$$\{\bar{u}\} = \{\bar{U}\}\, e^{j\omega t} \tag{5}$$

in eq. (3) yields

$$-\omega^2 [m] \{\bar{U}\} + j\omega [c(\omega)] \{\bar{U}\} + [k] \{\bar{U}\} = \{F\} \tag{6}$$

i.e.

$$\left[[k] - \omega^2 [m] + j\omega [c(\omega)]\right] \{\bar{U}\} = \{F\} \tag{7}$$

Eq. (7) may be written as

$$\{\bar{U}\} = [\bar{B}] \{F\} \tag{8}$$

where

$$[\bar{B}] = \left[[k] - \omega^2 [m] + j\omega [c(\omega)]\right]^{-1} \tag{9}$$

is called the complex admittance matrix. It can be written in its real and imaginary parts as

$$[\bar{B}] = [B'] + j[B''] \tag{10}$$

where $[B']$ is the real part of $[\bar{B}]$
and $[B'']$ is the imaginary part of $[\bar{B}]$

Substituting eq. (10) in eq. (8) yields

$$\{\bar{U}\} = [B'] \{F\} + j[B''] \{F\} \tag{11}$$

which implies that when the displacement response is in quadrature with the excitation force, it is required that

$$[B'] \{F\} = \{0\} \tag{12}$$

since the real parts of the displacement responses are zero. Eq. (12) represents a set of linear homogeneous equations which have a non-trivial solution if, and only if

$$|B'| = 0 \tag{13}$$

(giving a set of eigenvalues which are the undamped natural frequencies). As a corollary to this statement it can be concluded that "The response of a system will be in quadrature with the excitation if, and only if, the determinant of the real part of its complex admittance matrix $[B']$ is equal to zero". The corresponding force ratios required to isolate the modes will be given by eq. (12) using the eigenvalues obtained from eq. (13).

The theoretical results given by eqs. (12) and (13) were first stated by Asher. His method can be described with reference to these equations in the context of an idealized modal test on a discrete n degree of freedom linear structure with linear damping. The test is started by measuring the (n x n) complex admittance matrix over the frequency range of interest. This can be carried out by exciting and measuring the response at each degree of freedom, using sinusoidal sweeps. The real part of the admittance matrix, $[B']$ is extracted, and the determinant of $[B']$ is plotted as a function of frequency. It can be seen from eq. (13) that this determinant is exactly zero at an undamped natural frequency. Thus n zero-crossings will be found on the frequency axis, and the frequencies at which they occur, are the undamped natural frequencies, even though damping is present. The admittance matrix is again measured, but this time only at the undamped natural frequencies just found $[\bar{B}(\omega_0)]$. The force ratios necessary to excite these modes **individually** are then calculated from eq. (12). (Any one column of the adjoint matrix $[\hat{B}'(\omega_0)]$ represents the solution to eq. (12), i.e. it is identical to the force ratios (to within constant multiples)). Using these force ratios, the individual modes are tuned exactly for a multiple-shaker sine dwell test. For the i^{th} mode the shakers are set to produce the force ratios

$$\{F\} = \{F_i\} \sin \omega_i t \tag{14}$$

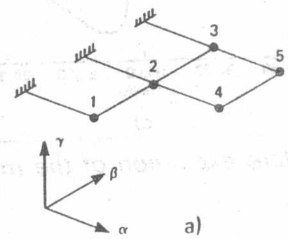
and the displacement response is measured to be

$$\{x\} = \{\phi_i\} \sin(\omega_i t - \pi/2) \tag{15}$$

where $\{\phi_i\}$ is the principal mode shape for the i^{th} undamped natural frequency, even though there is damping in the system. Generally a narrow-band sweep about ω_0 with fixed $\{F_i\}$ is more useful than a dwell, as it permits the plotting of Nyquist graphs from which mode shapes and modal damping values can be calculated and modal purity evaluated.

Simulation studies carried out by Hallauer and Stafford [3] on mathematical models will be described in the following to illustrate the effectiveness of Asher's method. As the parameters of a mathematical structural model are known precisely, and can also be designed to produce desired characteristics (such as high modal density), absolute values are available against which the simulated test results can be compared. Furthermore, they also aid in pin-pointing the specific causes for failures of the test method.

The structure used is a cantilevered plane grid shown in Fig.1 a, where five discrete masses are connected by identical massless bars at right angles in the α—β plane. The bars are flexible only in vertical bending and torsion, and the degrees of freedom are vertical (γ) translations of the masses.



a)

Undamped normal modes of models 5H and 5V

r		1	2	3	4	5
ω_r (rad/sec)		10.908	32.055	53.500	55.409	94.924
ϕ_r	(mass)					
	(1)	0.126	0.943	0.380	-0.595	0.121
	(2)	0.232	0.479	0.075	0.478	-0.712
	(3)	0.337	0.165	-0.703	0.320	0.217
	(4)	0.773	1.000	1.000	1.000	1.000
	(5)	1.000	-0.281	0.195	-0.342	-0.130

b)

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Fig.1 a) Structure of the model
b) Undamped normal modes of the model

By means of an optimization procedure, the structure was designed to have its third and fourth natural frequencies within 4% of each other. Damping values for the third and fourth modes were selected so that these modes were separated by less than one half-power bandwidth. The undamped natural frequencies and the corresponding normal modes are given in the table in Fig.1 b.

In Fig.2 the determinant of the real part of the admittance matrix, plotted as a function of frequency, is shown by solid lines when all the masses are excited, and by dashed lines when only masses 1 and 5 are excited. The normalization scheme used is such that the lowest mode dominates the higher modes which consequently may disappear in the resolution of the graph. Thus Fig.2 b and Fig.2 c are redrawn on an expanded scale. It can be seen that the curves have a similar form to the real part of a frequency response function near the resonant frequencies. When all the masses are excited, the curves in Fig.2 a and 2 c have zero-crossings exactly at the undamped natural frequencies as predicted by the theory.

The force-amplitude distribution required to excite the fourth mode (the higher of the two closely spaced modes) was calculated. This distribution when applied to the model in a 3 rad/s narrow band sweep gave a polar plot of the admittance X_i of each mass as shown in Fig.3 a. Tick marks on the response curves indicate equal frequency increments, and arrows indicate starting frequency for each response. Conjugates of the curves are plotted when appropriate to have all the curves on the same side of the real axis. The mode shape vector $\{\phi_4\}$ indicated in the figure, was calculated from the quadrature components of the response at the zero- crossing frequency of the determinant. In this case, it is the exact mode shape according to theory.

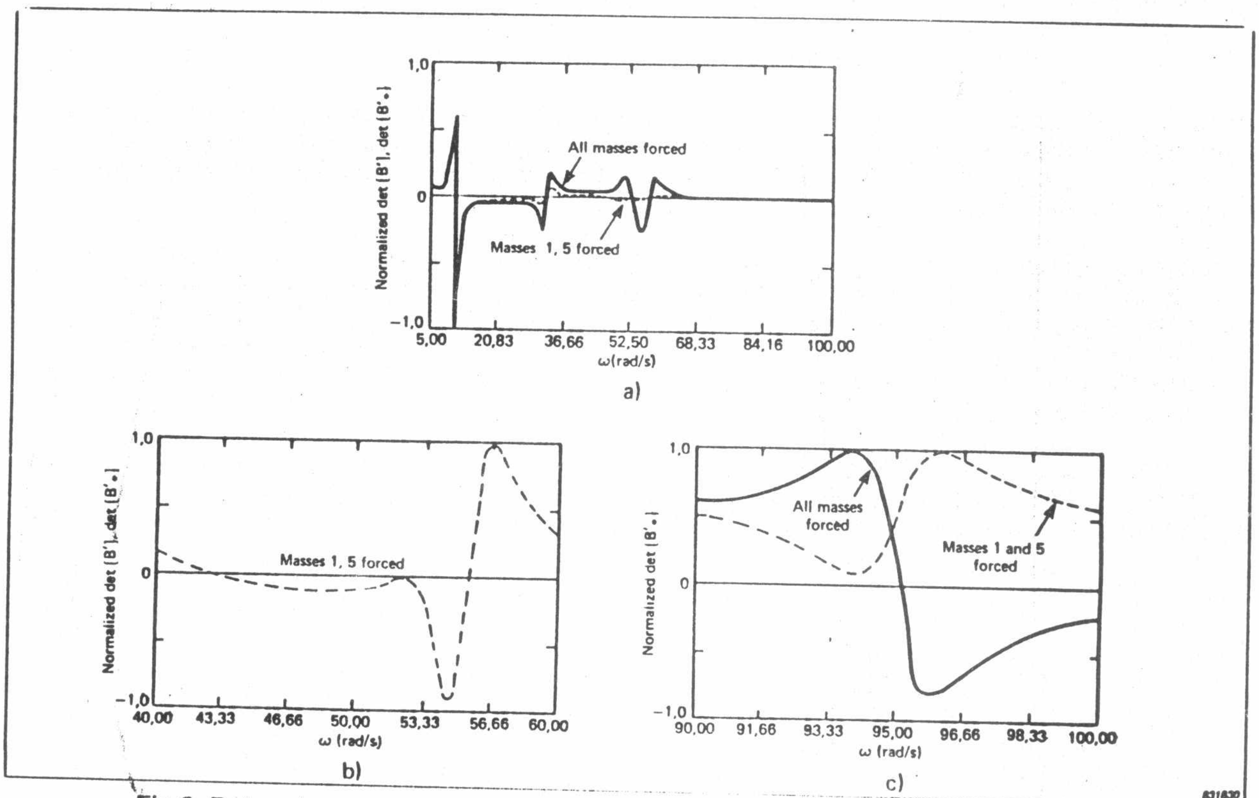

Fig.2 Determinant plots for complete and incomplete excitation of the model
a) 5-100rad/s
b) 40-60rad/s
c) 90-100rad/s

If the damping had been non-coupling, then according to Kennedy & Pancu [4], each curve of the polar plot would be an arc of a perfect circle centred on the imaginary axis with its highest point passing through the origin. However, coupling damping causes deviation from circular shape, which is evident in Fig.3 a where interference is introduced from the lower of the two closely spaced

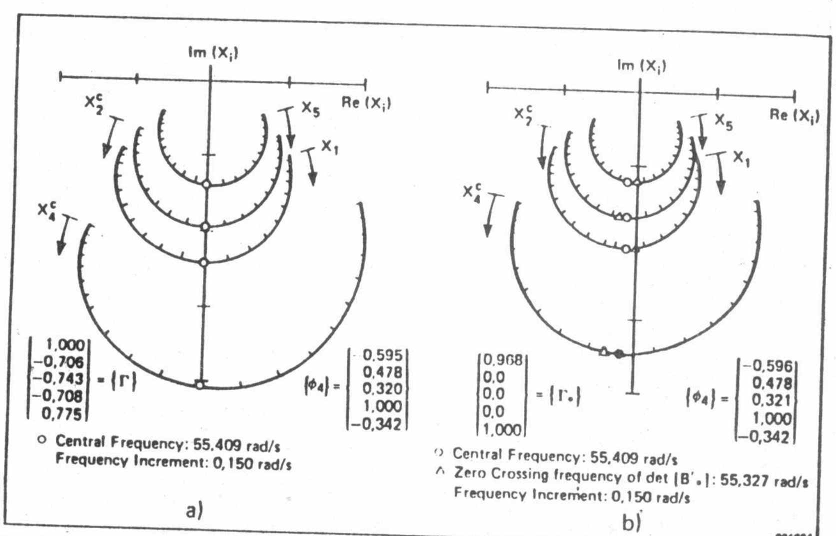

Fig.3 a) Perfect tuning with complete excitation: polar plot for the fourth mode with all masses forced in a 3rad/s narrow-band sine sweep
b) Excellent tuning with incomplete excitation: polar plot for the fourth mode with masses 1 and 5 forced

modes. As mentioned previously, according to theory perfect mode tuning is possible only at the undamped natural frequency for coupling damping, with responses in quadrature with the force, even though all degrees of freedom are excited.

From the table in Fig.1 it can be seen that for the first bending mode shape, all the responses are in phase. Thus intuitively one would expect all the tuning forces to be in phase as well. However, the normalized force vector for perfect tuning is given by

$$\{\Gamma\} = \begin{Bmatrix} -0{,}214 \\ -0{,}125 \\ 0{,}133 \\ 1{,}000 \\ 0{,}431 \end{Bmatrix}$$

On second thoughts the explanation is obvious, in that the appropriated forces required to tune the mode must cancel the damping forces, which are not generally distributed spatially as the mode shape.

The dashed curves of Fig.2 a, b, and c show zero-crossings at 10,909, 32,070, and 55,327 rad/s which are very close to the natural frequencies of the first, second and fourth modes respectively. However, there is also a false mode crossing at 44,524 rad/s which is characterised by its gentle slope. In general, for lightly damped structures, true modes are indicated by steep slopes at the zero-crossings, and false modes by relatively gentler slopes. This can again be seen on the dashed curves of Fig.2 b and c which do not have a zero-crossing for the third and the fifth modes, but nevertheless have a relatively steep slope in the vicinity of the natural frequency of the missed modes.

Fig.3 b shows the polar plots of the admittance X_i of each mass obtained using incomplete excitation. The tuning corresponds to the fourth mode zero-crossing of the determinant at 55,327 rad/s on Fig.2 a and b for masses 1 and 5 forced. The 5×1 force-amplitude distribution vector $\{F_*\}$ is derived from the 2×1 vector $\{\Gamma_*\}$ with zeros added for the unforced masses. In spite of incomplete excitation, the tuning is seen to be excellent which is often provided by Asher's method.

Instrumentation

Fig.4 shows an instrumentation set-up for testing of large structures using multiple shakers. The system is such, that it can be readily expanded to incorporate any number of shakers and accelerometer channels. All the equipment below the structure constitutes the excitation part of the system, while the instrumentation above the structure is used for data acquisition and further processing and analysis of the results.

On account of the multiplicity of shakers and accelerometers, automatic control of the system is imperative, although manual operation is possible. This can be achieved by the Digital System Controller, which could be, for example, a desk-top calculator having sufficient memory and speed commensurate with the sophistication of the analysis software.

The fundamental requirement on the excitation side is the accurate control of the amplitude and phase of the force, applied by each shaker to the test structure at a single frequency. A Precision Generator Type 1050 can be used to feed the signal to the shakers, via a power amplifier and a Level/Phase Controller. Whilst the latter controls the voltage input, and thus the force output from each shaker, via the IEC/IEEE Interface, monitoring of the forces into the structure is carried out by force transducers mounted between each shaker and the test structure.

On the data acquisition side, the acceleration signals have to be measured at several points on the structure, and conditioned, before they are fed to the Lissajous Display Type 5851. The Multiplier Type 5795 extracts the Real and the Imaginary components of the acceleration signals, while the Control Unit Type 5794 digitizes these signals, and controls the Group Selectors Type 5820 for outscanning of the system.

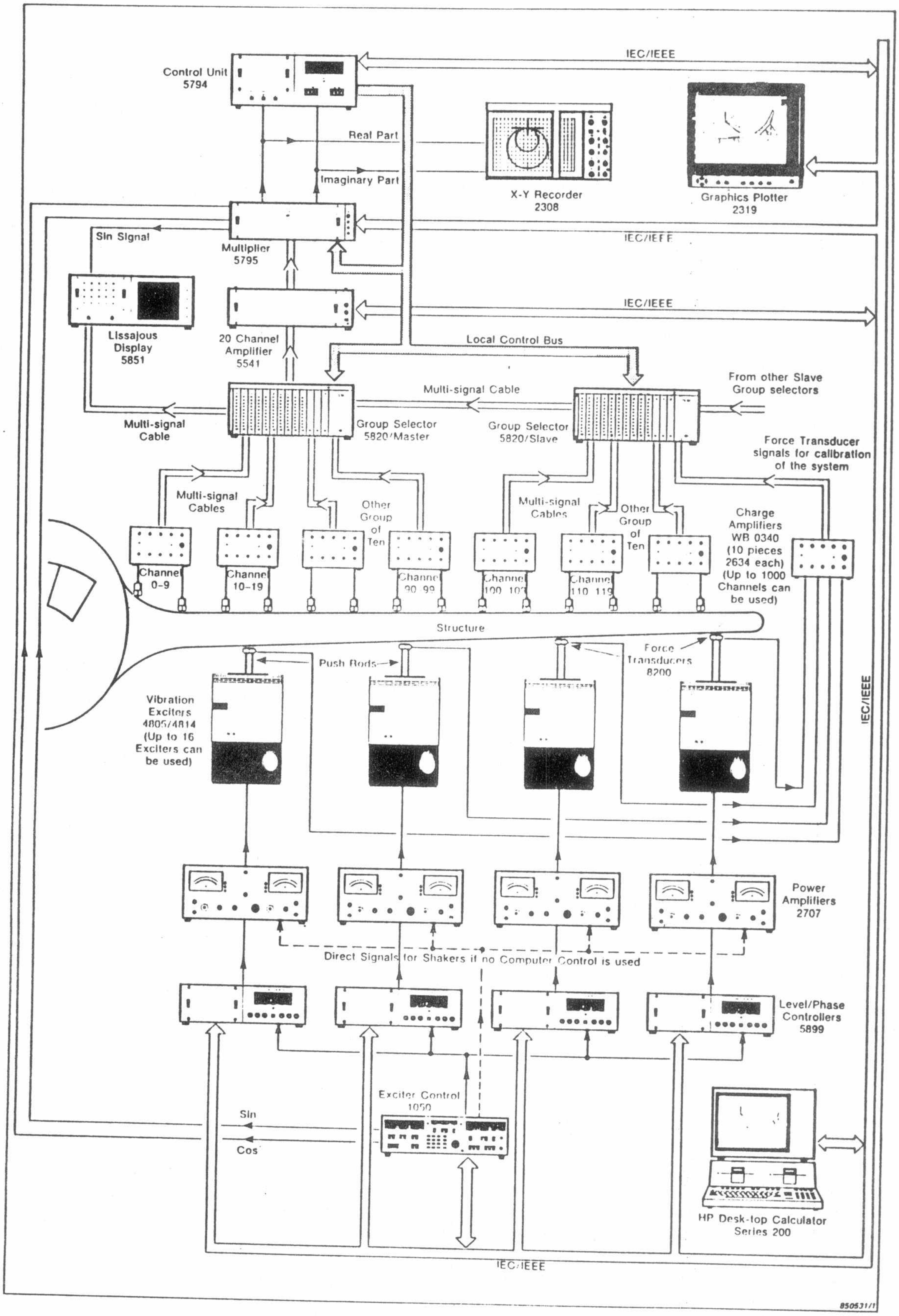

Fig.4 Instrumentation set-up for testing of large structures using multiple shakers

Extraction of Modal Parameters

Once the appropriated forces are calculated using Asher's method, a narrow band sweep around the resonant frequency is carried out using these forces, and the response obtained at several points on the structure are plotted, for example, as Nyquists plots. Since the structure would behave as a single degree of freedom system, the modal parameters can be calculated from these plots.

A more efficient way, however, to extract the modal parameters is to plot the real and the imaginary parts of the power, as the structure is excited around the resonant frequency using the appropriated forces.

$$\text{Real Power} = W' = \sum_{i=1}^{n} F_i v_i'$$

$$\text{Imag. Power} = W'' = \sum_{i=1}^{n} F_i v_i''$$

where v_i' and v_i'' are the real and the imaginary parts of the velocity response at the i^{th} location. Fig.5 shows typical curves of complex power from which the modal parameters can be calculated using the formulae shown in the figure.

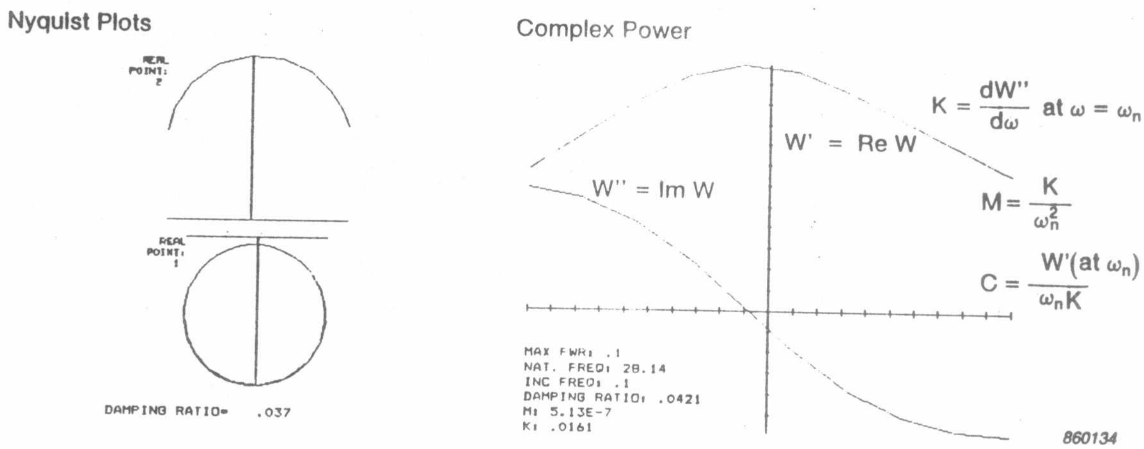

Fig.5 a) Nyquist plots
b) Complex power plots

REFERENCES

[1] LEWIS, R.D. and WRISLEY, D.L.: "A System for the Excitation of Pure Natural Modes of Complex Structures.", *J. Aeronaut Sci*, 17 (11), pp 705-722 (1950)

[2] ASHER, G.W.: "A Method of Normal Mode Excitation Utilizing Admittance Measurements," *Proc. National Specialists' Meeting on Dynamics and Aerolasticity, Ft. Worth, Inst. of Aeronautical Sciences*, 1958, pp. 69-76

[3] HALLAUER, W.L.Jr., and STAFFORD, J.F., "On the Distribution of Shaker Forces In Multiple-Shaker Modal Testing", *The Shock and Vibration Bulletin, Bull.* 48, Part 1, 1978, pp. 49-63

[4] KENNEDY, C.C. and PANCU, C.D.P. "Use of Vectors in Vibration Measurement and Analysis", *J. Aeronautical Sciences*, Vol. 14, No. 11, 1947, pp. 603-625