SCALING CRITERIA OF SOLIDIFICATION AND CASTING PROCESSES

by

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ABSTRACT

To ensure the quality of large size castinges, production of casting models on a laboratory scale is necessary as a preliminary step. The main object of this paper is to find, by performing a similarity analysis, the main criterial similarity groups to be preserved in the prototype and a small dimensioned model.

The given similarity approach is based on a comprehensive mathematical analysis of the solidification process, heat transfer mechanisms and temperature differentials through the governing differential equations of continuity, momentum and energy. Applying mathematical techniques on these equations, the following dimensionless numbers were found to have a valuable importance : phase change number, superheating number, drift flux number and firiction number. The physical significance of these numbers are discussed and the conditions imposed by them in design of model-casting process are evaluated.

The mentioned results are applied to a simple case for the process of casting simulation. Hence, interesting conclusions of the feasibility of this methed for modeling a casting process have been stated.

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1. INTRODUCTION :

Solidification during casting processes may be considered as a two phase phenomena in which the natural convection in molten metal movement(drift)of the solidified particles have the dominant role[1]. So, searching for scaling criteria in casting would be based on this concept.

The determination of scaling criteria for a natural convection phenomena in single phase processes is achieved through appropriate non-dimensionalization of the well established balance and constitutive equations as carried out by Singer [2] and Heisler [3]. However, the same approach for two phase processes encounters considerable difficulties due to the existing uncertainities in the basic formulation related to balance equations and two phase flow correlations.

The available methods to develop similarity criteria for a two-phase process where the natural convection is the dominant phenomena (natural drift) have been reviewed by Ishii and Jones [4]. In the present analysis, the results based on the local conservation equations and ones based on the perturbation method are utilized. The extension of the similarity analysis to a natural drift is achieved by considering the scaling for a small perturbation method and the steady state solution. For this purpose, the relatively well established drift flux model and constitutive relations [5,6] are used.

The above resluts are applied to the simulation of natural processes during solidification of metals in casting processes. Hence; similarity criteria for modeling of casting and solidification process for foundry have been concluded.

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2. BASIC FORMULATION

The similarity parameters for a natural convection movement under a two phase condition can be obtained from the integral effects of the local two phase balance equations. Under a natural drift conditions, the majority of transients are expected to be relatively slow. Furthermore, for developing system similarity laws, the response of the whole mixture is important rather than the detailed response of each phase and phase interaction [6,7]. The resulting transfer functions can be nondimensionalized. From these, the govering similarity parameters are obtained. Such method may give quite useful similarity laws.

For the derivation of system similarity under the natural convection conditions, the drift-flux model is appropiate because it can properly descripe the structure interactions [2,3,8]. So, the similarity criteria based on the drift flux model can be developed by two different methods. The first method is based on one-dimensional drift flux model by choosing proper scales for various parameters. It is obtained from the differential equation, so it has the characteristics of local scales. This method is useful in evaluating the relative importance of the existing physical effects and mechanisms. The second method is based on small perturbation techniques and consideration of the whole effective respones. The local responses of the main variables are obtained by solving the differential equations, then the integral effects are found.

In what follows, the combination of the results from the above two methods will be used to develop practical similarity criteria for a solidification process including fluid flow and a natural convection prenomena expressed as " natural drift " :

- Mixture continuity equation (mixture of the liquid and solidified metal) :

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial z} (\rho_m u_m) = 0 \qquad (1)$$

- Continuity equation for liquid metal :

$$\frac{\partial \alpha \rho}{\partial t} + \frac{\partial}{\partial z} (\alpha \rho u_m) = -\frac{\partial}{\partial z} \left(\frac{\alpha \rho_s \rho}{\rho_m} V_{1j}\right) - \Gamma_S \qquad (2)$$

- Mixture Momentum Equations :

$$\frac{\partial \rho_m u_m}{\partial t} + \frac{\partial}{\partial z} (\rho_m u_m^2) = - \frac{\partial P_m}{\partial z} - \rho_m g - \frac{\partial}{\partial z}\left(\frac{\alpha \rho \rho_s}{(1-\alpha)\rho_m} V_{1j}^2\right) -$$

$$\left[\frac{f_m}{z_D} + K.\delta.(z-z_i)\right] \rho_m u_m |u_m| \qquad (3)$$

- Mixture Enthalpy-Energy Equations (i^{th} section) :

$$\frac{\partial \rho_m H_m}{\partial t} + \frac{\partial}{\partial z} (\rho_m^2 u_m H_m) =$$

$$4 h_m (T_{sol} - T_w) - \frac{\partial}{\partial z} \left(\frac{\alpha \rho \rho_s}{\rho_m} \Delta H_{fs} V_{1j}\right) \qquad (4)$$

- Mold Energy Equation

$$\rho_w \cdot C_{P_w} \frac{\partial T_w}{\partial t} + K_w \nabla^2 T_w - q_w = 0 \qquad (5)$$

- Mold-Casting Boundary Conditions (i^{th} Section)

$$- K_w \frac{\partial T_w}{\partial y} = h_w (T_{sol} - T_w) \qquad (6)$$

Here V_{1j} is the drift velocity originated from the density difference between the solid and liquid particles during the solid fication process) :

$$V_{1j} = (1 - \alpha)(u_e - u_s). \qquad (7)$$

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The mixture friction factor and heat transfer coefficients are denoted by f_m and h_m respectively. The constitutive relations for the drift velocity, v_{lj} , and the solid source term Γ_s are specified in the above formulation. Under the thermal equilibrium condition, it can be shown that:

$$\Gamma_s = \frac{4}{d} \frac{h_m(T_{sol} - T_s)}{\Delta H_{fs}} \qquad (8)$$

Where h_m is the mixture heat transfer coefficient that can be found from the following equation:

$$N_u = 4,82 + 0,0185 \; (\mathrm{Re} \; \mathrm{Pr})^{0,83} \qquad (9)$$

The represertative constitutive equation (8) for the drift velocity is given by

$$V_{lj} = 0,2 \left(1 - \sqrt{\frac{\rho}{\rho}} \right) j + 1.4 \left(\frac{g \, \Delta\rho \, \mu}{\rho^2} \right)^{1/4} \qquad (10)$$

where the total flux (j) is given by

$$j = u_m + \frac{\alpha \, \Delta\rho}{\rho_m} \, V_j \qquad (11)$$

The relative motion between particles in both phases can be specified by a number of different forms. As an example of forms is the classical solid-liquid ratio that can be described by an equation of the form

$$\alpha = \frac{V_s}{V} = F \; (z) \qquad (12)$$

It is evident that the above balance equations is written for a mixture of liquid and solid metal during the process of pouring and solidification. It is based mainly on a hypothesis, as mentioned, stating that the dominant role here is due to change in densities as well as the mechanism of heat transfer and heat release pattern during the phase change process. So, the significance of different terms in these equations is based on this hypothesis and the analysis found in reference [5]. As an approach to deal with these equation for concluding the simulation parameters, the method of small perturbation is a convenient technique that is fully demenstrated in reference [6].

3. SIMILARITY CRITERIA

The similarity groups for the solidification process can be obtained from the mentioned set of balance equations by performing a perturbation analysis. The results obtained are sunmmarized as follows :

Phase change No: $N_{pch} = \frac{q''_o \, l_o}{d \, u_o} \, \frac{1}{\Delta H_{fs} \rho} \, \frac{\Delta \rho}{\rho} = \frac{\text{Flux through wall}}{\text{Flux due to phase-change}}$

Overheating No: $N_{over} = \frac{H_{ov}}{H_{fs}} \, \frac{\Delta \rho}{\rho} = \frac{\text{Over heating}}{\text{Latent heat}}$

Froud No : $N_{fr} = \frac{u_o^2}{g l_o} \, \frac{\Delta \rho}{\rho} = \frac{\text{Inertia force}}{\text{Gravity force}}$

Density ratio : $N_{\rho} = \frac{\rho_s}{\rho} = \frac{\text{Solidus density}}{\text{liquidus density}}$

Friction No : $N_f = \frac{d}{fe} \, \frac{(1+\frac{\mu}{u\rho d})}{(1+\frac{\Delta \rho}{\rho})} \left(\frac{a_g}{a_i}\right) = \frac{\text{Viscous Friction}}{\text{Mold Friction}}$

Gate No : $N_G = \left(1+\frac{\Delta \rho}{\rho}\right)\left(\frac{a_g}{a_i}\right) = \frac{\text{Gate effective area}}{\text{Mean effective area}}$

Time Datio No : $N_T = \frac{\alpha_M}{S_M^2} \, \frac{l_o}{u_o} = \frac{\text{Transport time}}{\text{Conduction time}}$

Stress No : $N_{\sigma} = \frac{(1-\nu)\sigma}{E.\beta.(T_{sat}-T_w)}$

Physically, the phase change number is the amount of coolling provided by the mold during the solidification process. Where as, the over heating number is the scale for molten metal temperature during the mold filling process.

4. SPECIAL SIMILARITY ANALYSIS :

The similarity criteria between two different systems can be obtained from a detailed consideration of the similarity groups developed above together with necessary constitutive relations. In a similarity analysis , subscipt R denotes the ratio between the model and prototype thus :

$$\psi_R = \frac{\psi_m}{\psi_p} = \frac{\text{For model}}{\text{For prototype}}$$

In general, the mould materials need not be the same between the model and prototype. However, for simplicity the use of the same mould materials is assumed in the present analysis. This implies :

$$\alpha_{MR} = K_{MR} = C_{PMR} = S_{MR} = 1$$

The most fundamental requirement for similarity is concerned with the geometrical similarity criteria. It is evident from the continuity equation that for a complete kinematic similarity the geometrical similarity for the flow area should be satisfied, thus

$$A_{iR} = \frac{(ai/ao)m}{(ai/ao)p} = 1$$

On the other hand, for the dynamic similarity, it is necessary that

$$(L_i/A_i)_R = 1$$

A special case of a seale model with the same metal is now examined because of its obivious importanc. In this case, all the fluid properties can be considered the same in the model and prototype, thus

$$\rho_R = \rho_{SR} = \beta_R = C_{PR} = K_R = \mu_R = \nu_{gR} = H_{fgR} = 1$$

Under the above conditions, the similarity criteria become

$$(N_{Pch})_R = \frac{q''_R}{d_R} \frac{l_R}{u_R} = 1$$

$$(N_{sup})_R = (\Delta H_{sup})_R = 1$$

$$(N_{Fr})_R = \frac{u_R^2}{l_R} = 1$$

$$(N_f)_R = \left(\frac{fl}{d}\right)_R \left(\frac{a_g}{a_i}\right)_R^2 = 1$$

$$(N_\sigma)_R = \varphi_R\left(\frac{a_g}{a_i}\right)_R^2 = 1$$

So, the similarity criteria developed above reduce to the following equations

$$u_R = \sqrt{l_R}$$

$$d_R = \sqrt{\frac{l_R}{u_R}}$$

$$a_R = \frac{1}{\sqrt{l_R}}$$

$$\left(\frac{fl}{d}\right)_R = 1$$

and $\left(\frac{a_o}{a_i}\right)_R = 1$

However, the real time simulation cannot be achieved in this case of two-phase flow due to the additional conditions imposed on the system.

5. SAMPLE CALCULATION :

As an example, a preliminary consideration on the simulation of a prototttype by a model is presented. In this case, as stated above it is not possible to operate in real scale. Therefore, the time scale should be distorted in order to have meaningful simulation. If we have between an ingot, as a prototype, and model a length scale ratio of

$$L_R = 0,4$$

Hence, from the dervied equations, the similarity criteria require that

$$U_R = 0,63$$

$$q_R = 1,58$$

$$d_R = 0,8$$

6. CONCLUSIONS

A Similarity criteria which characterizes casting processes have been derived. It is based on writing the mathematical model of the whole process. Such criteria needs to be verified through an experimental work. This task is to be performed by foundaries in Helwan.

7. NOMENCLATURE

a	Flow area
a_s	Wall cross sectional area
A	Non-dimensional area
C_p	Fluid heat capacity
C_{ps}	Solid heat capacity
d	hydraulic diameter
E	Modules of elesticity of solidified metal
f	friction factor
g	Gravity
h	Heat transfer coefficient
H_m	Enthalpy of mixture
j	Total volumetric flux
k	Conductivity of liquid metal
k_s	Conductivity of solidified metal
k	Orifice inlet conditions

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	Axial length
Nu	Nusselt number
N_{pch}	Phase change number
N	Gate number
N_{Fr}	Froude Number
N	Density ratio
N_f	Friction number (two phase)
N	Stress number
N_T	Time ratio-number
q''	heat flux through the walls
t	Time
T_{sol}	Solidus temperature
u	liquid metal velocity
u_s	Solidified particles velocity
u_m	Mixture velocity
V	Nondimensional velocity
V_{sj}	Solid drift velocity

Greek Symobls :

α	Solid fraction
β	Thermal expansion coefficient of metal
ΔT	Characteristic temperature rise
H_{fs}	Latent heat of solidification
H_{ov}	Orer (super) heating
$\Delta \rho$	Density difference
$\Delta \mu$	Viscosity difference
θ	Non-dimensional temperature
ρ	Density of liquid metal
ρ_s	Density of solidified metal
σ	Tensile strength
τ	Non-dimensional time

Subscripts :

o	Reference section (half the mold)
m	Mixture
M	Model
P	Prototype
r_R	Representative variable
s	Solidified metal
w	Mold walls

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8. References

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