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APPLICATION OF HERTZ ANALYSIS TO THE CONTACT OF SURFACES COVERED  
BY SOFT METAL FILMS

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ABSTRACT

Understanding of the behaviour of Hertzian contact is basic to the design of the tribological contacts such as rolling bearings, gears and cams. This paper describes an investigation of the static contact between a steel ball on a lead plated steel flat. Both the normal approach of these surfaces and their area of contact have been measured as functions of normal loads and the thickness of the lead film. It is shown that at high loads all the coated surfaces behaved elastically. It is also shown that the Hertz solution can be applied to calculate both normal approach and contact area when using the suitable value of the elastic modulus. An empirical equation has been given to calculate the equivalent elastic modulus which is shown to be function of both, elastic properties of the contacting surfaces together with the film thickness.

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## INTRODUCTION

For many years, Hertzian analysis has been applied to studies of asperity contact between engineering surfaces in static contact (1) and in sliding contact (2).

More recently, Hertzian analysis has been extended to model the contact of systems where one surface is covered by a layer of material having elastic properties different from those of the substrate (3). These models are also relevant to any tribological systems where solid/solid contact can occur, due to the presence of some sort of film on almost all engineering materials.

The experimental program has been made to investigate the contact behaviour of soft metal-film coated surfaces in case of static contact, with the aim of providing the designer with the contact data by application of the simple Hertz solution on the static contact of two surfaces one of which is covered by a thin soft-metal film.

## EXPERIMENTAL DETAILS

### SPECIMEN PREPARATION

Flat annular discs of SAE 52100 steel, 90 mm OD, 40 mm ID and 10 mm thick were ground and polished to a surface finish  $0.17 \mu\text{m } R_A$ . The test specimens were coated with thin films of lead using the ion-plating process, which gives excellent adhesion between the film and the substrate. Specimens were prepared with film- Thicknesses of 1.5, 3, 6, 9 and 12  $\mu\text{m}$ .

### EXPERIMENTAL PROCEDURE

The apparatus illustrated schematically in Fig.1, was designed to measure the normal approach of the surfaces when three spheres equally spaced around a circle are compressed between two similar flat surfaces.

Measurements were made using loading masses of 1, 2, 3, 4, and 5 Kg, and ball diameters of 7.935 mm and 3.175 mm.

The capacitance probes used for measurement of the normal approach consisted of a 25.4 mm diameter steel ball, and the end face of a 25.4 mm disk steel cylinder. Both probes were located in insulating material contained in earthed steel cups. The capacitance was measured using a standard capacitance bridge, and was found to obey the empirical relationship shown in Fig.3. In the present work, the diameter of the contact area was also measured. This contact area could be seen quite clearly as it was more lustrous than the surrounding surface, due to plastic deformation of some



asperities within the contact areas.

The contact diameters were measured using the measuring system of a Leitz microhardness tester.

RESULTS AND DISCUSSION

When an elastic sphere is loaded against an elastic half space the radius of the contact circle and the normal approach, according to Hertz solution, are calculated as follows :

$$a = \left( \frac{3Nr}{4E'} \right)^{1/3} \quad (1)$$

$$\delta = \left( \frac{9N^2}{16E'^2 r} \right)^{1/3} \quad (2)$$

$$k = \frac{1}{E'} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} = K_1 + K_2 \quad (3)$$

where a = radius of contact circle

δ = normal approach

N = normal load

r = ball radius

E' = equivalent elastic modulus defined by the relation (3)

E<sub>i</sub>, ν<sub>i</sub>, K<sub>i</sub> = Youngs modulus, Poissons ratio and modulus of rigidity of material i.

A plot of log a Vs log N should, therefore, give a linear relationship having a slope of 1/3 for elastic deformation.

Simelrly, a plot of log δ VS log N should give a linear relationship having a slope 2/3.

Graphs for log a VS log N are shown, for both ball diameters and various film thicknesses, in Fig.4 . It can be seen that at higher loads, all surfaces give slopes of 1/3 indicating elastic deformation. At lower loads this relationship was maintained for the thinnest films only.

Graphs for log δ VS log N are also shown in Fig.5 giving similer results with slope 2/3.

It can also be seen that the graphs for different film thickness do not coincide, where the contact radius and the normal approach at any load increasing with film thickness.

This observation suggests, at first sight, that the modulus of a coated

surface decreases with increasing film thickness. Although this conclusion has not yet been proven to be correct, the experimental results have been used, backwards, to calculate the equivalent modulus of rigidity  $K$  of the coated surfaces behaved elastically assuming that Hertz solution is applicable. Assuming that the film/substrate system can be treated as new composite material, having modulus of rigidity  $K_c$ , knowing the values of the contact radius at different loading conditions, we would be able, by applying Hertz equations, to calculate the corresponding values of  $K_c$  (equations 1 and 2).

From equation 3 we get

$$K_c = K_2 = K - K_1$$

where

$$K = \frac{1-\nu_1^2}{3Nr} = \left( \frac{16.5^3 r}{9N^2} \right)^{1/2}$$

$$K_1 = \frac{1-\nu_1^2}{E_1} \quad \text{for the steel ball.}$$

Fig.6 shows the graphs of  $K_c$  VS  $t/a_0$  where  $t$  is the film thickness and  $a_0$  is the contact radius of the ball with a flat substrate material as calculated from Hertz solution.

The experimental results show that the value of  $K_c$  approaches the value of that of the substrate material  $K_s$  when  $t/a_0$  tends to zero, and approaches the value of that of the film material when the value of  $t/a_0$  tends to infinity.

The following empirical equation has been established to fulfill these requirements

$$K_c = K_s \cdot \frac{1+A \cdot K_f \cdot t/a_0}{1+A \cdot K_s \cdot t/a_0} \quad (4)$$

Equation (4) is shown as dashed line in Fig.6 to fit the experimental results and it showed a good correlation when the value of  $A=7.6 \cdot 10^5$ .

Where in equation (4),  $K_s$  and  $K_f$  are the modulus of rigidity of substrate and film materials respectively.

El Sherbiny and Halling (3), introduced a theoretical solution for the problem of deformation of thin elastic film on a rigid substrate. A computer program was established to calculate the values of  $K_c$  by applying Hertz solution on their calculated values of the contact radius  $a$ , for

different values of film thickness. The results are also plotted in Fig-6 and shown as full line. It is shown that the theoretical solution gave lower values of  $K_c$  than the experimental results. This difference should be due to ignoring the elasticity of the substrate by assuming it to be rigid.

#### CONCLUSION

Thin soft films on hard substrate behave mainly elastically under the application of high values of normal loads. The contact parameters, needed by the designer could be approximately calculated using Hertz solution in which a modified modulus of rigidity is used. This modulus of rigidity is calculated from the given empirical equation and is shown to be function of the elastic properties of both film and substrate materials, and the film thickness.

#### REFERENCES

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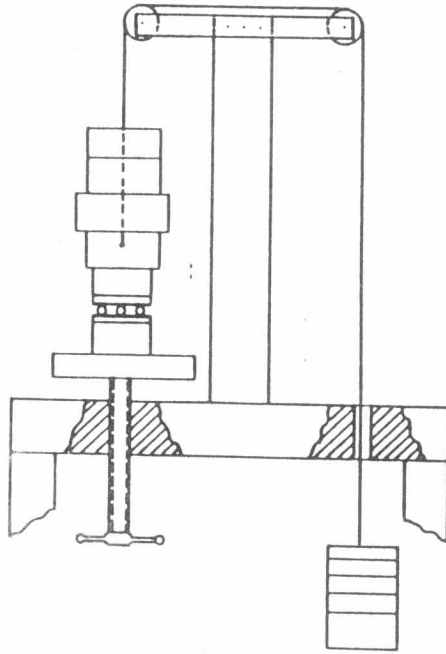


Figure (1) Normal approach test rig

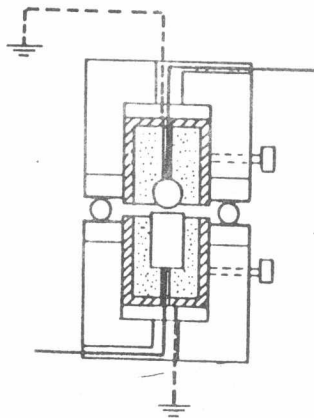


Figure (2) Capacitance probes

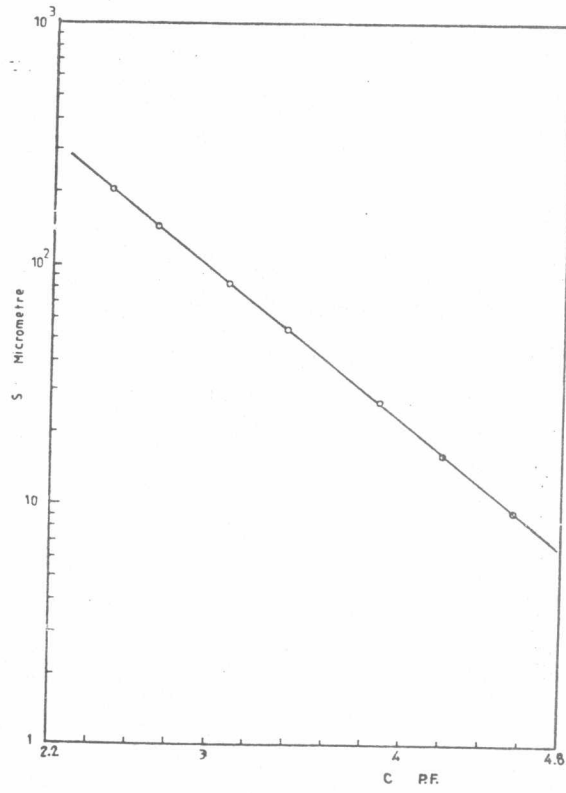


Figure (3) Calibration chart for the capacitance probes

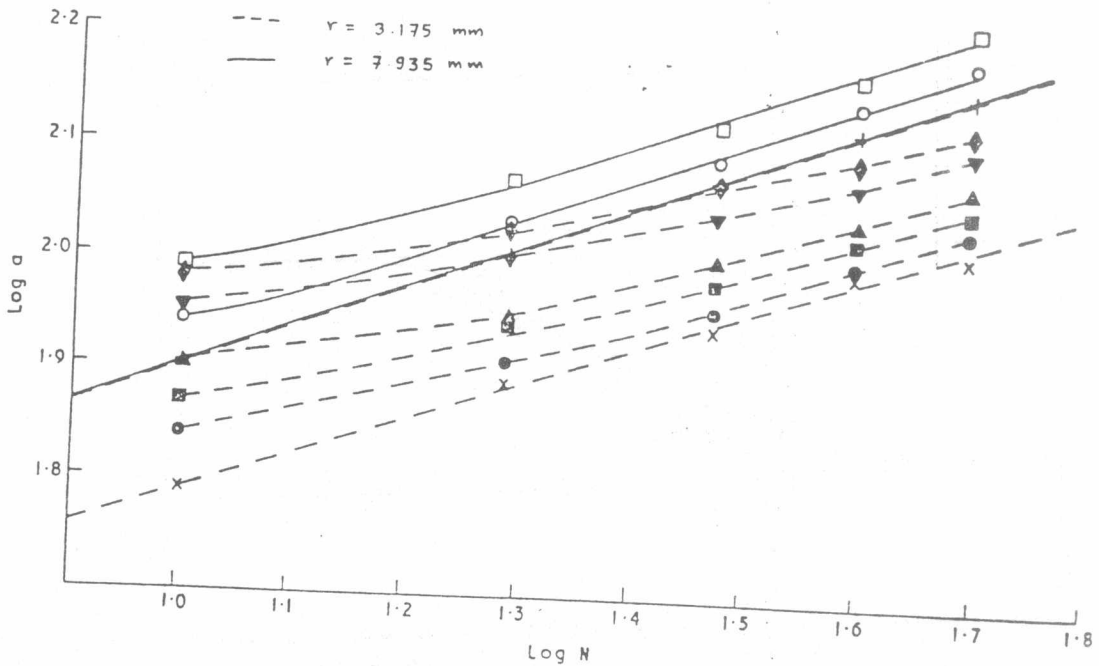


Figure (4)

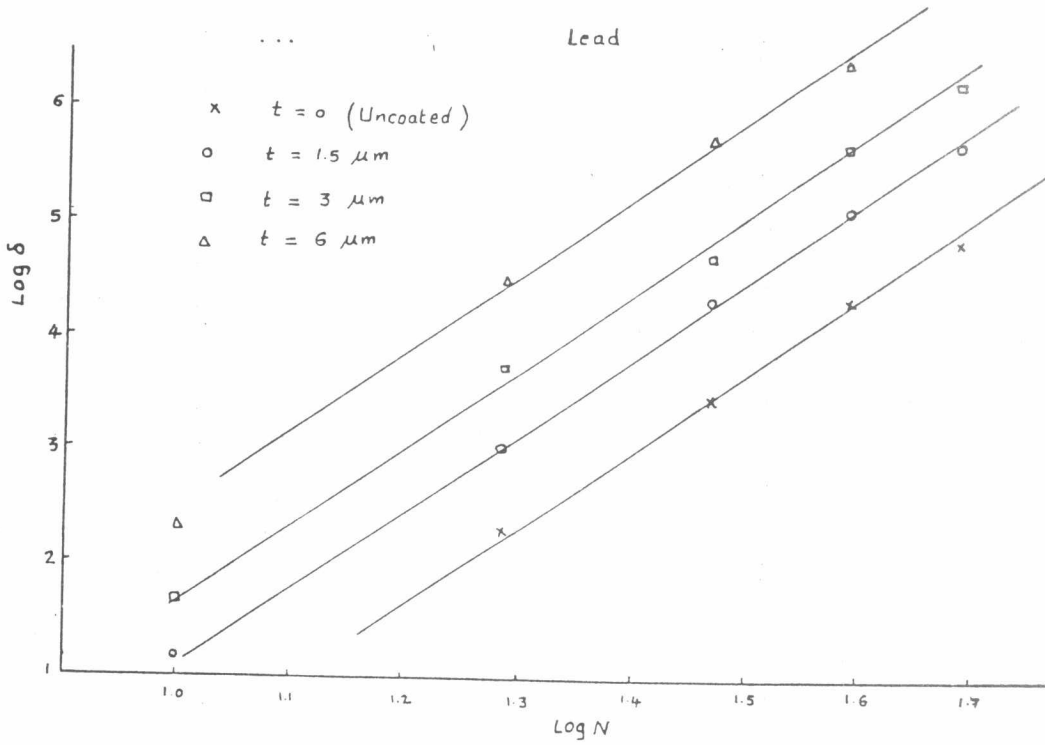


Figure (5)

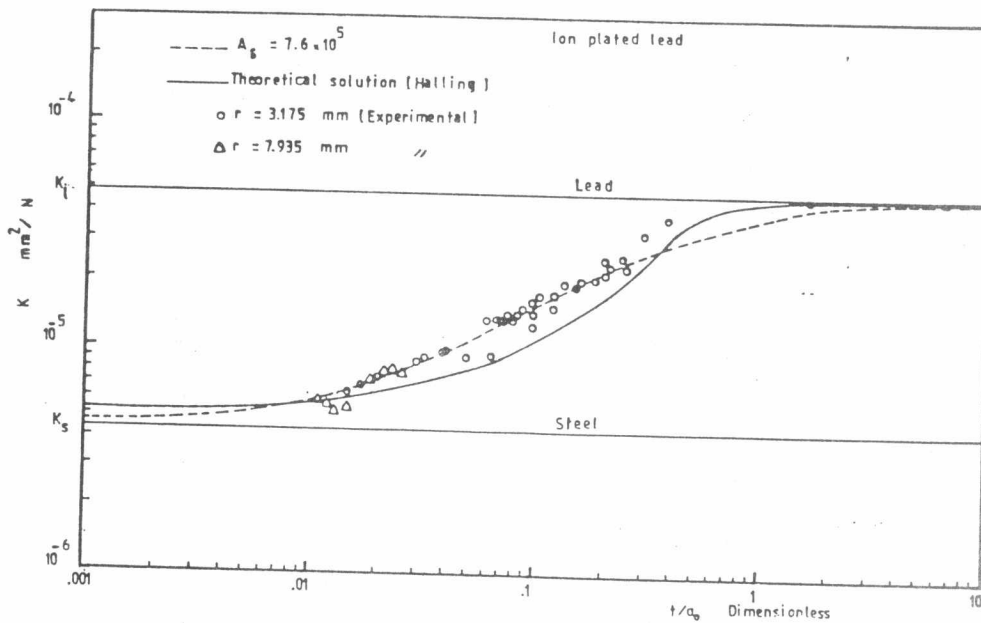


Figure (6) Relationship between  $K$  and  $t/a_0$  for ion plated lead films on steel substrate. Equation (4) is shown by dashed line for  $A = 7.6 \times 10^5$  for Lead