A.M.E.

MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

OPTIMAL CONTROL OF A TURBOJET ENGINE A DYNAMIC PROGRAMMING APPROACH.

MOHTASSEM BILLAH KADDAH **

ABSTRACT

This paper discusses the application of dynamic programming techniques in the domain of turbojet engine control system design.

Based on a known detailed model of a by-pass double-spool turbojet engine, derived in previous works, we obtained an optimum control law governing the acceleration of the engine.

The work discusses the adaptation of dynamic programming techniques to the specified problem. A detailed analysis of the problem characteristics is included and a discussion of computer implementation requirements is presented.

** AERONAUTICAL DEPARTMENT - MILITARY TECHNICAL COLLEGE
CAIRO - EGYPT

1. INTRODUCTION

The dynamic programming (DP) technique represents a valuable tool for optimal control systems studies. However, in spite of the extended development of its theoretical background, its practical implementation is generally difficult. This difficulty increases in case of complicated models such as variable geometry turbojet engines.

In this study, we have introduced the basic concepts of dynamic programming and showed in detail the steps adopted in order to apply it to the optimal acceleration problem of a double-spool by-pass turbojet engine.

2. PROBLEM DEFINITION

2.1 ENGINE MODEL

During our previous works [4,5], we have developed a detailed turbojet engine model specially adapted for control systems studies. The engine is a double-spool by-pass turbojet engine with variable exit nozzle area (fig. 1).
Under the considered assumptions, the basic state parameters of the engine would be:

NLP...... low pressure rotor rpm
NHP...... high pressure rotor rpm
(neglecting the pnumatic effects)

and the basic input parameters are:

C .... rate of fuel flow
S .... exit nozzle area
(neglecting the actuators dynamics)

several outputs may be computed from the model, the most importanty are:

P_i , T_i .. pressure and temperature at individual engine sections (i)
F .... engine thrust
MLF ..... low pressure compressor (LPC) surge margin
MLH ..... high pressure compressor (HPC) surge margin

The engine was represented by the following model:

$$\dot{\underline{X}} = \underline{f} (\underline{X} , \underline{U})$$
$$\underline{Y} = \underline{g} (\underline{X} , \underline{U}) \qquad (2.1)$$

where

$$\underline{X}^T = (NLP , NHP)$$
$$\underline{U}^T = (C , S)$$
$$\underline{Y}^T = (T5 , MLF, MLH, F)$$

6

Due the pecularities of the considered model, equations (2.1) are non-linear implicit equations where f cannot be obtained in an analytical form.

The solution of these equations is obtained through an itterative scheme based on conditions of common operation of individual engine components. The components are themselves represented by non-linear imperical or theoretical characteristics.

2.2 ENGINE OPERATIONAL CONSTRAINTS

In general depending on the aircraft type, the nature of missions and the needed engine performance characteristics, the following criteria should be clearly defined:

- Engine protection : temperature limits, revolutions number, pressures, ...
- Engine stability : performance fluctuations, surge margins, ...
- Steady state performance : thrust, fuel consumption, ...
- Transient performance : thrust variations, combustion stability, transient durations, ...

The general character of these criterea is the same for every type of engine but their priorities may differ from one engine type to the other. These priorities are determined through a deep understanding of the physical phenomena occuring within the engine.

These criteria are further translated into a set of constraints which generally define extreme limits for the different engine parameters: state (NLP, NHP) ; inputs (C, S) and outputs (F, MLF, MHP, T5 , ...)

$$\underline{q}\ (\underline{X}, \underline{U}) \leqslant 0 \qquad \begin{cases} S_{min} - S \leqslant 0 \\ S - S_{max} \leqslant 0 \\ T - T5_{max} \leqslant 0 \\ MLF - MLF_{max} \leqslant 0 \\ MLH - MLH_{max} \leqslant 0 \\ NLP - NLP_{max} \leqslant 0 \\ NHP - NHP_{max} \leqslant 0 \end{cases}$$

. . .

2.3 CONTROL PROBLEM DEFINITION

The aim of this study is to determine an optimal control law applicable to the acceleration of the engine from idle to maximum regimes and for the flight conditions (H = 0 , M = 0).

An optimal control law is determined for a given optimality criterion. We have chosen the integral of the thrust over the transient period to express the optimality criterion :

$$J = \int_0^T F\ (\ \underline{X},\ \underline{U}\ )\ dt$$

A discussion of the basis of this choice was presented in previous works [1,4,5]

The optimal control problem may thus be expressed as :

Given the state equations..... $\dot{\underline{X}} = \underline{f}\ (\ \underline{X},\ \underline{U}\ )$

$\underline{Y} = \underline{g}\ (\ \underline{X},\ \underline{U}\ )$

Find the control law $\underline{U}$,
maximizing the criterion...... $J = \int_0^T F\ (\ \underline{X},\ \underline{U}\ )\ dt$
Assuming the system is
subject to the constraints.... $\underline{q}\ (\ \underline{X},\ \underline{U}\ ) \leqslant 0$

3. SOLUTION APPROACHES

Two basic methods may be used for the solution of this problem :

a- HAMILTONIAN APPROACH

This approach results in general in an open loop control law . It was discussed in detail in our previous work [5].

b- DYNAMIC PROGRAMMING APPROACH [2]

This approach results in a closed loop control law, but its application is time consuming and necessitates the processing of large amounts of data.

3.1 COMPARISON BETWEEN THE TWO APPROACHES

Considering the state space representation of the engine, the aim of control system computations is to determine a set of trajectories, within this space, covering the specified domain of engine operation (fig. 2c)

The Hamiltonian approach is based on a repeated computational procedure allowing each time to calculate an optimal trajectory (fig. 2a)

The dynamic programming (DP) approach, on the other hand, allows an immediate solution for a given zone of the state space and resulting in a set of optimal trajectories within this zone. The procedure is then repeated for other zones until all the domain of operation is covered(FIG. 2B).

. . .

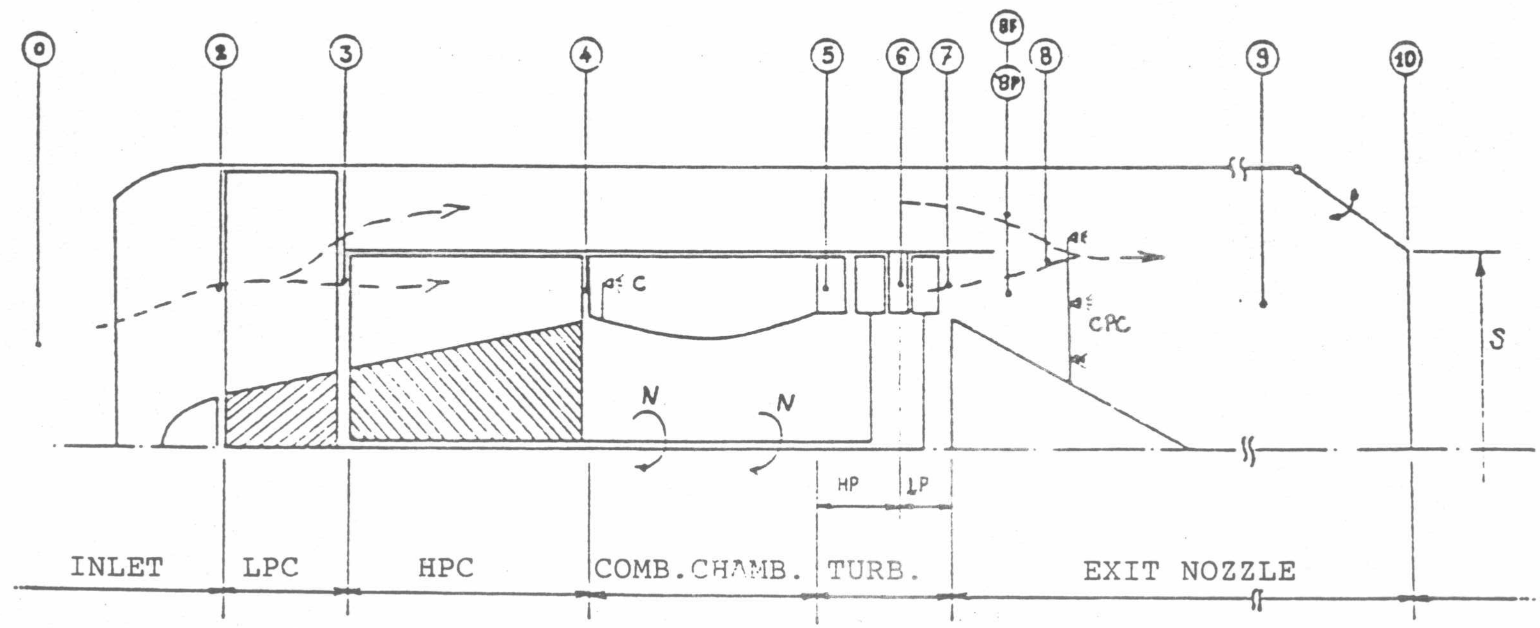
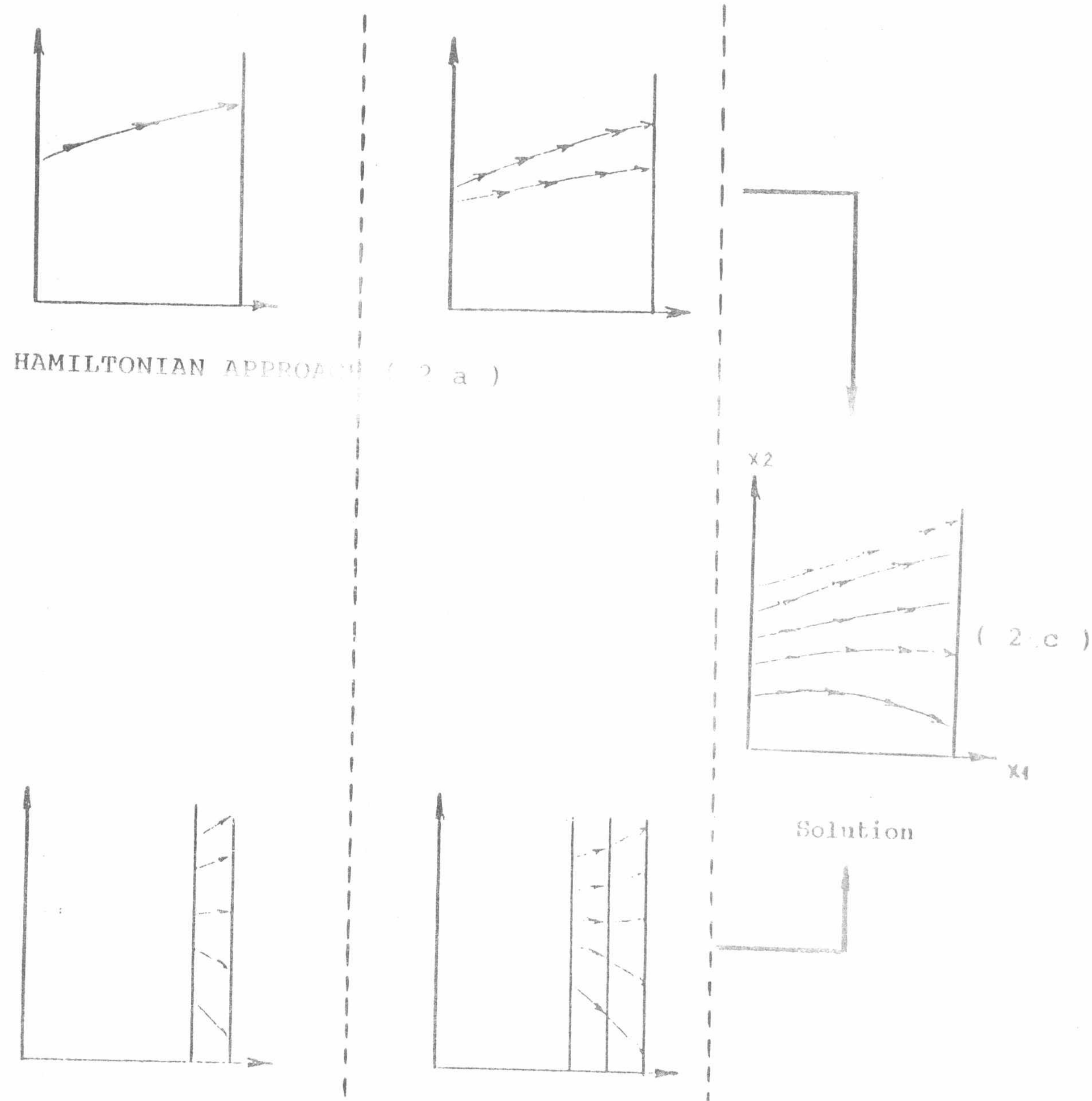

0
2
3
4
5
6
7
8F
8P
8
9
10
C
CPC
N
N
S
HP
LP
INLET
LPC
HPC
COMB.CHAMB.
TURB.
EXIT NOZZLE

FIG. 1



DYNAMIC PROGRAMMING (2 b)

FIG. 2

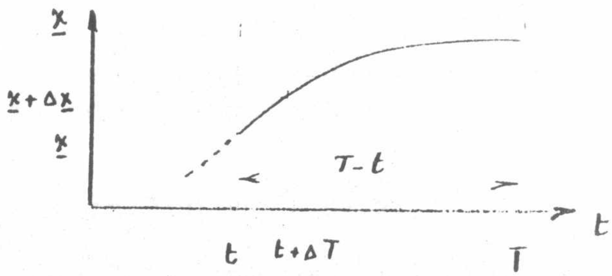
Within the scope of this work, we shall introduce and implement the dynamic programming approach. The results of application of the Hamiltonian approach will help overcome some of the difficulties related to DP, specially storage requirements.

4. DYNAMIC PROGRAMMING APPROACH

4.1 General

Referring to the engine model (equ. 2.1) and assuming an initial state $\underline{X}$ at a certain time t

fig. 3



Considering a given trajectory, we may represent the criterion J as:

$$J = \int_0^t F (\underline{X}, \underline{U}) \, d\tau + \int_t^T F (\underline{X}, \underline{U}) \, d\tau \qquad (4.1)$$

at this instant t, we may only compute trajectories for the remaining period (T-t), i.e. we can only compute the the second term of the equation (4.1).
assuming a general control law

$$\left\{ U(\tau) \;\middle|\; \tau \in (t, T) \right\}$$

for which the second term may be determined as

$$R (\underline{X}, t ; \underline{U}) = \int_t^T F (\underline{X}(\tau), \underline{U}(\tau)) \, d\tau$$

taking into consideration that

$$\dot{\underline{X}} = \underline{f} (\underline{X}, \underline{U}) \quad \text{and} \quad \underline{X}(t) = \underline{X}$$

for an optimal control law, this may be expressed as

$$\hat{R} (\underline{X}, t) = \underset{\{U(\tau) | \tau \in (t,T)\}}{\text{maximum}} R (\underline{X}, t ; \underline{U})$$

defining a second point at a time = t + Δ T, and assuming Δ T to be suuficiently small, we may compute the new state X+ Δ X from the state equation, where

$$\Delta X = \int_t^{t+\Delta T} f (\underline{X}, \underline{U}, \tau) \, d\tau$$

we can also express

$$R(\underline{X}, t; \underline{U}) = \int_t^{t+\Delta T} F(\underline{X}(\tau), \underline{U}(\tau)) \, d\tau + \int_{t+\Delta T}^{T} F(\underline{X}(\tau), \underline{U}(\tau)) \, d\tau$$

$$= F(\underline{X}, \underline{U}(t)) . \Delta T + R(\underline{X}+\Delta\underline{X}, t+\Delta T; \underline{U})$$

Further, assume that we have already determined an optimal trajectory starting at the point $(X+\Delta X, t+\Delta T)$ then

$$R(\underline{X},t;\underline{U}) = F(\underline{X},\underline{U}(t)).\Delta T + \hat{R}(\underline{X}+\Delta\underline{X},\ t+\Delta T)$$

The optimum trajectory starting at the point (X,t) would then be

$$\hat{R}(\underline{X},t) = \underset{\{U(\tau)\,|\,\tau\in(t,T)\}}{\text{maximum}} R(\underline{X},t\ ;\ \underline{U})$$

i.e.

$$\hat{R}(X,t) = \underset{\underline{U}(t)}{\text{maximum}} \left[F(\underline{X},\underline{U}(t)).\Delta T + \hat{R}(\underline{X}+\Delta\underline{X}, t+\Delta T) \right] \qquad (4.2)$$

From this relation we may conclude that the optimal trajectories may be computed backwards in time starting at the final point $t = T$, since

at $t = T$ $\qquad \hat{R}(\underline{X},T) = 0 \qquad \forall X$ (by definition of R)

Assuming suitable time intervals ΔT we may apply equation (4.2) for all the points $(X, T-\Delta T)$ and then repeat the procedure backwards until $t=0$.

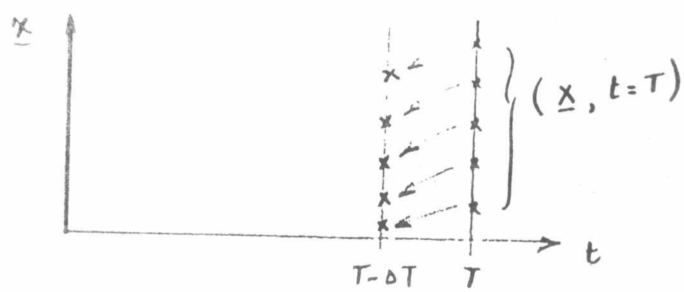


fig. 4

4.2 APPLICATION OF THE DP TO THE TJE PROBLEM

4.2.1 PROBLEM SIMPLIFICATION

Choosing a time horizon T sufficiently large, we notice that the engine regimes(NLP,NHP) reach their final steady state (max. regime) a long period before the time T.

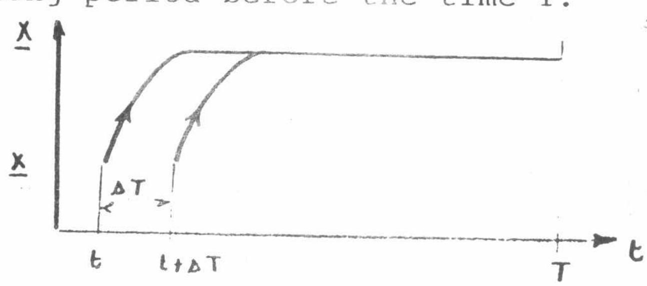


fig. 5

As seen from fig. (5), the optimal trajectory passing by the point $(X, t+\Delta T)$ may be considered similar to that passing by the point (X,t) but with a time delay ΔT, then, if

$$\hat{R}(\underline{X},t) = \max_{u(t)} \int_t^T F(\underline{X}(\tau), \underline{U}(\tau))\, d\tau$$

and

$$\hat{R}(\underline{X},t+\Delta T) = \max \int_{t+\Delta T}^{T} F(\underline{X}(\tau), \underline{U}(\tau))\, d\tau$$

we may express $\hat{R}(X, t) = \hat{R}(X, t+\Delta T) + F_{max} \cdot \Delta T$

where F_{max} is the thrust obtained at the final steady state point (known design value).

Further we may consider a trajectory at t=0, and get

$$\hat{R}_o(\underline{X}) = \hat{R}(\underline{X}, 0) = \hat{R}(\underline{X}, t) + F_{max} \cdot t \quad t \ll T$$

i.e.

$$\hat{R}(\underline{X}, t) = \hat{R}_o(\underline{X}) - F_{max} \cdot t$$

4.2.2 EXPRESSION OF OPTIMALITY CONDITION

The optimal criterion was expressed as:

$$\hat{R}(\underline{X}, t) = \underset{\underline{U}(t)}{\text{maximum}} \left\{ F(\underline{X}, \underline{U}) \cdot \Delta T + \hat{R}(\underline{X} + \Delta X, t + \Delta T) \right\}$$

applying the simplification assumptions, we get

$$R(\underline{X}) = \underset{\underline{U}(t)}{\text{maximum}} \left\{ [F(\underline{X}, \underline{U}) - F_{max}] \cdot \Delta T + \hat{R}_o(\underline{X} + \Delta \underline{X}) \right\}$$

where $\Delta \underline{X}$ and ΔT are related by

$$\Delta \underline{X} = \underline{f}(\underline{X}, \underline{U}) \cdot \Delta T$$

It is clear that this relation is a reccurrent relation that may be initiated at the final steady state regime (t=T ; X=X) where $\hat{R}_o(\underline{X}_{final}) = F_{max} \cdot \Delta T$

4.3 PRACTICAL IMPLEMENTATION

We have used the results obtained during the application of the Hamiltonian approach [5] to simplify the implementation of the DP and reduce the volume of processed data to a reasonable amount.

4.3.1 COMPUTATIONAL GRID

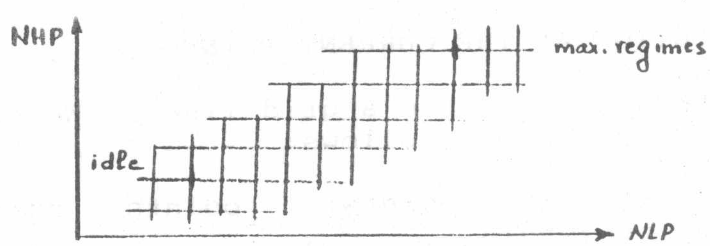


fig. 6

We have computed the optimum trajectories through a scanning scheme in the state space. For this purpose, we have defined a grid as follows:

- Step within the state variables = 200 rpm.
- Grid is limited to a zone defined in the viscinity of the steady state operating line.

. . .

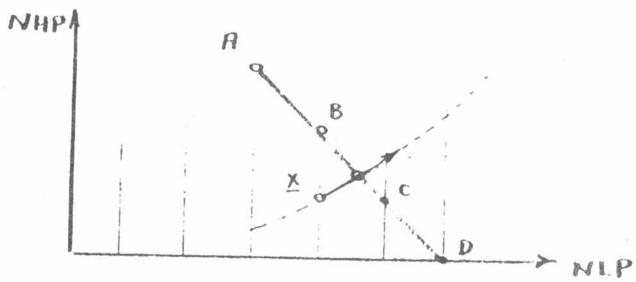
4.3.2 EXPLORATION LINES

In order to solve the optimality equation at a point $\underline{X}$, it should have already been solved at all reachable points $\underline{X}+\Delta\underline{X}$ applying admissible values of commands $\underline{U}$.

In case of jet engine acceleration, we can neglect all the points for which both the state variables decrease. Thus when looking for a solution at point $\underline{X}$, we shall limit our exploration lines to directions where at least one of the two state variables increases.

Further, consider that an optimal trajectory originating at point $\underline{X}$ passes by the line AD

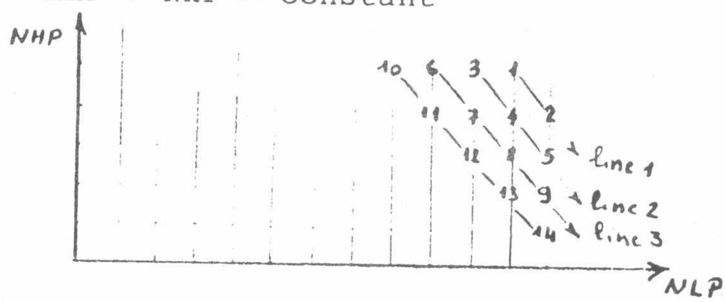
fig. 7



the point $\underline{X}+\Delta\underline{X}$ will be the intersection of the optimal trajectory and the line AD. It is not a point on the chosen grid but it may be easily computed knowing A, B, C, D.
We have thus chosen the grid points lying on exploration lines defined by

$$NLP + NHP = \text{constant}$$

fig. 8



4.3.3 SOLUTION AT THE CURRENT POINT

Assume the exploration has reached a grid line i. We examine all its points and classify them as follows:

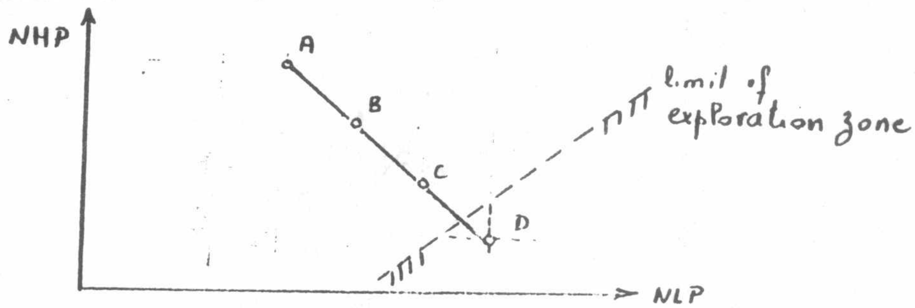
- Non admissible points: points outside the exploration zone

- Admissible points: points within the exploration zone, these are memorized and saved with the parameters of the corresponding optimal solution.

a- Domain of possible successors:

Consider a point $\underline{X}$ on the next exploration line i+1. The points A, B, C, D were already processed during the exploration of the line i. Some of these points may be non-admissible.

fig. 9



We could limit the domain of possible successors to the segment AC. This would lead to erroneous results, since the obtained optimal trajectory is now constrained to pass between A and C. We have thus decided to extrapolate the results obtained in A, B, and C to obtain the point D.

b- Solution of the optimality equation

At a given point X, we scan all possible input values by considering a constant value of nozzle area S and stepwise varying the rate of fuel flow($\Delta C = 0.01$ kg/s.), then by changing the nozzle area by a step($\Delta S = 0.005$ m)

For each couple (C,S) we have used the engine model to compute the needed engine parameters(T5 , MLF, MLH, F, NLP, NHP). In case the considered couple is admissible, we calculate the time step T and then determine the value of the criterion

$$J (C,S) = (F - F_{max}) . \Delta T + \hat{R}_o (\underline{X} + \Delta \underline{X})$$

After scanning all couples (C,S) we come to one of the following conclusions :

- None of the couples is admissible, then the corresponding point X is memorized as non-admissible.

- At least one of the couples is admissible, then that which results in the maximum value of J(C,S) is the optimal couple, it is memorized with the criterion

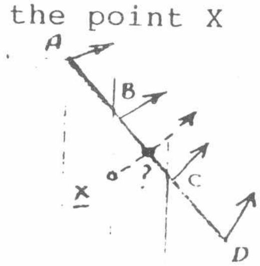
$$\hat{R}_o (\underline{X}) = \max_{C,S} J (C,S) = J (\hat{C},\hat{S})$$

We then proceed to the following point of the exploration line.

4.3.4 IMPROVEMENT OF THE ALGORITHM

A considerable improvement may be reached if we use an improved scheme for the scanning of the input couples (C,S). If we consider the known optimal accelerations, we may compute a "most probable" successor for the point X

FIG. 10



The value of the couple (C,S) obtained by interpolation of the known couples at A, B, C, D represent a good initialization for the scanning scheme.

4.4 RESULTS

The algorithm devised in the previous section was implemented on an IBM 360 44 mainframe.

The simplification assumptions and the results obtained during the application of the Hamiltonian approach enabled us to use a scanning zone of width 400 rpm around the steady state operating points of the engine. This has limited the number of points defining the operational zone within the state space to only 200 points and consequently overcomming the problem of storage requirements. Nevertheless, the algorithm needed 90 minutes to run an acceleration.

A sample of the preliminary results is shown in figures 11 & 12:

Fig.11 represents the vectors NLP, NHP in each point within the exploration domain. These values are the basis for calculating the optimal trajectories and they represent a measure of the transient behaviour of the engine within this zone.

In fig. 12 we may compare simulations of engine accelerations (idle - max. regime) starting at two different points.

5. CONCLUSIONS

Through a deep knowledge of the engine performance characteristics and using the preliminary results obtained by the Hamiltonian approach, we have succeeded in overcomming several of the implementation problems of dynamic programming.

The obtained algorithm will allow us to investigate in a thorough way the problem of optimal control of engine. This investigation will necessarily lead to adjustments both in the engine model and the computational algorithm until the final phases of engine conception.

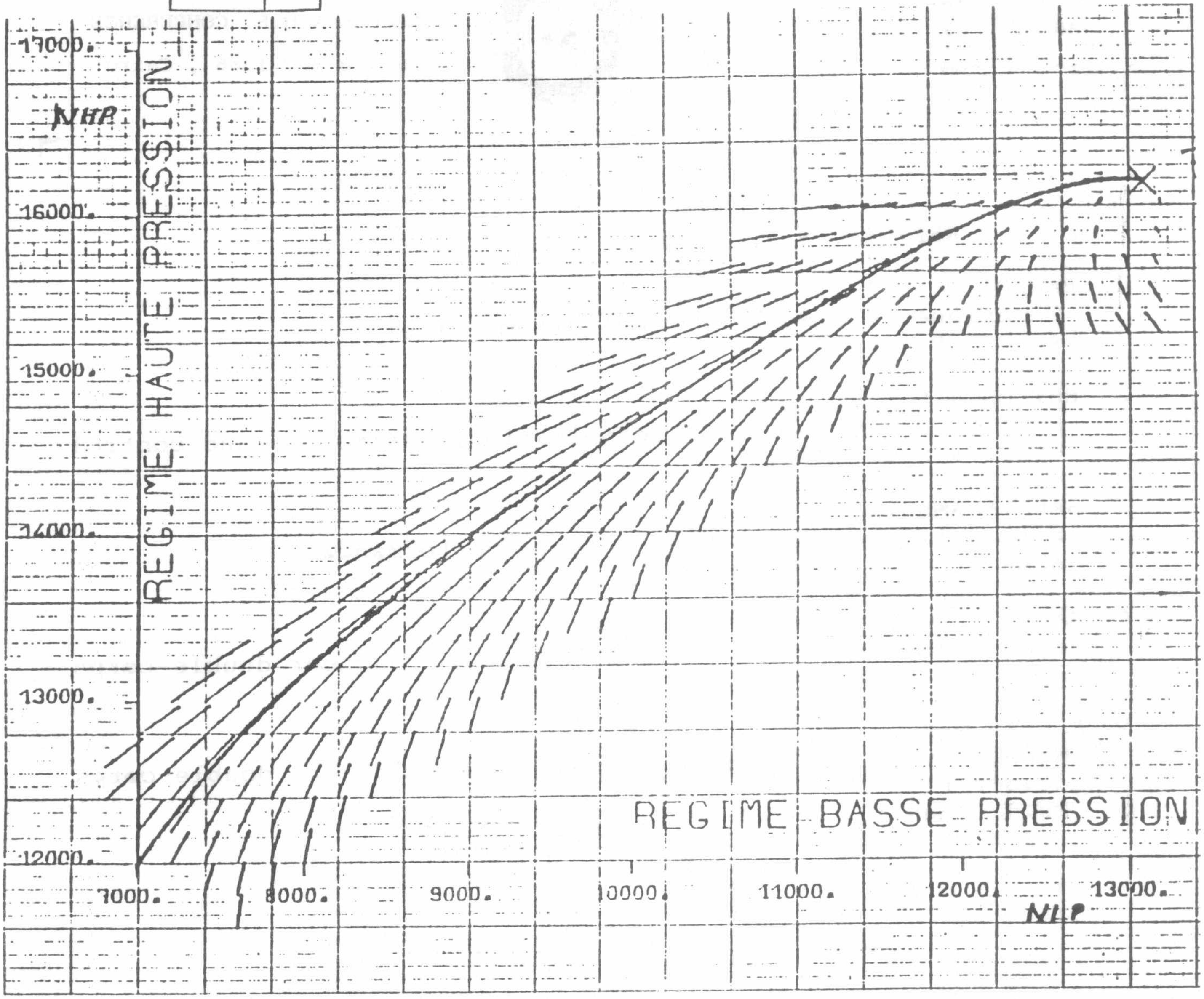


fig. 11 Vectors NLP and NHP

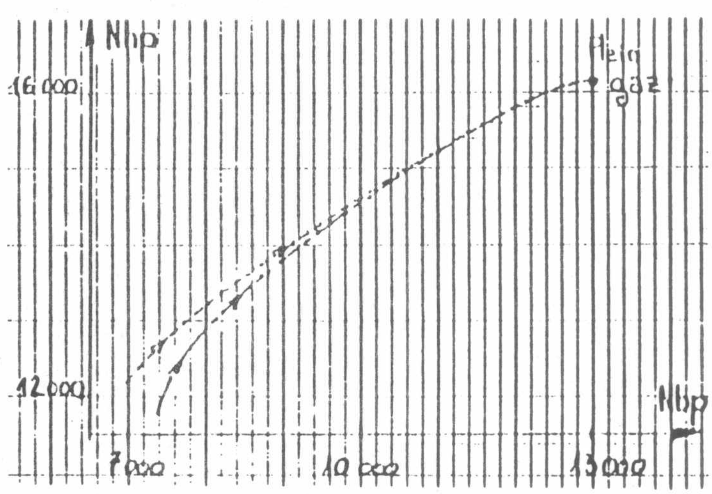


fig. 12 Optimal trajectories.

REFERENCES

1. BARBOT A.
Technique de simulation des turbomachines
AGARD CP-151,1974

2. BRYSON A.E., HO Y.C.
Applied optimal control-Optimization, estimation and control
WILEY & SONS, N.Y. 1975

3. SARAVANAMUTTO H., FAWKE A.J.
Simulation of gas turbine dynamic performance
ASME paper no. 70-GT-23

4. BARROUIL C., KADDAH M.B.
Modelisation et commande optimale d'un reacteur double-corps
Repport CERT/DERA no. 1/7222 - 1979

5. KADDAH M.B.
Modelisation et commande d'un turboreacteur double-corps double-flux.
Ph.D. Thesis , ENSAE/CERT , Toulouse 1980