THEORETICAL REDUCTION OF RATE OF DEPOSITION OF SUSPENSIONS IN TURBULENT FLOW OVER A TILTED FLAT PLATE BY APPLYING AN EXTERNAL ELECTRIC FIELD

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ABSTRACT

Particle deposition from turbulent flow of suspensions on inclined surfaces is related to many practical problems such as attenuation of solar energy collected by solar flat plate collectors due to dust deposition. In this paper, the effect of external electric field on the deposition of suspensions in turbulent flow over a tilted flat plate is investigated. The external electric field is obtained by a number of equally spaced parallel cylinders having the same voltage and height above the plate. The case of a constant external electric field is also considered.
A mathematical model for the flow of suspensions using the combined equation of continuity of fluid and particulate phases with the application of Fick's law and the equations of Poisson, external electric field and rate of deposition along with the proper boundary conditions are given. Numerical solution is obtained using the finite difference method.

The effects of the intensity of the external electric field, the number of charged cylinders and their height above the plate on the rate of deposition of the solid particles are studied.

The theoretical results indicate that an externally applied electric field obtained by an array of charged cylinders placed parallel to the plate can considerably reduce the rate of particle deposition from turbulent flow of suspensions over a tilted flat plate if the potential on the electrodes of the charged cylinders has an opposite polarity with respect to the polarity of the charge on the particles. In addition, increasing the number of the cylinders for the same applied voltage results in more reduction in rate of deposition. Moreover, for a given applied voltage, there is an optimum height of the array of cylinders that minimizes the total rate of deposition.

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INTRODUCTION

Particle deposition from turbulent flow of suspensions on inclined surfaces is related to many practical problems such as attenuation of solar energy collected by flat plate collectors due to dust deposition [1]. External flows of suspensions have been investigated by many authors [2-7]. Studies flow characteristics without deposition such as particle streamlines and density distributions [3-6] and studies on particle deposition on an ellipsoid of revolution and filtering walls [2-7] have been reported.

In addition, a lot of research work has been done in internal flows of suspensions [8-19]. The effects of diffusion, gravity and induced electricity on the rate of deposition in both laminar and turbulent flows for pipes, channels and diffusers have been investigated [8-13, 15-18].

The electrostatic precipitator is one of the conventional particulate control devices that effectively control the emmissions of the particles [20]. The collection mechanism is dependent upon the electrical force that results from the action of an electric field on electrically charged particles. However, the electrostatic precipitator is concerned with collection of solid particles of small size from a low speed flow of suspensions.

An induced electric field is generated by collision among the solid particles themselves and also by collision between these particles and the plate. Experiments by Kunkel [21] indicated that all dust particles become electrostatically charged upon being dispersed into a cloud. Also measurements of electrostatic charge on particles have been conducted by many investigators [17, 21-23]. It is worth noticing that the presence of charge on the particles [21] makes it possible to pull them up by means of an external electric field thus reducing the rate of deposition. This reduction is dependent on the external field intensity and distribution.

Deposition of suspensions in turbulent flow over a tilted flat plate owing to electrostatic charge, gravity and diffusion effects was studied [24]. The effects of different flow parameters on the rate of deposition and particle concentration were investigated.

in this paper, theoretical investigation for reduction of rate of deposition of suspensions in turbulent flow over a tilted flat plate by applying an external electric field is presented. The effect of different external electric field configurations on the rate of deposition, particle concentration and electric field intensity is studied. The external electric field is obtained either by a number of equally spaced parallel cylinders having the same voltage and height above the plate, or by a parallel charged plate. The first arrangement of negligible perturbation of fluid flow characteristics and other aspects related to the problem.

ANALYSIS

Simplifying Assumptions

Referring to Fig. 1, the following assumptions were made to simplify and limit the analysis of the problem :

(1) Incompressible, steady flow.

(2) Two dimensional turbulent flow.
(3) The flow is parallel to the plate.
(4) The velocity of the fluid phase is approximately constant.
(5) The thickness of the boundary layer δ is approximately constant (shear flow) .
(6) Dilute suspension.
(7) Particle-particle interaction is negligible.
(8) Fluid-particle interaction is negligible.
(9) The thickness of the layer of deposit is much smaller than the boundary layer thickness.
(10) The effect of surface adhesion is neglected.
(11) Negligible induced electric field outside the boundary layer.
(12) Negligible axial componenets of the induced elctric field.
(13) The external electric field is generated by a number of equally spaced parallel cylinders having the same voltage and height above the plate as shown in Fig. 1.
(14) The cylinders radii are much smaller than both the spacing between them and their height above the plate.
(15) The effect of the particle charges in the calculations of the external electric field is neglected.

Governing Equations

Considering the aforementioned assumptions, the governing equatiions for the particulate phase will be [10-12, 16] :

$$u \frac{\partial C}{\partial x} = D_p \frac{\partial^2 C}{\partial y^2} + \left(1 - \frac{\rho}{\rho_p}\right) g \tau \cos\theta \frac{\partial C}{\partial y} - \tau \left(\frac{q}{m_p}\right) \frac{\partial\, [\rho_p (E_i + E_e)]}{\partial y} \qquad (1)$$

$$\frac{\partial E_i}{\partial y} = \left(\frac{C}{\varepsilon_o}\right) \left(\frac{q}{m_p}\right) \qquad (2)$$

$$E_e = \sum_{n=1}^{N} - K_n V \left(\frac{h-y}{r_n^2} + \frac{h+y}{(r_n')^2} \right) \qquad (3)$$

$$\dot{m} = \sigma \tau C_w \left[\left(1 - \frac{\rho}{\rho_p}\right) g \cos\theta - \left(\frac{q}{m_p}\right)(E_{iw} + E_{ew}) \right] \qquad (4)$$

Where :

$$K_n = \frac{C_{ii}}{2\pi\varepsilon_o} = \frac{1}{\ln\left(\frac{2h}{a}\right)}$$

$$r_n^2 = (x - x_n)^2 + (h-y)^2$$

$$r_n'^2 = (x - x_n)^2 + (h+y)^2$$

Equation (1) is the combined equation of continuity of fluid and particulate phases with the application of Fick's law. Equation (2) is the Poisson equation. The left hand side of Eq. (2) does not contain the external electric field since its divergence is zero within the boundary layer. Equations (3) and (4) represent the external electric field and rate of deposition of solid particles respectively.

Boundary Conditions :

at x = 0 (at inlet)

$$\left.\begin{array}{l} C\,(o,y) = C_o \\ E_i\,(o,y) = 0 \end{array}\right] \quad (5)$$

at y = 0 (at the plate)

$$D_p \frac{\partial\, C(x,0)}{\partial y} = (1-\sigma)\ \tau\, C_w\ [\ (\frac{q}{m_p})\ (E_{iw}+E_{ew}) - (1-\frac{\rho}{\rho_p})\ g \cos\theta\ ] \quad (6)$$

at y = δ (at the boundary layer edge)

$$\left.\begin{array}{l} \dfrac{\partial\, C(x,\delta)}{\partial y} = 0 \\ E_i\ (x,\delta) = 0 \end{array}\right] \quad (7)$$

Using the normalized quantities given in the nomenclature, the above equations can be written as :

$$\beta \frac{\partial C^*}{\partial x} = \frac{\partial^2 C^*}{\partial Y^2} + \eta \frac{\partial C^*}{\partial Y} - \frac{\partial C^* (E_i^* + E_e^*)}{\partial Y} \quad (8)$$

$$\frac{\partial E_i^*}{\partial Y} = 4\, \alpha\, C^* \quad (9)$$

$$E_e^* = \sum_{n=1}^{N} K_n\ V^*\ [\ \frac{(\ H-Y\ )}{R_n^2} + \frac{(\ H+Y\ )}{R'^2_n}\ ] \quad (10)$$

$$\dot{m}^* = \sigma\ C_w^*\ (\eta - E_{iw}^* - E_{ew}^*\ )\ /\beta \quad (11)$$

With boundary conditions :

$$\underline{\text{at } X = 0} \qquad \left.\begin{aligned} & C^*(0,Y) = 1 \\ & E_j^*(0,Y) = 0 \end{aligned}\right] \tag{12}$$

$$\underline{\text{at } Y = 0} \qquad \partial C^*(X,0)/\partial Y = (1-\sigma)C_w^*(E_{iw}^* + E_{ew}^* - \eta) \tag{13}$$

$$\underline{\text{at } Y = 1} \qquad \left.\begin{aligned} & \partial C^*(X,1)/\partial Y = 0 \\ & E_i^*(X,1) = 0 \end{aligned}\right] \tag{14}$$

NUMERICAL ALGORITHM

Equations (8) to (11) with boundary conditions (12) to (14) can be solved for the unknowns C^*, E_i^* and $\dot{m}^*$ by applying the finite difference technique [25,26]. The finite difference equations take the form of the matrix equation AX=B which can be solved at any axial position once the elements of the matrix A and the column vector B are known at the previous location. The size of the matrix A was (40x40) and the resulting equations were solved simultaneously by the Gauss elimination technique.

RESULTS AND DISCUSSIONS

The results presented in this paper are based on typical values for the parameters (α, β, η and σ) obtained from previous investigations [16, 17, 22-24]. These values are $\alpha = 1$, $\beta = 1000$, $\eta = 1$, and $\sigma = 0.5$ for a particle $d = 13.5\ \mu m$ and $\tau = 1.818\times10^{-3}$ S, $D_p = 20\times10^{-6}\ m^2/s$, $C_o = 0.39\ Kg/m^3$, $u = 2 m/s$ and $(q/m_p) = 10^{-4}$ Coul/Kg. These values were considered constant througout the computations for Figs. 3 to 8 since the main objective of this paper is to investigate the effect of the external electric field on the rate of deposition of the particles on the plate.

The diameter of the charged cylinders was taken as 1/5 of the boundary layer thickness (D= 0.20). The assigned range for the number of cylinders, voltage and height is N=1 to 5, $V^* = 0$ to 300 and H= 2 to 20 .

The effect of the external electric field E_e^* on the rate of deposition $\dot{m}^*$ for the case of three cylinders with $V^* = 100$ each (at X=17, 51 and 85) and H=10 is shown in Fig. 3. It is obvious that the rate of deposition curve becomes wavy with points of minima at the axial positions of the charged cylinders. Also it is clear that the external electric field reduces the deposition rate of the particles on the plate.

Figure 4 shows the influence of the height H on the relative rate of deposition $\dot{m}_r$ defined as the ratio of the integral of the rate of deposition over the plate with the external electric field applied to its value without the external elctric field for four cases (N=3, $V^* = 100$ and N =4 , $V^* = 100$ and 200). One can see that $\dot{m}^*_r$ has a minimum value at H=8 for $V^* = 100$ and at H =12 for $V^* = 200$ irrespective of the number of cylinders. Also, it is clear that increasing V^* decreases $\dot{m}^*_r$ for the same H.

The effect of the number of charged cylinders N on the relative rate of deposition for different values of voltage V^* ($V^* = 50$, 100 and 200) at H = 10 is depicted in Fig. 5. It can be noticed tgat as N increases, $\dot{m}^*_r$ decreases as expected.

Figure 6 illustrates the effect of V^* on $\dot{m}^*_r$ for both parallel charged plate and three equally charged cylinders (located at X=17 , 51 and 85) at H =10. It is clear that for $V^* = 50$, $\dot{m}^*_r = 0$ for the case of charged plate and $\dot{m}^*_r = 0.596$ for the case of three charged cylinders, thus charged plate results in more reduction in particle rate of deposition.

Figure 7 indicates the effect of external electric field E^*_e on particle concentration C^* . It is clear that E^*_e makes the distribution of C^* closer to a uniform one, i.e. less distortion near the wall. Also it can be noticed that as V^* increases, C^*_w decreases and hence reduction in $\dot{m}^*$ occurs.

The distributions of the y-components of the total elctric field ($E^*_i + E^*_e$) and the induced field for the case of N=3. $V^* = 100$ at H=10 are shown in Fig. 8. Since H >> , then the external electric field is almost constant within the boundary layer at a given X. Thus the total electric field will have the same shape as the induced field but shifted positively by an almost constant amount depending on X.

The external electric field can reduce the deposition rate through the force it exerts on the charged particles. If $(q/m_p) = 0$, deposition is due to gravity only and the external electric field would have no effect since it would not produce on non charged particles. However, as (q/m_p) increases, the external field becomes more effective in pulling the particles upward and hence reducing the rate of deposition. It is to be noted that as (q/m_p) increases, the induced electric field also increases tending to increase the rate of deposition.

Figure 9 shows the effect of on the total deposition rate for different values of external applied voltage. Such dependence of the total deposition rate on the electrostatic charge parameter is to be considered when determining the proper value of the applied voltage.

CONCLUSIONS

From this theoretical investigation, it is concluded that an externally applied electric field obtained by an array of charged cylinders placed parallel to the plate can considerably reduce the rate of particle deposition from turbulent flow of suspensions over a tilted flat plate if the potential of the charged cylinders has an opposite sign with respect to the polarity of the charge on the particles. In addition, increasing the number of cylinders for the same applied voltage results in more reduction in rate of deposition. Moreover, for a given applied voltage, there is an optimum height of the array of cylinders that minimizes the total rate of deposition. Further experimental study is needed to support the theoretical results presented in this paper.

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NOMENCLATURE

C	particle concentration by mass
C_o	inlet particle concentration
C^*	dimensionless particle concentration, $C^* = C/C_o$
d	diameter of a charged cylinder
D	dimensionless diameter of a charged cylinder, $D = d/\delta$
D_p	eddy diffusivity of particles
E_i	induced electric field intensity, $E_i = E_{iy}$
E_i^*	dimensionless induced electric field intensity, $E_i^* = \left(\frac{q}{m_p}\right)\left(\frac{\tau\delta}{D_p}\right) E_i$
E_e	the y-component of the external electric field intensity given by Eq. (3)
E_e^*	dimensionless y-component of the external electric field intensity given by Eq. (10)
g	gravitational acceleration
h	height of charged cylinders above the plate
H	dimensionless height of charged cylinders, $H = h/\delta$
m_p	mass of a particle
$\dot{m}$	mass flow rate of the particles deposited on the plate, given by Eq. (4)
$\dot{m}^*$	dimensionless rate of deposition of particles, given by Eq. (11)
$\dot{m}_r$	relative rate of deposition of particles, defined as the ratio of the integral of $\dot{m}^*$ over the plate with external electric field to its value without the external field. Integration is done over the plate normalized length L along the flow direction
N	number of equally charged cylinders
q	electric charge per particle
r, r'	distances from a point (x,y) to a charged cylinder and its image respectively as shown in Fig. 2
R, R'	dimensionless distances,
u, v	axial and vertical components of fluid velocity
u_p, v_p	axial and vertical components of particle velocity

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u_o	inlet velocity (uniform)
V	applied voltage on each cylinder
V*	normalized voltage, $V^* = -(q/m_p)(\tau V/D_p)$
x, y	axial and vertical coordinates respectively
X, Y	dimensionless axial and vertical coordinates, X=x/ ,Y=y/

Greek Letters

α	electrostatic charge parameter , $\alpha = \left(\frac{C_o}{4\ \varepsilon_o}\right)\left(\frac{q}{m_p}\right)^2 \left(\frac{\tau\delta^2}{D_p}\right)$
β	diffusive Peclet number , $\beta = u_o\ \delta/D_p$
δ	boundary layer thickness (considered constant)
ε_o	permittivity of free space , $\varepsilon_o = 8.85434 \times 10^{-12}$ (Coul)^2/N.m^2
η	gravity parameter , $\eta = (1-\rho/\rho_p)\ g\ \delta\tau\cos\theta/D_p$
θ	tilt angle of flat plate
ρ	density of the material constituting the fluid
ρ_p	density of the material constituting the particles
σ	sticking probability accounts for electro-viscous and gravity forces
τ	ordinary relaxation time

Superscripts

*	dimensionless quantities

Subscripts

e	for external electric field
i	for induced electric field
o	initial condition
p	for wall condition
δ	for boundary layer edge condition

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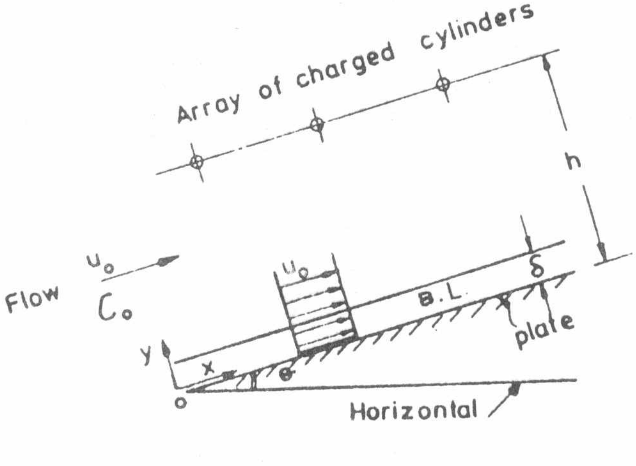

Fig.1 Flow configuration

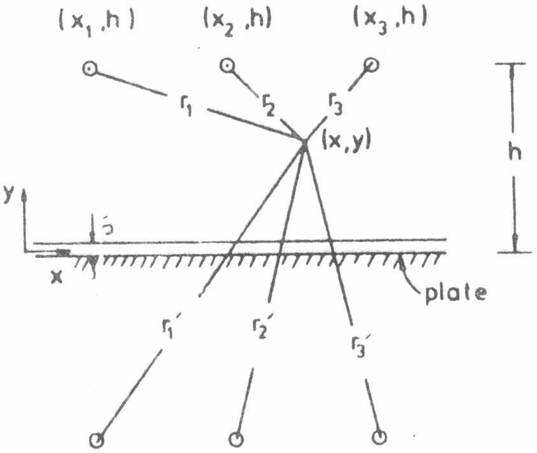

Fig.2 External electric field geometry

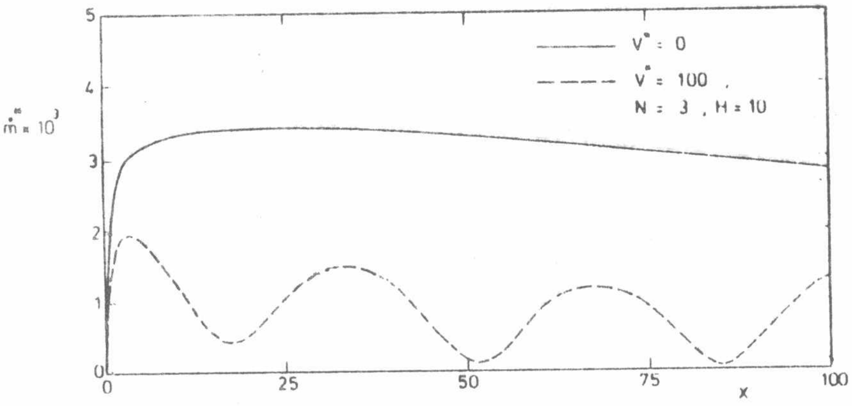

Fig.3 EFFECT OF EXTERNAL ELECTRIC FIELD ON RATE OF DEPOSITION

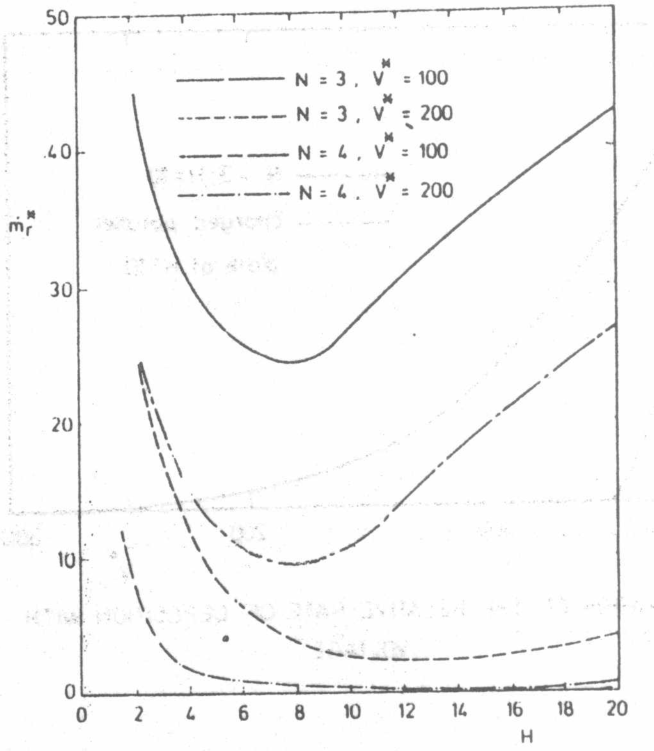

Fig. 4 VARIATION OF THE RELATIVE RATE OF DEPOSITION WITH HEIGHT OF CYLINDERS

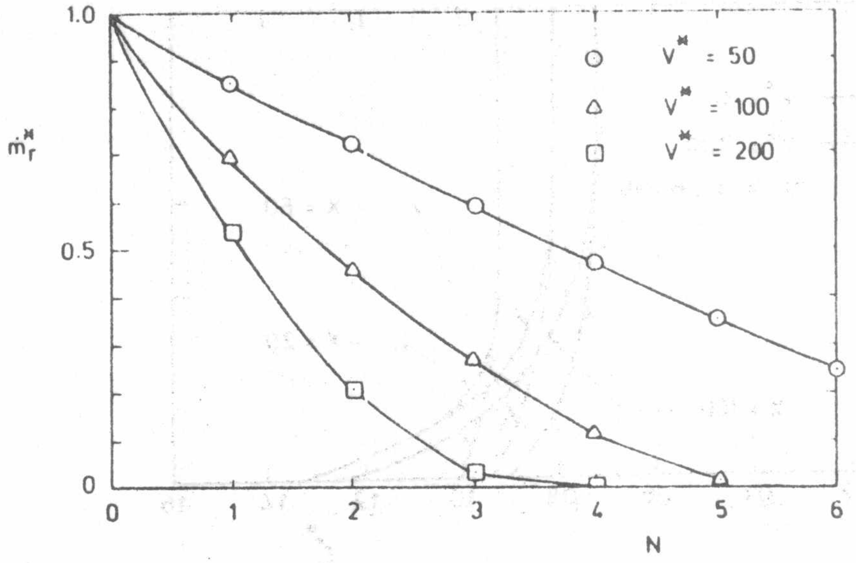

Fig. 5 EFFECT OF NUMBER OF CYLINDERS ON THE RELATIVE RATE OF DEPOSITION (H = 10)

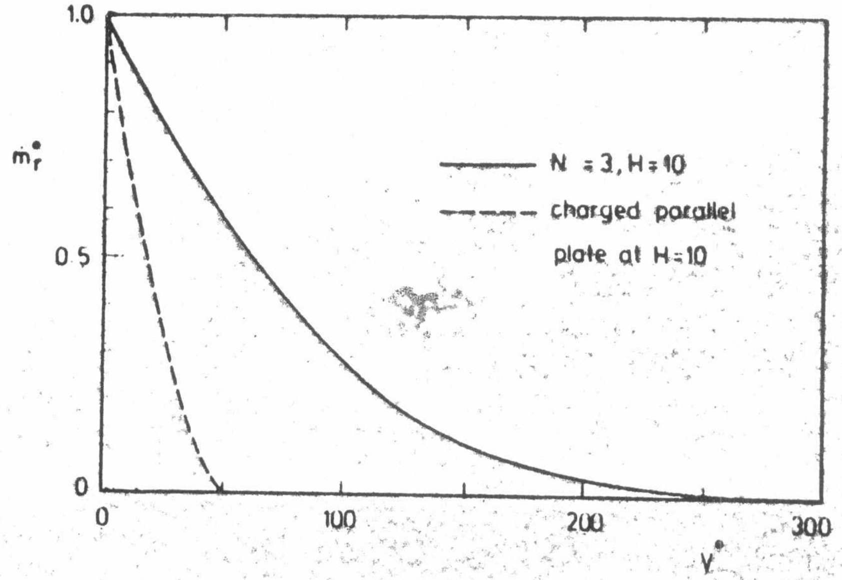

Fig.6 VARIATION OF THE RELATIVE RATE OF DEPOSITION WITH VOLTAGE

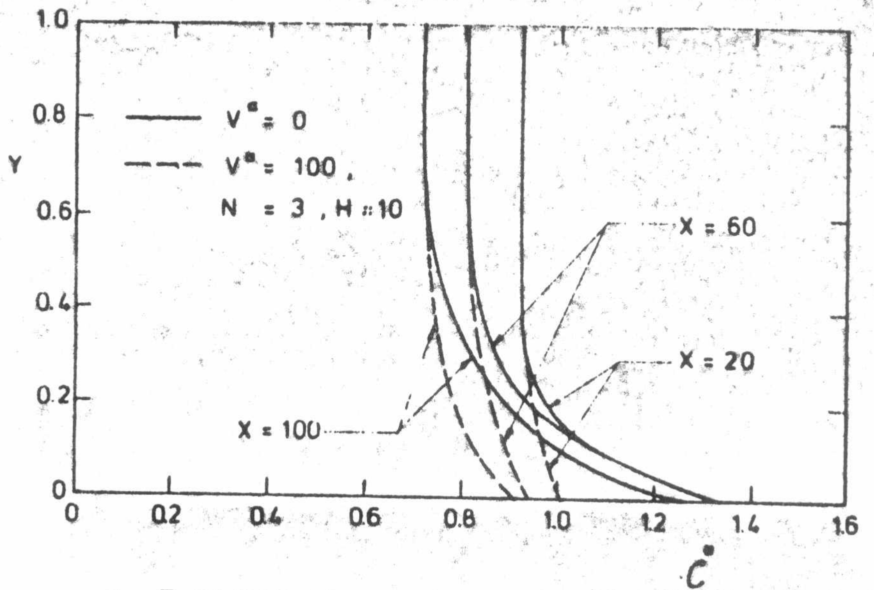

Fig.7 EFFECT OF EXTERNAL ELECTRIC FIELD ON PARTICLE CONCENTRATION

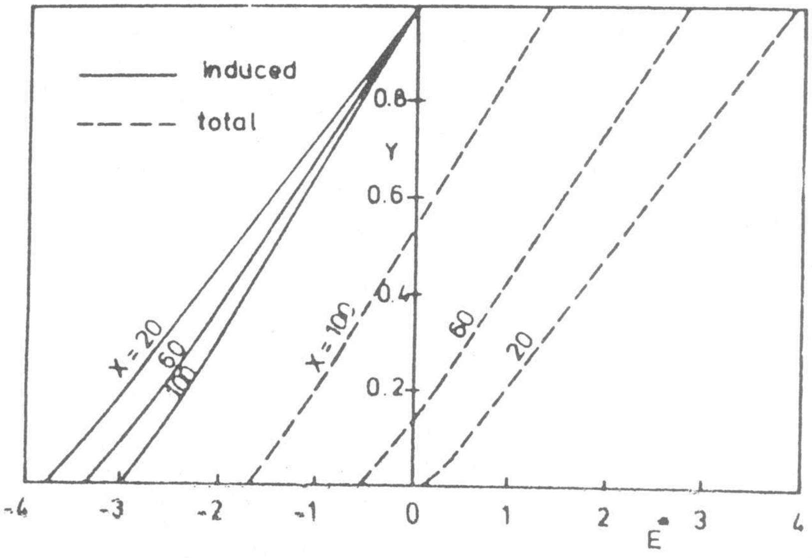

Fig. 8 INDUCED AND TOTAL ELECTRIC FIELD DISTRIBUTIONS
($N = 3$, $V^{\bullet} = 100$, $H = 10$)

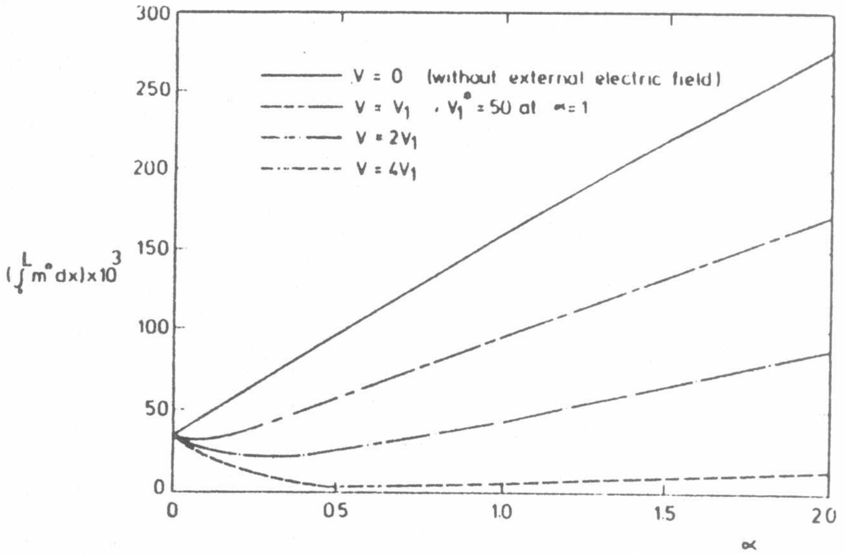

Fig. 9 EFFECT OF ELECTROSTATIC CHARGE PARAMETER ON TOTAL RATE OF DEPOSITION ($N = 3$, $H = 10$)