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Low Cycle Fatigue and Crack Initiation Prediction at the Root of a Notch

M.GABRA* G.RABIE* M.F.E.IBRAHIM**

ABSTRACT

Conventional fracture mechanics have made it possible to quantify the slow propagation of a crack subjected to cyclic loadings,as well as the fracture phase. Unfortunately,it is still impossible to quantify seperately the initiation phase of a crack. Fatigue crack initiation was investigated by testing 2124T351 Aluminum alloy with 12 mm thick ASTM compact tension specimens having various notch-root radii between 0.1 and 10 mm . Crack initiation was detected by electric potential method. In order to calculate the number of cycles required to initiate a crack,notch root behavior was investigated through two main approaches: Elastic and Elasto-plastic analysis of stress concentration under fatigue.

The effect of confined plasticity at the notch root was analysed. Consideration of this plasticity would ameliorate to a great extent the crack initiation analysis and prediction.

* Lecturers,Dept. of Aeronautical Engineering,Military Technical College,CAIRO.

** Lecturer,Dept. of Machine Design,Military Technical College,CAIRO.

INTRODUCTION

Engineering structures are usually designed for nominally elastic stresses and do not exhibit any gross plastic deformations. However,plastic strains may occur locally at points of high stress concentration. Unfortunately,the local stresses and strains,existing at critical locations, govern the fatigue behavior of the entire structure. They are not simply related to each other and to the applied structural loads or deflections[1]. It is well known that small,sharp notches have less effect in fatigue than indicated by the theoritical stress concentration factor,K_t. Therefore,a fatigue notch factor K_f which accounts for size effect is used in place of K_t when dealing with fatigue problems. Some of the noteworthy approaches to relate K_f to material and geometric variables are due to Neuber[2],Petrson[3],Harris[4],Heywood[5] and Grover[6]. Several analysis were developed to describe the nonlinear stress-strain behavior at notches. These analysis relate the cyclic load range on a notched member to the actual stress or strain range at the notch root and then estimate the life of the notched member from the stress versus life or strain versus life plots obtained from smooth specimen testing. It is difficult to distiguish clearly how these theories can be applied to complex notch geometries. That is,how the critically stressed volume in which flaws are present is related to specific geometries and stress gradients. Further difficulty is encountered when these methods,which give constant value of K_f, are applied over a range of stess or life where K_f has been obsereved to vary.

An alternative approch is presented in this paper which does not require to solve for actual stress or strain at the notch root. Instead, a new fatigue notch factor, K_{fp}, which accounts for plasticity effect is used to convert smooth specimen data taken from 2124T351 Aluminum alloy into a life plot which can be used to estimate the number of cycles required to initiate a crack in any notched member made of same alloy.

ANALYSIS

To account for the fatigue behavior of notches,Peterson[7] and Neuber[2] equations are very popular and widely used.In general their equations may be expressed as:

$$K_f = 1 + \frac{K_t - 1}{1 + (A/\rho)^{\beta}} \qquad (1)$$

Where β = 1 for Peterson's equation ,β = 0.5 for Neuber's eq., A is a material constant and ρ is the notch root radius.

Neuber stated that the theoritical stress concentration factor K_t is equal to the geometrical mean of the actual stress and strain concentration factors K_σ and K_ε , where($K_\sigma = \Delta\sigma_{actual}/\Delta\sigma_{nom.}$, $K_\varepsilon = \Delta\varepsilon_{actual}/\Delta\varepsilon_{nom.}$) such that $K_t = (K_\sigma . K_\varepsilon)^{1/2}$. In applying Neuber's rule to the notch fatigue problems, Topper et al[8] used K_f in place of K_t, and K_σ and K_ε are written in terms of ranges of stresses and strains to take the form :

$$K_f = \left(\frac{\Delta\sigma \cdot \Delta\varepsilon}{\Delta\sigma_{nom.} \cdot \Delta\varepsilon_{nom.}}\right)^{\frac{1}{2}} \qquad (2)$$

When yielding occurs, K_σ decreases and K_ε increases such that the product $K\sigma$ and K_ε may be considered constant. This would show that the volume of metal located at the notch root undergoes "stress-strain" cycles which comply with following rule:

$$\Delta\sigma \times \Delta\varepsilon = \text{Constant} \qquad (3)$$

For specimens tested at $\Delta\sigma_{nom.}$ less than the elastic limit,

$$K_f^2 = \frac{E \cdot \Delta\sigma \cdot \Delta\varepsilon}{\Delta\sigma^2_{nom.}} \qquad (4)$$

From which :

$$K_f \cdot \Delta\sigma_{nom.} = \sqrt{E \cdot \Delta\sigma \cdot \Delta\varepsilon} \qquad (5)$$

The quantity $\sqrt{E.\Delta\sigma.\Delta\varepsilon}$ is chosen as a criterion which expresses a function of damage and would determine the fatigue behavior of notched specimens provided that the crack propagation is negligible in relation to the life of the specimen. The low-cycle fatigue data(see appendix),namely Manson-Coffin curve and the cyclic stress-strain curve,make it possible to establish a correlation between the number of initiation cylces N_i and the value $\sqrt{E.\Delta\sigma.\Delta\varepsilon}$ in accordance with fig.1. This theoritical correlation is represented graphically by the elasto-plastic theoritical curve fig.2.

CRITICAL -LOCATION APPROACH

Although the previous model is considered as a good tool to determine the number of initiation cycles, N_i, for determined values of $\sqrt{E.\Delta\sigma.\Delta\varepsilon}$, it is still inconveient for mechanical engineering applications. The inconveience of this model lays in the difficulty to accurately determine the real values of $\Delta\sigma$ and $\Delta\varepsilon$ at the critical location (notch root). Besides, the fact that K_f does vary would emply its experimental determination.

Surveying Neuber's rule once more, eq.(1) we can see that the only factor that causes the variation of K_f is A ,which Neuber considered as a material constant. Studying the units of right hand side, A would represents a characteristic distance from the notch root, beyond which there is no stress gradient and reflect then the dependence of K_f upon plasticity. Under cyclic loads, the material at the notch root undergoes either cyclic hardening or softening within the plastic zone generated proportional(in size), r_y , to the square of applied load range (or square of stress intensity factor range Δ K):

$$r_y = \frac{1}{6}\left(\frac{\Delta K}{\sigma_y}\right)^2 \text{ in plane stress} \qquad (6)$$

Thus, we can define A as a function of material and r_y, where r_y is the monotonic plastic zone zise at the root of a notch of length a mm.

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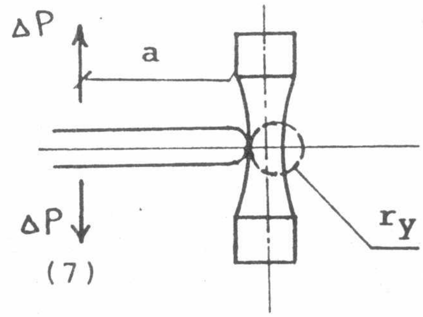

Consequently,, Neuber's rule can be modified for the same material to become :

$$K_{fp} = 1 + \frac{K_t - 1}{1 + [\ A(r_y)/\rho\ ]^{1/2}} \qquad (7)$$

From which :

$$A(r_y) = \left(\frac{K_t - K_f}{K_f - 1} \right)^2 . \rho \qquad (8)$$

and from eq. (5) of function of damage adopted previousely:

$$K_f = \frac{\sqrt{E\ .\ \Delta\sigma\ .\ \Delta\varepsilon}}{\Delta\sigma_{nom.}} \qquad (9)$$

Equations (8) and (9) together with r_y calcualtion for different load values and notch root radii would enable us to plot the relation A as $A(r_y)$. The modified fatigue notch factor K_{fp} can then be recalculated by eq.(7). Finally ,plotting $K_{fp} \cdot \Delta\sigma_{nom.}$ versus number of cycles to initiation N_i would provide an elaso-plastic theoritical curve enabling to predict crack initiation at the root of a defect showing a radius ρ and subjected to cyclic loading, without the need to determine real stress and strain ranges ($\Delta\sigma$, $\Delta\varepsilon$) at the critical location .

EXPERIMENTAL WORK

Tested compact specimens are chosen with a width W of 75 mm and a thickness B of 12 mm. Five notch - root radii ranging between 0.1 mm and 10 mm are tested to give different values of stress concentration,while the notch length is kept constant in all cases, $(a/w) = 0.5$.
Considering the linear fracture mechanics and according to Clark [9],:

$$\Delta\sigma_{nom.} = \frac{\Delta P}{B(W-a)} \left[1 + 3 \frac{(W+a)}{(W-a)} \right]$$

and

$$\Delta\sigma_{max.} = \frac{2\,\Delta K}{\sqrt{\pi\rho}}$$

where ΔK is calculated by : $\Delta K = \frac{\Delta P}{B\sqrt{W}} . f(a/W).$

Both the effect of crack length and notch depth are considered in a polynomial expression for f(a/W).

Theoritical stress concentration factor $K_t = \Delta\sigma_{max.} / \Delta\varepsilon_{nom.}$

Specimens were loaded in tension by a servo-valve MTS machine of ± 150 KN. capacity. Sinesoidal signals of 10 Hz frequency are chosen to study different domains of initiation for total life duration ranging from 10^3 to 10^5 cycles.
The electrical potential method is applied to detect the crack initiation. For the initiation criterion, we considered the number of cycles corresponding to a measureable deviation of electric energy during the experiment would indicate the begin of the microcrack to open in mode I and thus corresponds to the

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end of initiation phase [V_o = 82 mV, V=0.2mV, $\Delta V/V_o$ =0.0024] Experimental results and preliminary calculations are listed in table 1.

ANALYSIS OF EXPERIMENTAL RESULTS

In a previous work [10], the crack initiation was analysed in terms of the elastic parameters $\Delta\sigma_{nom.}$ and $\Delta K/\sqrt{\rho}$ as shown by fig.3 and fig.4. This analysis showed that : 1) for the same notch root radius, the relation is linear. 2) for the same number of cycles to initiation N_i, a higher load amplitude must be applied as the notch -root radius increases. and 3) the dispersal of results is reduced for the analysis in terms of $\Delta K/\sqrt{\rho}$ but still giving different linear relations for different notch-root radii.

The interpretation and application of modified Neuber's rule with the notation of K_{fp} are explained hereafter. Experimental results presented in table 2 together with calcualted parameters enabled to establish:

1) Plot of $A = A(r_y)$ shown in fig.5 ,expressed as:

$$A = 0.285 \; e^{4.95 \, r_y}$$

2) Calculation of the modified fatigue notch factor K_{fp}
3) Plot of $K_{fp} \cdot \Delta\sigma_{nom.}$ vesus N_i Fig.6, resulting in a theoritical elasto-plastic curve, approximated by the relation:

$$N_i = \left(\frac{695.5}{K_{fp} \cdot \Delta\sigma_{nom.}} \right)^{-0.072}$$

Which is a unique relation for all studied notch-root radii with a minimum scatter behavior.

Fig. 7 summerizes the procedure scheme to estimate the initiation phase for any notch-root radius in 2124 T 351 Aluminum alloy.

DISCUSSION AND CONCLUSIONS

Initiation data from Reference[11] were used to check the applicability of this elasto-plastic approach together with the nottation of K_{fp} variation for different load values. Fig. 8 shows the acceptable coincedance between experimental and predicted number of load cycles to crack initiation for different notch-root radii.

The scatter band is still to be narrowed, this can be achieved through further investigations that would consider :

1) The influence of structural geometries and dimensions, namely the thickness affecting to a great extent the estimation of the plastic zone size r_y (plane stress or plane strain cases)
2) The roll of the reversed plastic zone
3) Application of highly preecised techniques in detecting crack initiation cycles.

4) The notation of critical notch-root radius, below of which the number of cycles to initiation is no more depending on the notch - root radius .
In this concern, we think that for notches of redii less than $\rho_{critical}$, the initiated microcracks are developed directly in mode I crack propagation (opening mode) without a distinguished mode II propagation (shear mode).

Nevertheless, the elasto-plastic analysis corresponds to a theoritical law of the initiation behavior of notched specimens and is derived from the conventional laws of low-cycle fatigue behavior of the material used.

ACKNOWLEDGEMENT

The Authors are very gratefull to Prof. C.Bathias of U.T.C. France for his encouragement and valuable suggestions . Thanks to Aerospacial- France for supply of the specimens .

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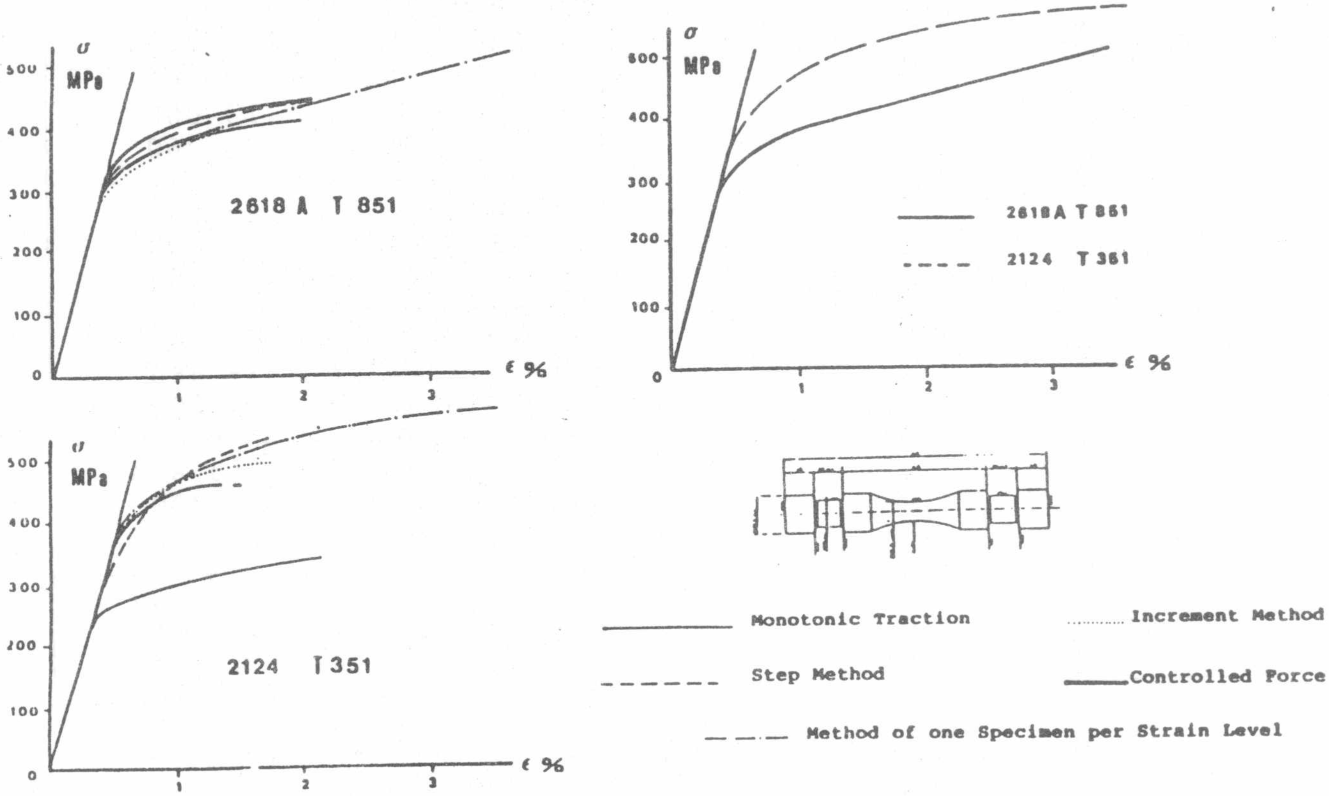
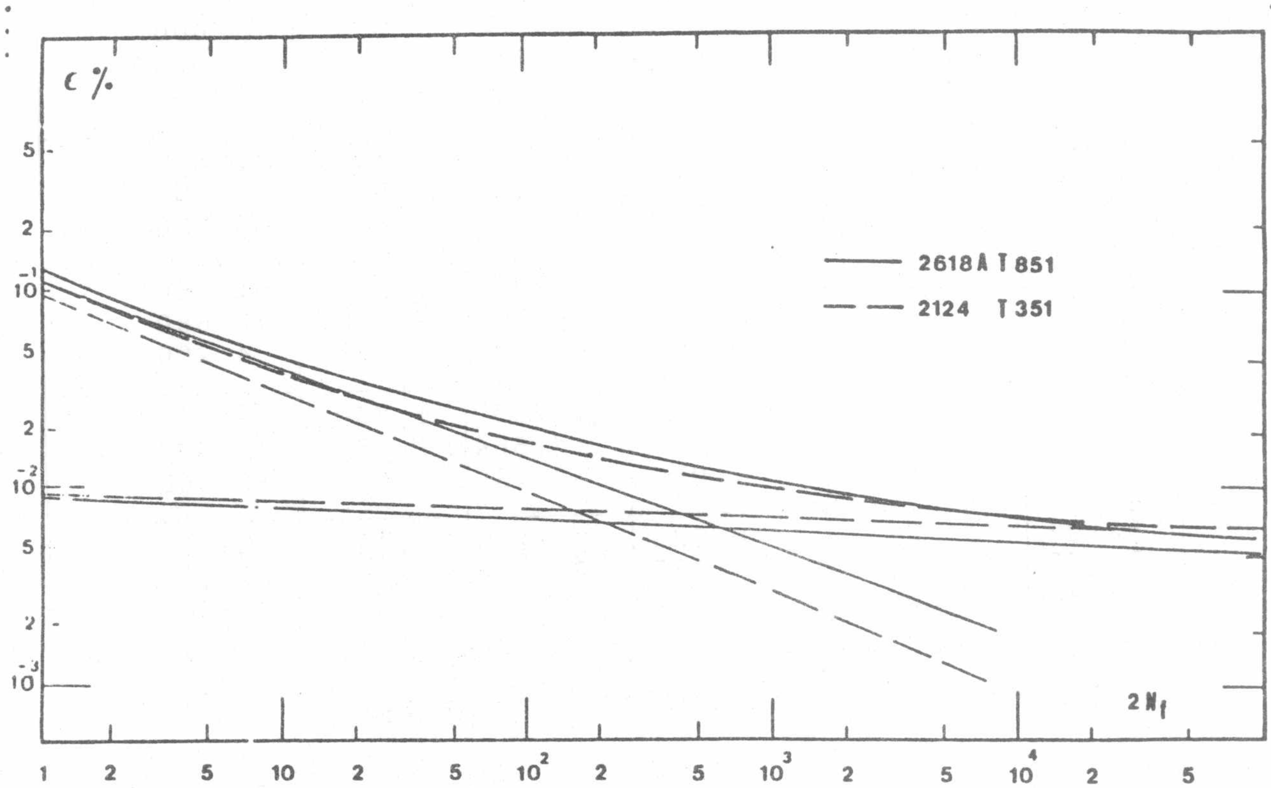


Appendix a) Cyclie Stress-Strain Curve Ref [10]



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ρ [mm]	ΔP [N]	ΔK [MPa m]	Δσ nom [Mpa]	K_t	$\Delta K/\sqrt{\rho}$ [Mpa]	$\sqrt{E.\Delta\sigma.\Delta\varepsilon}$ [MPa]	N_i [Cycles]
0.1	2380 3090 4500	6.95 9.03 13.15	52.9 68.6 100	14.85	720 930 1370	342.1 349.6 412.5	17 600 13 000 1 760
0.5	3760 4400 5820	10.98 12.85 17.00	83.6 98.0 129.3	6.62	511.2 600.8 793.7	319.2 344.9 377.6	49 000 15 700 4 800
1	4310 4900 6000	12.59 14.31 17.53	95.8 108.8 133	2.69	414.2 471.2 575.5	314.5 323.6 381.7	61 560 39 680 4 200
5	7840 11230 13400	22.9 32.80 39.14	174.2 249.6 297.8	2.1	336.5 482.2 575.5	302.5 344.9 350.4	113 850 15 700 12 600
10	9310 12470 15040	27.19 36.42 43.93	260.8 277 334	1.48	283 379 457	296.3 329.7 356.9	159 800 30 000 9 850

Table (1) Test Results and Primary Calculations

ρ [mm]	ΔP [N]	ΔK [MPa m]	r_y [mm]	A [mm]	K_{fp}	N_i [Cycles]
0.1	2380 3090 4500	6.95 9.03 13.15	0.034 0.057 0.122	0.235 0.558 1.182	6.47 5.12 4.12	17 600 13 000 1 760
0.5	3760 4400 5820	10.98 12.85 17.00	0.085 0.116 0.204	0.494 0.758 1.857	3.82 3.52 2.92	49 000 15 700 4 800
1	4310 4900 6000	12.59 14.31 17.53	0.112 0.145 0.217	0.379 0.756 0.966	3.283 2.980 2.861	61 560 39 680 4 200
5	7840 11230 13400	22.90 32.80 39.14	0.370 0.760 1.080	1.223 17.664 137.80	1.740 1.38 1.180	113 850 15 700 12 600
10	9210 12470 15040	27.19 36.42 43.93	0.522 0.937 1.364	0.123 23.296 367.10	1.43 1.19 1.07	159 800 30 000 9 850

Table (2) Elaso-Plastic Notch Factor K_{fp}

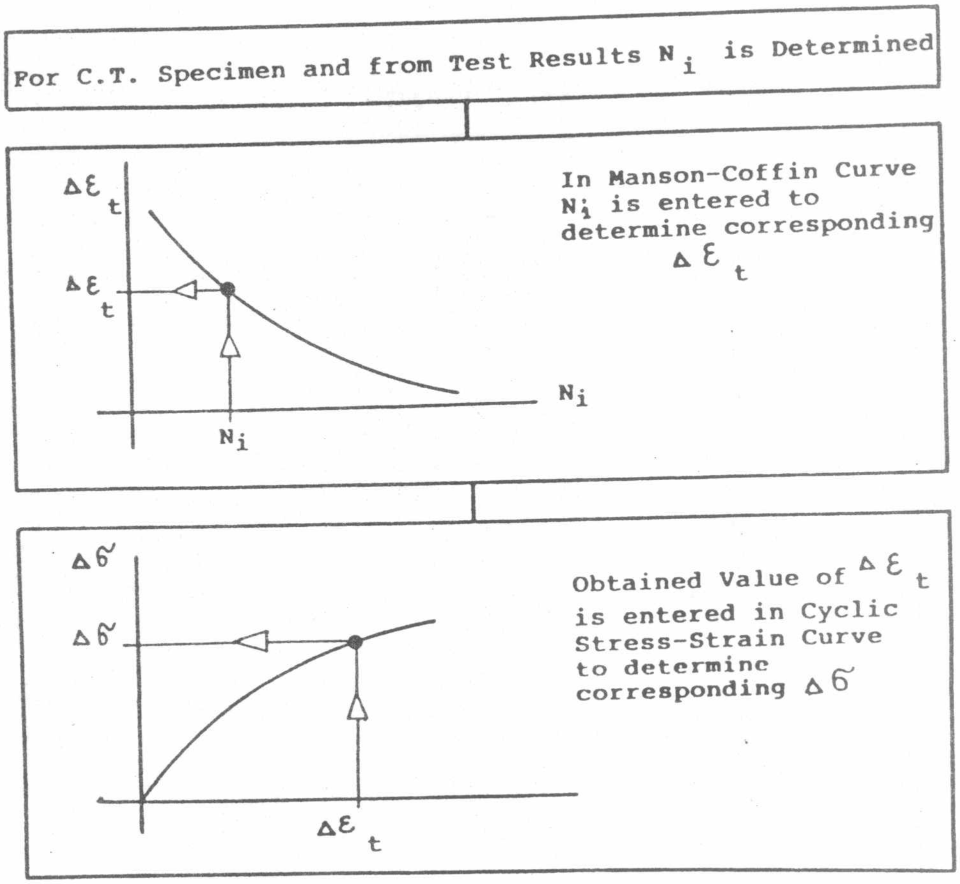


Figure (1) Theoritical Approach to correlate $\sqrt{E.\Delta\sigma.\Delta\varepsilon}$ and N_i

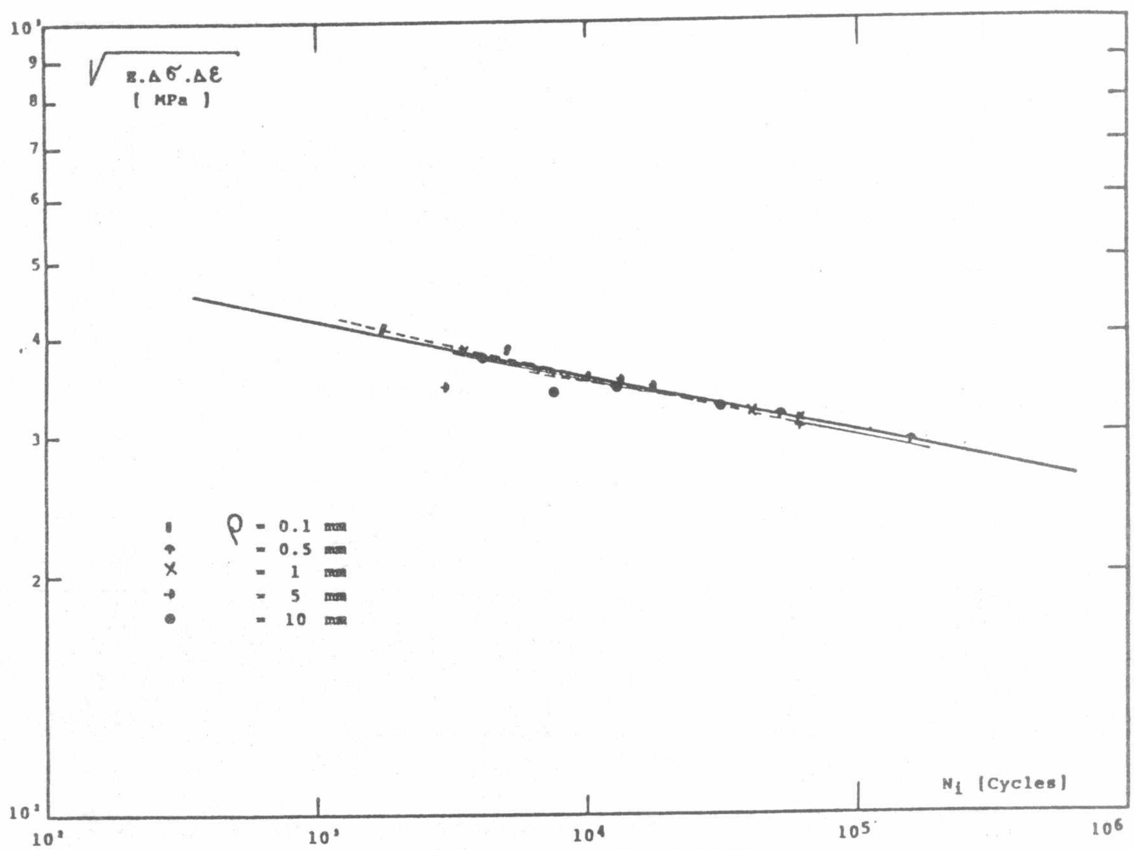


Figure (2) Elasto-Plastic Theoritical Curve

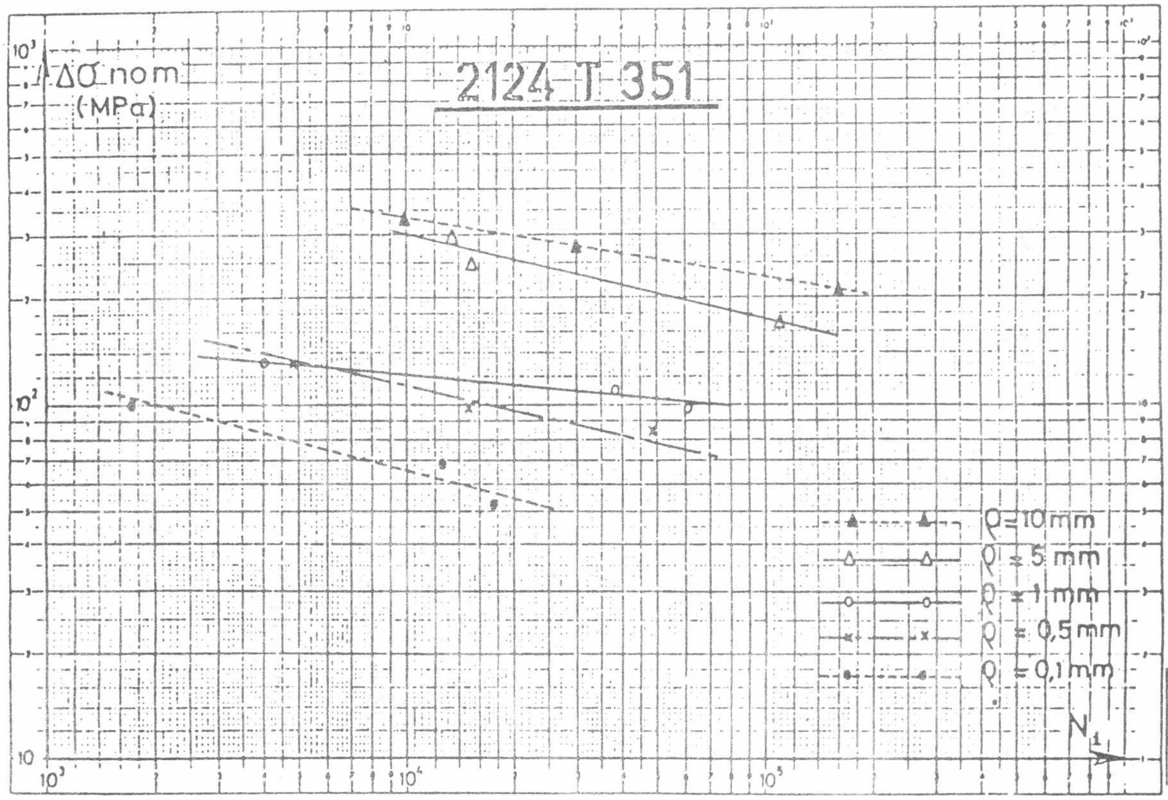


Figure (3) Analysis of Initiation in Terms of $\Delta\sigma_{nom}$.

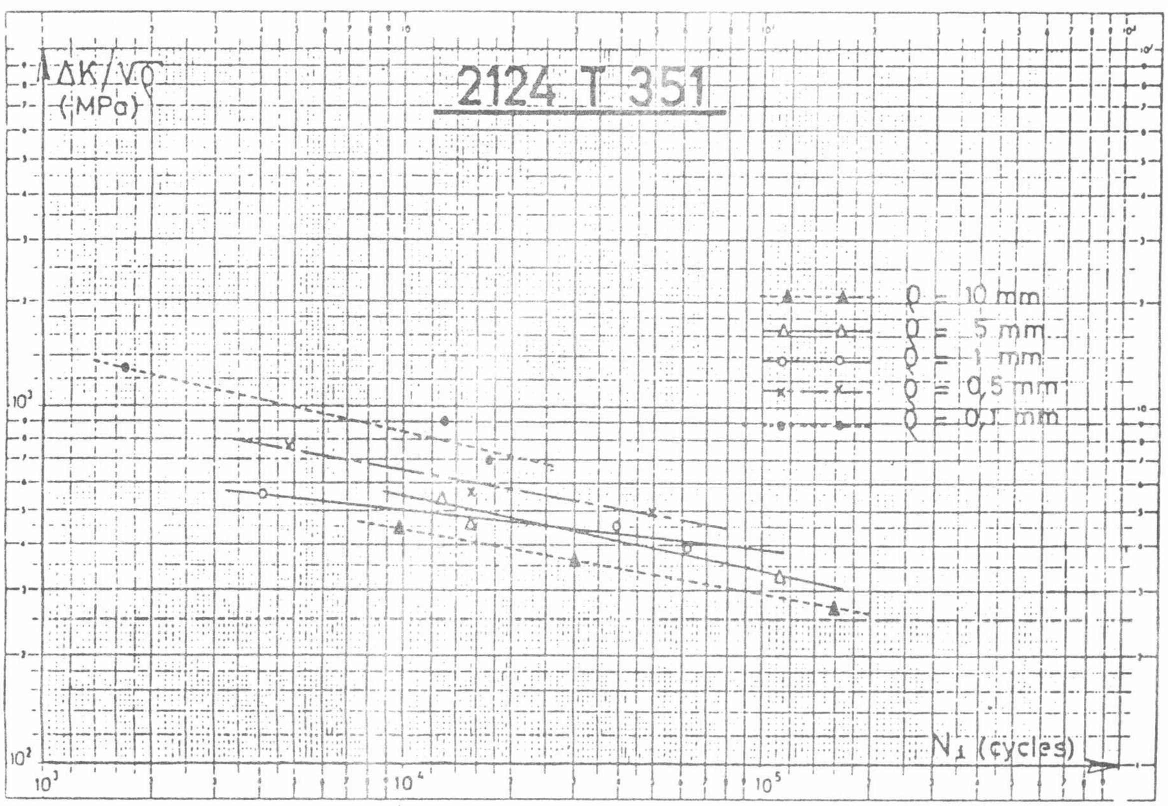


Figure (4) Analysis of Initiation in Terms of $\Delta\sigma_{max.}$ or $\Delta k/\sqrt{\rho}$

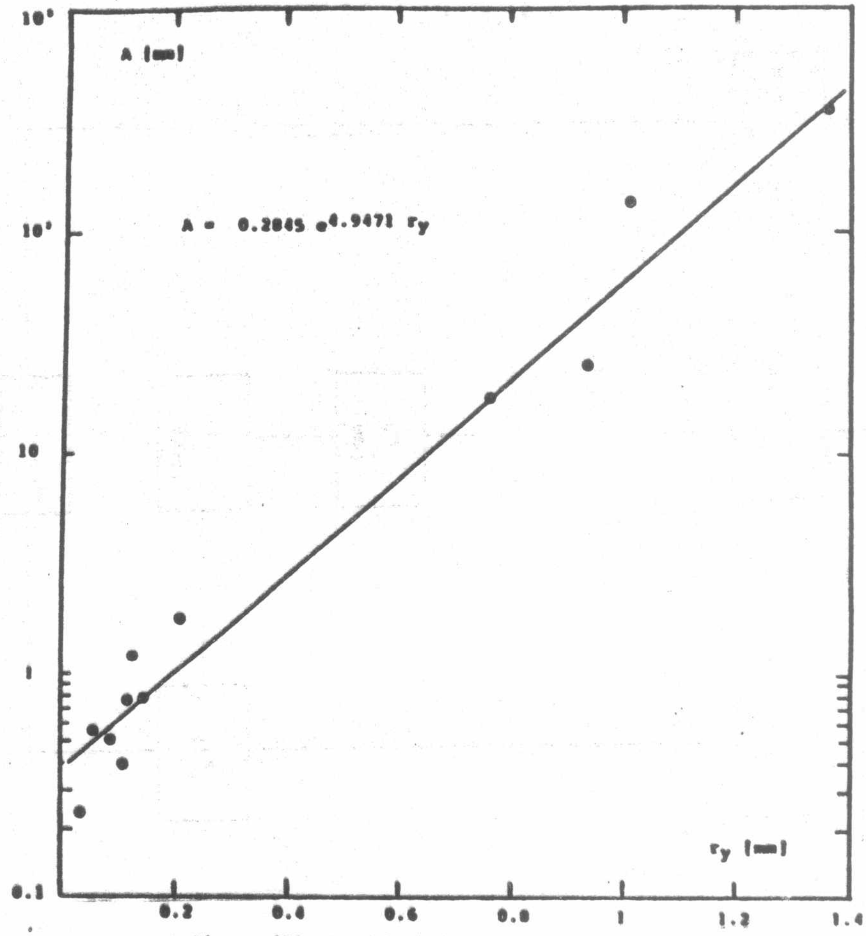


Figure (5) A as function of Plastic zone size ry

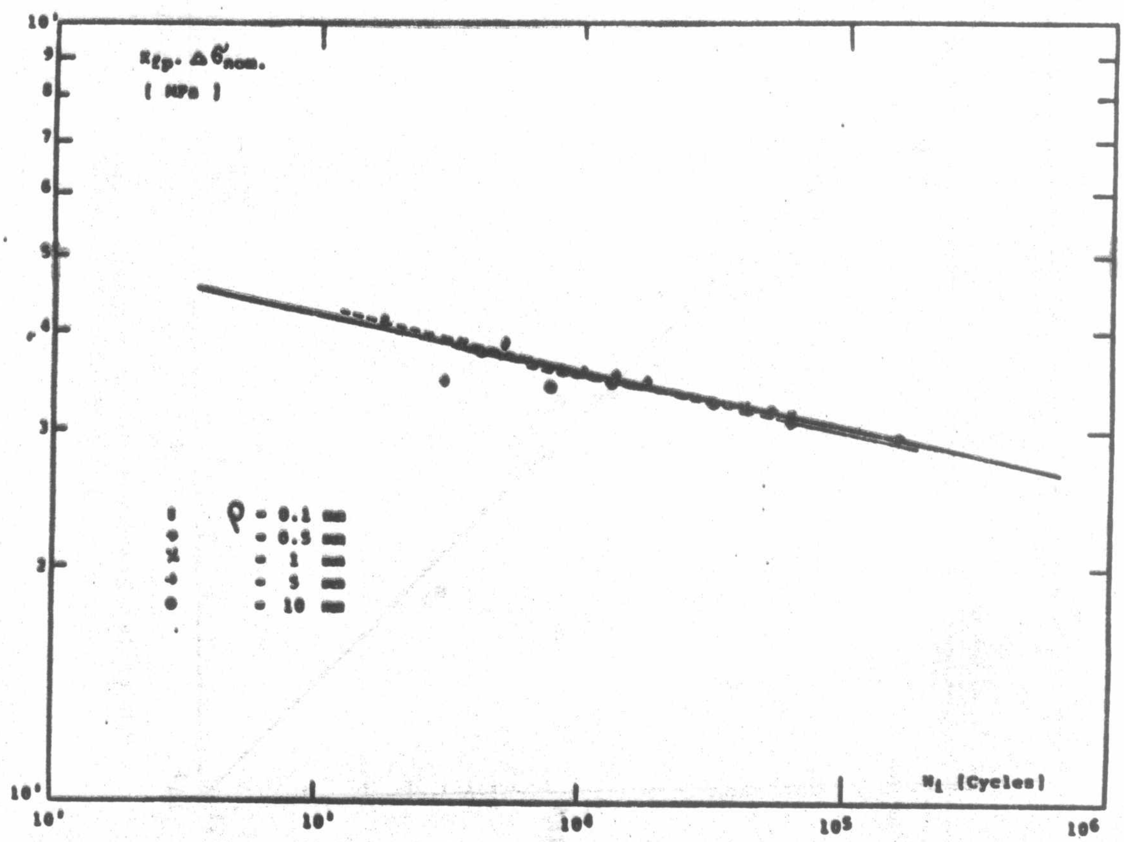


Figure (6) Theoritical Elaso-Plastic Correlation(Kfp. nom.)

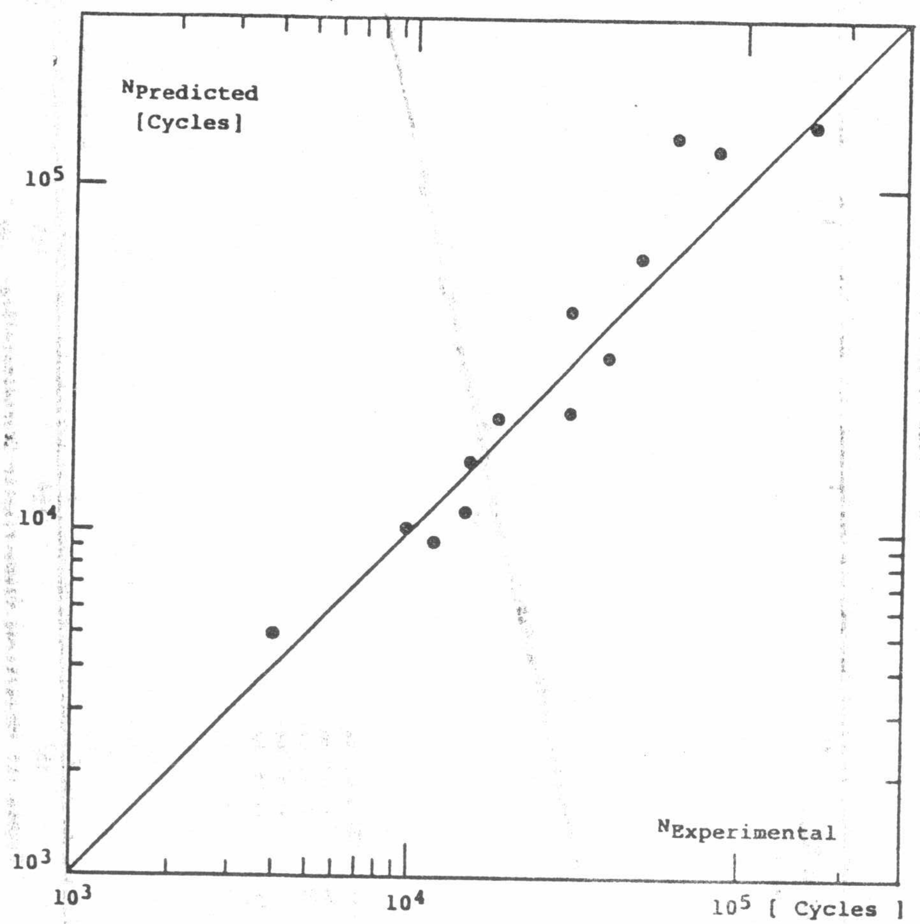
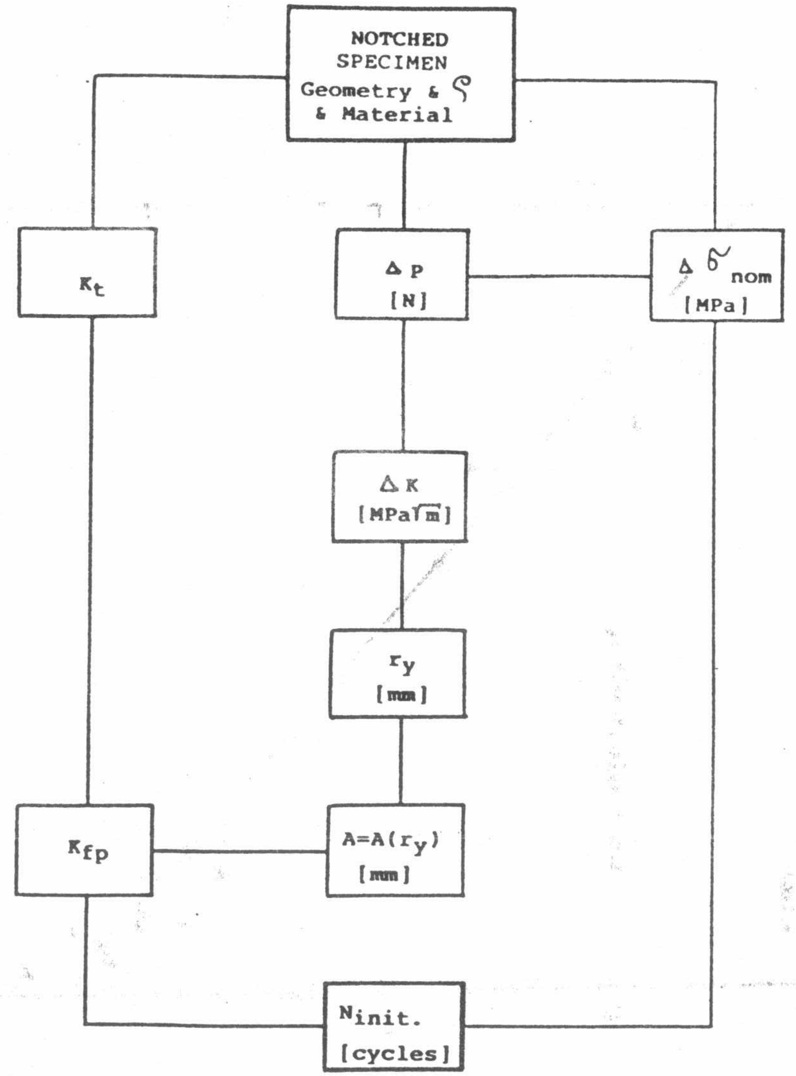


Figure (8) Predicted and Experimental Number of Cycles to Crack Initiation



Figure(7) Procedure Scheme to estimate $N_{initiation}$