MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

ELECTRO-HYDRODYNAMIC ANALOGY SOLUTION OF CONFORMAL MAPPING OF EXTERNAL DOMAIN OF AN AIRPLANE FUSELAGE

O . E . ABDELHAMID *

ABSTRACT

Determination of bilinear transformation functions that mutually mapp conformally the exterior of an arbitrarily shaped given contour to exterior of unit circle is of practical importance in several fields of engineering. For solution of this problem,certain numerical techniques are available, suffering from either low accuracy or complicated procedures and long computer time disadvantages.In presented paper is applied an experimental computational technique based on the electro-hydrodynamic analogy.and method of trigonometric interpolation. Required mapping functions are assumed in form of power series with coefficients determined using images of points equally deviding the circle contour.Image points on given contour are determined by electro analogy on electrically conductive paper .The method is applied for conformal mapping of exterior of airplane fuselage sections with and without its horizontal tail surfaces. Mapping series proved to be convergent and fluently changing in harmony with fuselage body streamlining. The method is simple and economic with results of acceptable accuracy. Moreover ,it is the only practical solution of conformal mapping problems of real non-idealized actual cross-sectional shapes of modern complicated wing-body combinations.Due to its relation to velocity potential and pressure distributions,longitudinal behaviour of mapping functions with fuselage stations can depict positions of unfavourable pressure gradient and assist its streamlining.

INTRODUCTION

In aerodynamics,potential flow theory of nonviscous fluid is applied in the solution of flow problems in different regiemes of flow velocities. In this advent ,the theory of two dimensional incompressible flow plays a leading role where the complex variable theory and conformal mapping directly enter the aerodynamic scene.Conformal mapping enables finding flow patterns and pressure distributions around shapes of practical interest from that of simple or standerd flow patterns.A problem of special interest in external

* Department of Aeronautical Engineering , Military Technical College , Cairo , EGYPT

aerodynamics ,is the determination of mapping functions that map exterior of unit circle to exterior of given connected contour and vice versa , keeping conditions of flow at infinity in both domains unchanged.Several analytical and numerical techniques are applied for conformal mapping,both directly or indirectly.Joukowsky and Karman Trefftz transformations enabled definition of classes of airfoil shapes for various aerodynamic applications by mapping the circle exterior domain.The direct problem is a more complicated one,where is required the mapping function for a given specified contour. For this problem , different methods are available in literature. Theodorsen-Garrick method[1]solves the problem in two steps,firstly by transforming the given contour into a near circle ,then the near circle into circle.Schwarz-Cristoffel transformation approximates given contour by a polygon of finite number of vertices for further mapping to circle exterior[2]. Other works attempted generalization of Schwarz-Cristoffel to convex ovals [3] or substituting the general contour by circular arcs [4] .Details of other numerical methods in conformal mapping such as variational methods,method of integral equations and mapping of adjacent regions , are given in [5] .General features of these methods are mathematical complixity and need of large capacity computers and long computation time runs . Method of trigonometric interpolation [6] avoids these shortcomings.The mapping function is assumed in the form of power series whose coefficients are easily determined from the coordinates of images on the given contour of points equally deviding the unit circle circumference.The presented work applies this method for different cross-sections of a complete airplane fuselage where coordinates of image points are obtained using the electro-hydrodynamic analogy.

METHOD OF TRIGONOMETRIC INTERPOLATION

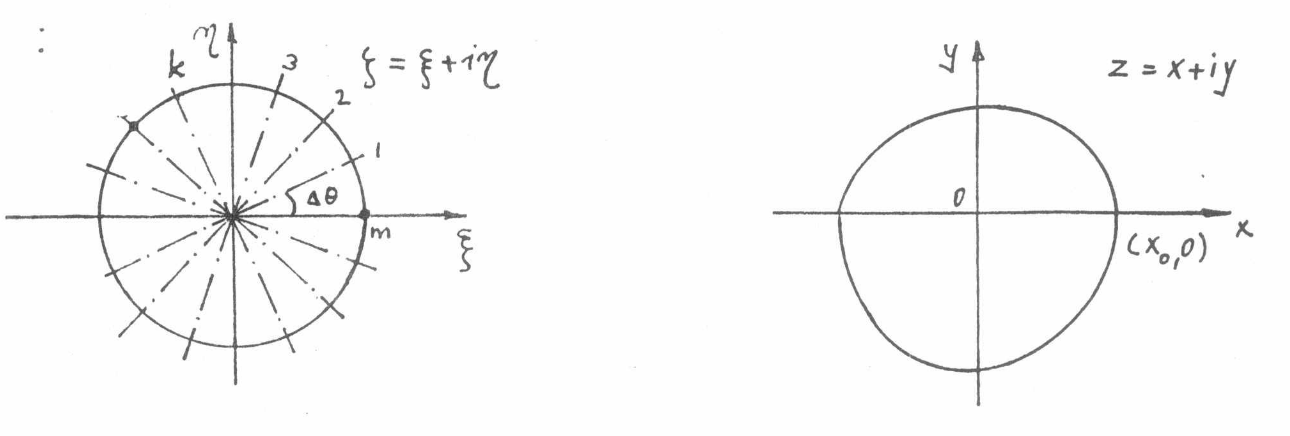
Detailed derivations of the method are given in [6],here are given its relations for mapping of symmetrical contours with respect to one axis,which is the case corresponding to considerd one of airplane body.Mapping of exterior domains is subject to condition of flow field preservation far from the body

$$\lim_{z\to\infty} \zeta = z \quad , \quad \lim_{z\to\infty} \frac{d\zeta}{dz} = 1 \tag{1}$$

this implies that the mapping function be in the form

$$z = f(\zeta) = \sum_{n=-1}^{\infty} C_n \zeta^{-n} = C_{-1}\zeta + C_0 + \frac{C_1}{\zeta} + \cdots + \frac{A_n}{\zeta^n} + \cdots \tag{2}$$

where C_n are all complex numbers $C_n = A_n + i B_n$



6

Condition (1) is combined with the other boundary condition that stagnation points of flow field in both planes should be corresponding. For symmetrical domains ,this condition is realized by mapping point $\zeta = 1$ on the unit circle contour to the point $z = x_o$ on contour S of domain L lying on real x-axis. For symmetrical domains with respect to x-axis ,coefficients of series (2)are all real numbers

$$z = f(\zeta) = \sum_{n=-1}^{\infty} A_n \zeta^{-n} = A_{-1}\zeta + A_0 + \frac{A_1}{\zeta} + \dots + \frac{A_n}{\zeta^n} + \dots \tag{3}$$

For points $\zeta = \rho e^{i\theta_k}$ equally deviding the unit circle contour

$$\Delta\theta = \frac{2\pi}{m} \quad , \rho = 1 \quad , \theta_k = k\Delta\theta = \frac{2\pi}{m}k \tag{4}$$

we get

$$\zeta = \cos\theta_k + i\sin\theta_k \quad \& \quad \zeta^{-n} = \cos n\theta_k - i\sin n\theta_k \tag{5}$$

Substituting above relations in series(2)and considering only the terms up to $n = m-1$,by seperation of real and imajinary parts we get

$$\begin{aligned} x_k &= A_{-1}\cos\theta + A_0 + A_1\cos\theta_k + A_2\cos 2\theta_k + \dots + A_{m-1}\cos(m-1)\theta_k \\ y_k &= A_{-1}\sin\theta_k + 0 + A_1\sin\theta_k - A_2\sin 2\theta_k - \dots - A_{m-1}\sin(m-1)\theta_k \end{aligned} \tag{6}$$

According to (4),for the angle θ_k we have

$$\cos\theta_k = \cos(m-1)\theta_k \quad , \quad \sin\theta_k = -\sin(m-1)\theta_k \tag{7}$$

Inserting in (6)we get

$$\begin{aligned} x_k &= A_0 + A_1\cos\theta_k + A_2\cos 2\theta_k + \dots + (A_{m-1} + A_{-1})\cos(m-1)\theta_k \\ -y_k &= A_1\sin\theta_k + A_2\sin 2\theta_k + \dots + (A_{m-1} + A_{-1})\sin(m-1)\theta_k \end{aligned} \tag{8}$$

Trigonometric functions of discrete argument,in case of equal intervals defined by (4)have the property of orthognality,that is :

$$\begin{aligned} &\sum_{k=1}^{m} \sin j\theta_k \sin h\theta_k = \begin{cases} 0 & \text{for } j \neq h \\ \frac{m}{2} & \text{for } j = h \end{cases} \\ &\sum_{k=1}^{m} \cos j\theta_k \cosh\theta_k = \begin{cases} 0 & \text{for } j \neq h \\ \frac{m}{2} & \text{for } j = h \end{cases} \\ &\sum_{k=1}^{m} \sin j\theta_k \cosh\theta_k = 0 \end{aligned} \tag{9}$$

Applying orthognality condition,for domains symmetric with respect to x-axis the unknown coefficients A_n are given by

$$A_n = \frac{2}{m}\sum_{k=1}^{m/2} x_k \cos n\theta_k - y_k \sin n\theta_k \tag{10}$$

where $n = 1,2,\dots\dots,m-2$; while for n=m-1 we get instead of the coefficient A_{m-1} the sum $A_{m-1} + A_{-1}$

$$A_{m-1} + A_{-1} = \frac{2}{m}\sum_{k=1}^{m/2} x_k\cos(m-1)\theta_k - y_k\sin(m-1)\theta_k \tag{11}$$

From convergence criteria of series (2), for $m \to \infty$ the condition $A_{m-1} < \epsilon$ must be satisfied for sufficiently large values of m . Hence, within ranges of practical accuracy it can be written

$$A_{-1} = \frac{2}{m} \sum_{k=1}^{m/2} x_k \cos(m-1)\theta_k - y_k \sin(m-1)\theta_k \qquad (12)$$

Coefficient A_0 is determined from equation

$$x_0 = A_{-1} + A_0 + A_1 + \cdots + A_n \qquad (13)$$

obtained by substitution of $\zeta = 1$ and the second boundary condition of stagnation points correspondancy on both contours. From equations (10) and (12) it is clear that for calculating coefficients of mapping functions it is necessary to determine the positions of image (basic) points .

ELECTRO-HYDRODYNAMIC ANALOGY (EHA)

Coordinates of basic points z_k ($k = 1,\ldots,m$) may be determined either numerically or experimentally using the electro-hydrodynamic analogy. Numerical procedure[4]&[6], is very laborious and time consuming, specially when applied to general contours. Theory of EHA is explained in detail in [7]&[8]. It is based on similarity of equations of two dimensional steady potential fluid flow $\nabla^2\phi = 0$ and the equation of two dimensional steady electrical flow field $\nabla^2 V = 0$. Problems in both two fields ,having same boundary shapes and boundary conditions ,will have same solutions. As measuring techniques in electrical flow fields are much simpler and well developed EHA enables solution of potential flow boundary value problems as long as analogy is preserved. Hence , from uniqueness of conformal mapping ,orthogonal set of equipotentials and streamlines exterior to the given contour of general shape corresponds to set of equipotentials and lines of electric force formed by electric flow around same contour through electrically conductive medium. Electrically conductive paper is very suitable for such type of experiments. It has a base of normal paper provided on one side by a nonconductive paint and covered on other side by very thin conductive layer of alluminium, graphite or similar materials. It is produced by specialized factories of analogue models. The nonconductive boundary is formed on the paper by cutting. Straight copper produced electrodes simulating equipotentials far from body, are fixed to the coductive paper at reasonably sufficient distance from the cut contour and fed with either AC or DC voltages. For determining the shape of equipotential curves a hand or automatically driven sond is used to follow and spot the course of constant electric voltage on the conductive paper. Schematic diagram of experimental setup is shown in Fig.2. Required image points of those equally deviding the circle contour are obtained from intersection of cut contour with equipotentials spaced at constant voltage. Voltage spacing is given by potential difference between points on axis of symmetry devided by number of circle divisions.
It is important to note that main sources of error in this experimental solution are due to conductive paper nonhomogenity and placing the straight constant potential electrodes at distances not sufficiently far from the cut contour. A test was supposed to investigate the extent of these effects on obtained results. The potential V on circle contour is known from theory to be linearly proportional to cosine of angular argument θ

$$V = const \cdot \cos\theta \qquad (14)$$

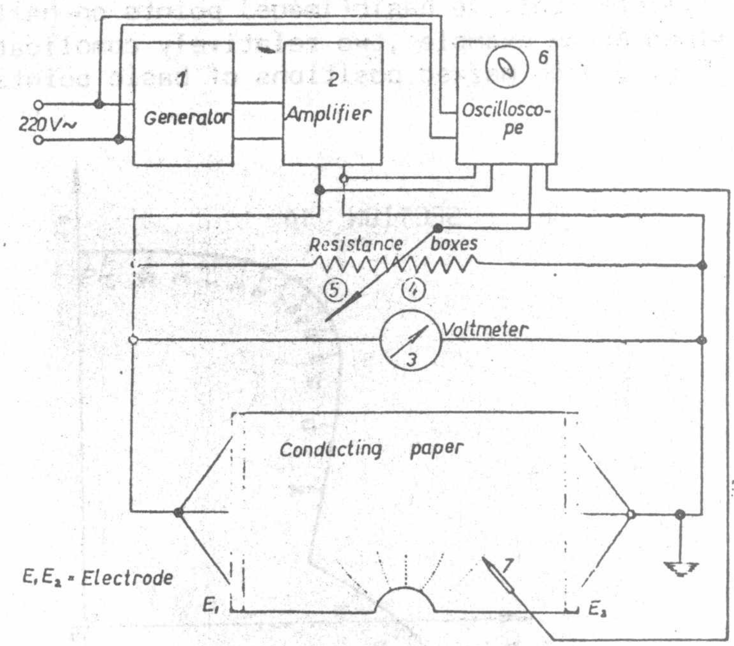


Fig.2 Schematic diagram of experimental setup

Using the designed experimental setup for measuring this voltage variations along contours of circles with diameters in range of dimensions of considered sections, fitted curves were obtained to be

-for circle of diameter 153 [mm]

$$V=K(160 \cos\theta + 0.26\ 10^{-4} \sin\theta - 2.01 \cos 2\theta + 0.157\ 10^{-4} \sin 2\theta + 1.35 \cos 3\theta - 0.408\ 10^{-5} \sin 3\theta + 0.307 \cos 4\theta - 0.199\ 10^{-3} \sin 4\theta + \ldots)$$

-for circle of diameter 75.8 [mm]

$$V=K(86.9 \cos\theta + 0.259\ 10^{-4} \sin\theta + 0.413\ 10^{-1} \cos 2\theta + 0.102\ 10^{-4} \sin 2\theta + \ldots\ldots)$$

where K is a constant . (15)

Above relations show that relative error in cosine terms is about 1.25% for the large circle ,while it is much less (= 0.5%) for the smaller one. Coefficients of sine terms are approximately negligible . These results suggests that any corrections for the obtained coordinates of image points are not necessary.

APPLICATION TO THE EXTERIOR OF AN AIRPLANE FUSELAGE

Above explained technique is applied for determining basic points on contours of 25 different cross-sections selected along an airplane fuselage axis, without its wings. Sections varied from nearly circular at noze to sections of general shape at positions of pilot canopy, engine air intakes and of the rear mounted horizontal tail panels. For getting the most accurate results with this simple available technique, mapping functions are choosen to be of

72 terms.By virtue of symmetry,only 36 basic(image) points on halfs of sections are to be determined.As an example ,two relatively complicated cross-sections are given in Fig. 3 with marked positions of basic points.

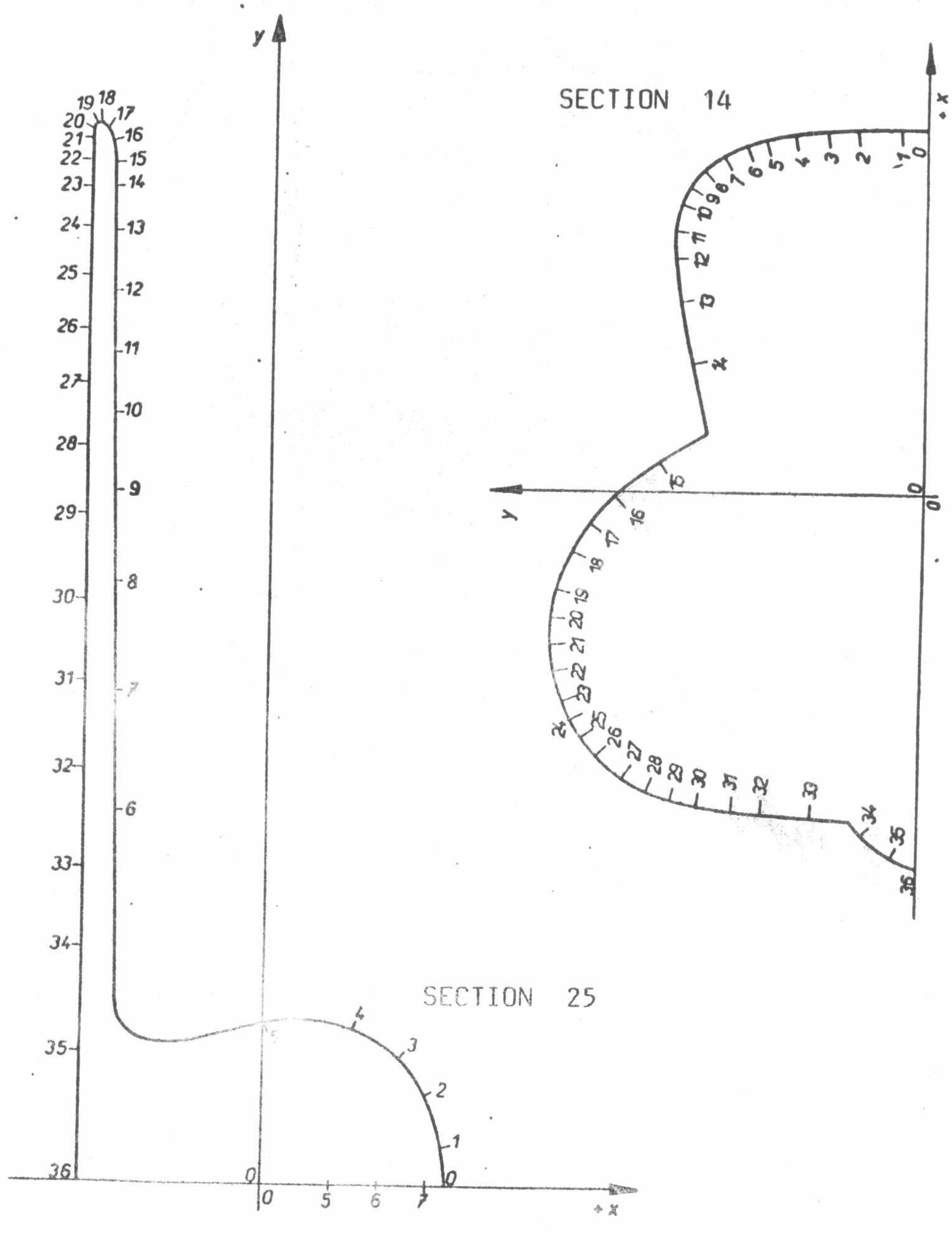
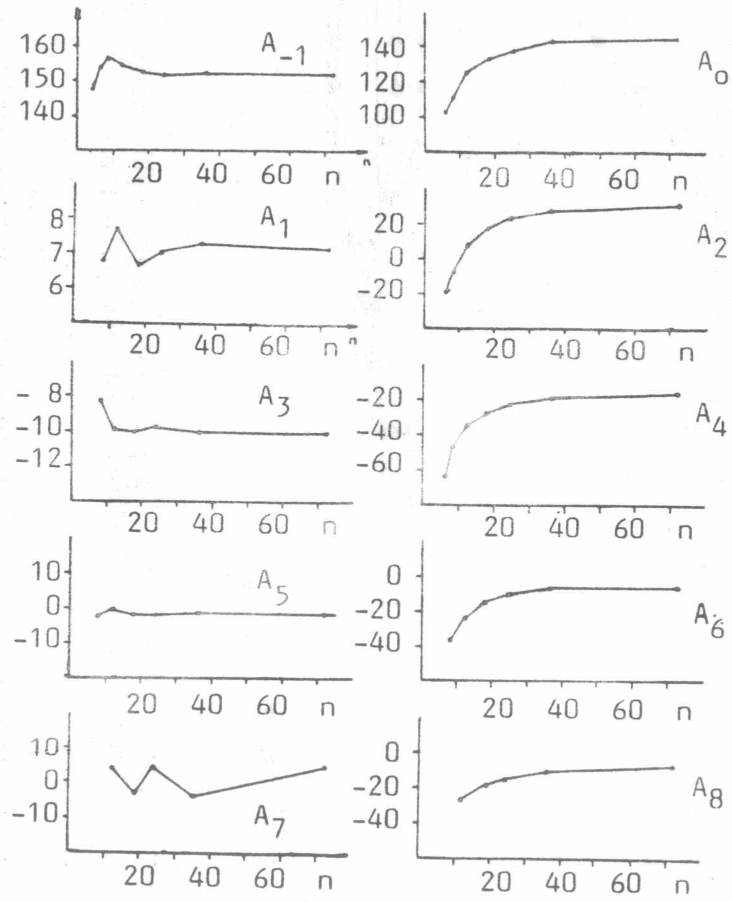
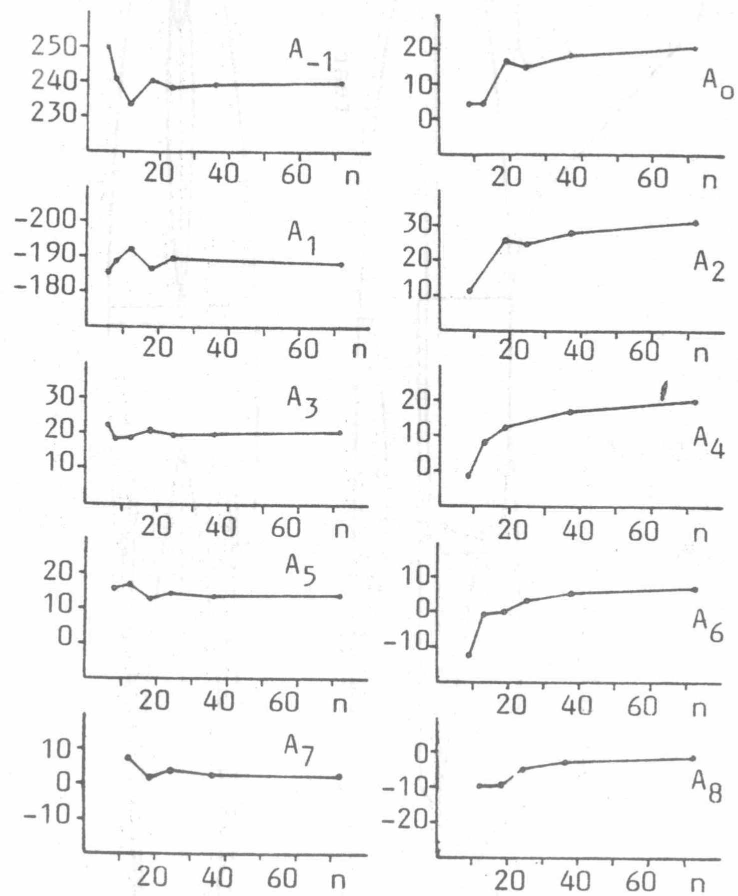


Fig.3 Sections with marked positions of image points

For checking the convergence of mapping functions,values of first ten coefficients are calculated for different number of series terms(8,12,18, 24,36 and 72). Convergence behaviour of coefficients for both types of sections is shown in Fig.4. Results show that covergence of coefficients is well satisfactorily. However , for higher accuracy , larger number of terms is recommended. Results also show that positions of significant points for values of coefficients ,are not necessarily equidistant.Considered sections are at engine air intake position and at fuselage rear with tail surfase .

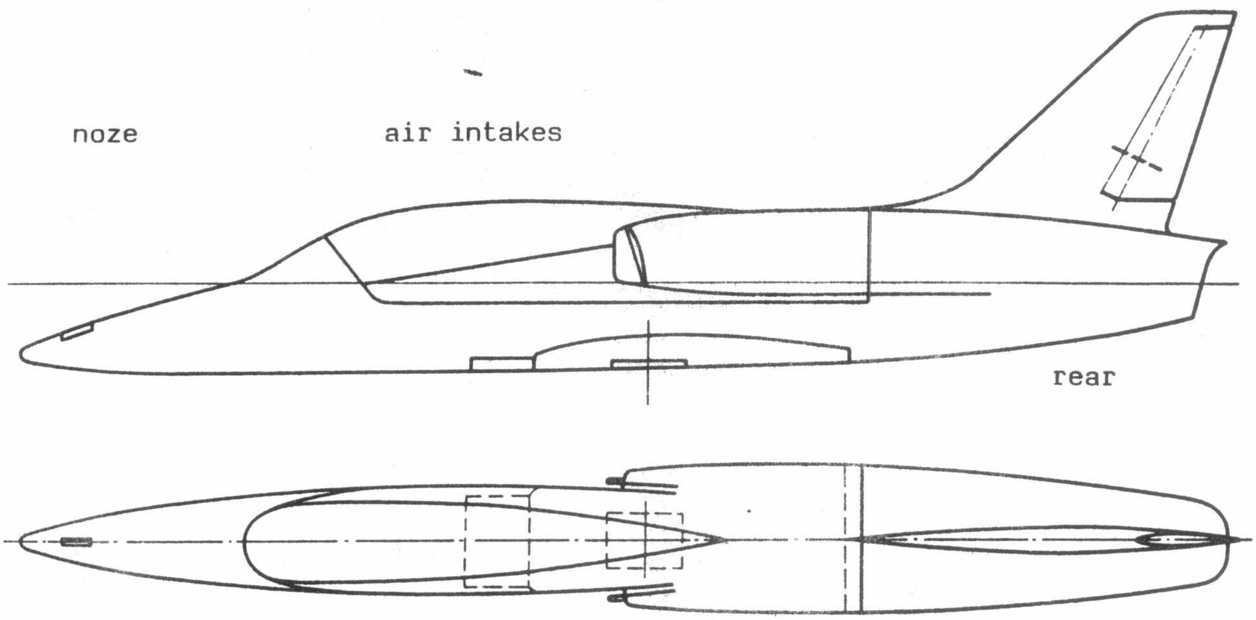


a- SECTION 14



b- SECTION 25

Fig. 4 Convergence behaviour of series coefficients with number of series terms



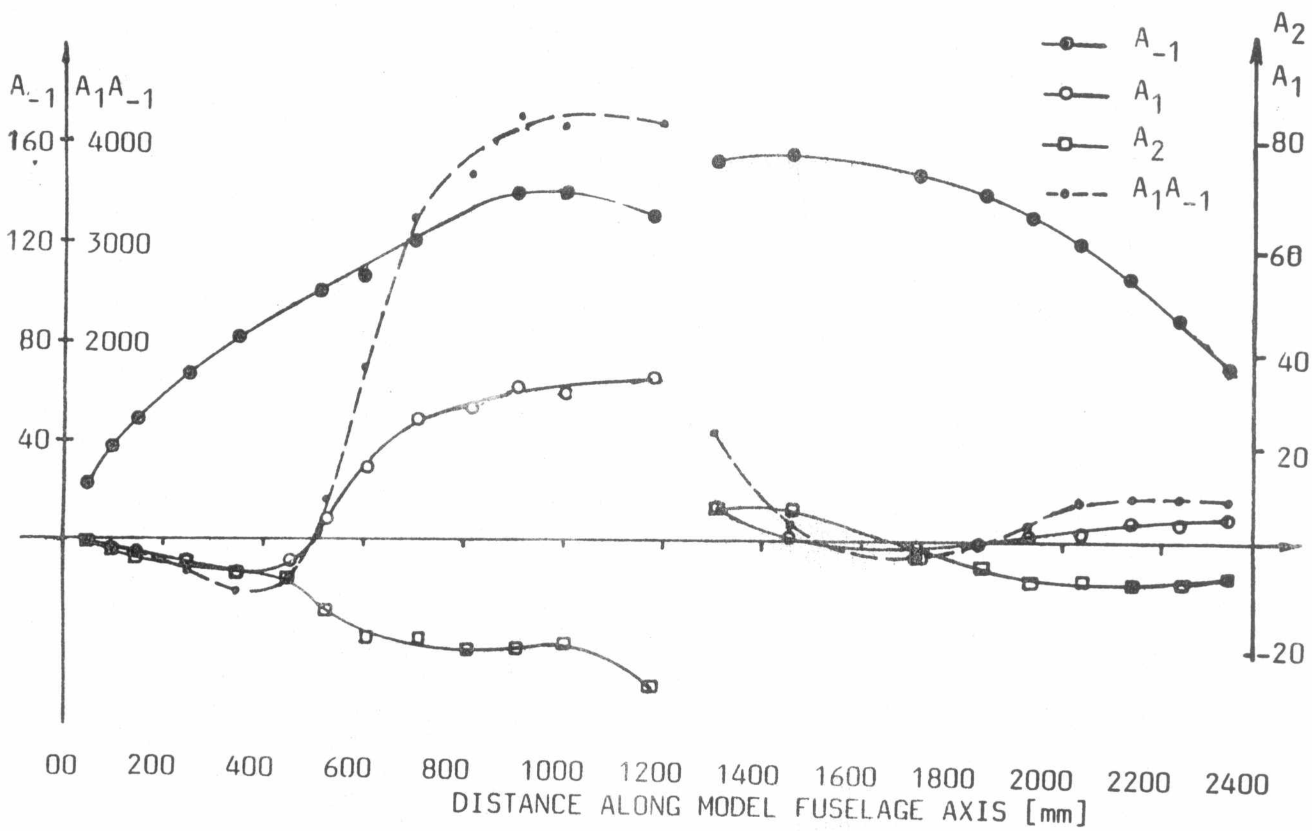


Fig.5 Variation of mapping series coefficients along fuselage axis.

6

An interesting presentation is the variation of determined values of mapping series coefficients along fuselage axis.In Fig.5 , are plotted these variations for the coefficients A_1 , A_{-1} and A_2 together with the considered fuselage plan and side views.Residue of mapping function is a value of mathematical and aerodynamical significance .Its value equals $A_1 A_{-1}$ and is also plotted in Fig.5. Coefficients and the residue are noticed to vary smoothly and fluently for this type of well streamlined fuselage . Mapping functions of fuselage section exterior fields are closely related to their cross-flow velocity and pressure distributions[9],[10]. Hence,their irregular variations may depict positions of unfavourable pressure gradients leading to boundary layer early seperation and excessive friction and pressure drags. Previous remark suggests the possibility of application of these mapping functions for improving fuselage streamlining and avoiding some of the expensive wind tunnel measurments.

CONCLUSIONS

Application of method of trigonometric interpolation and electro-hydrodynamic analogy for determination of conformal mapping functions of exterior domains prooved to be practical and less money and effort consuming.Obtained results are of sufficient degree of accuracy for engineering purposes. Mapping of fuselage sections proved the ability of the technique to deal with complicated irregular shapes. Variations of coefficients with fuselage axis are smooth and fluent which suggests the application of mapping functios for checking and improving airplane body streamlining.

REFERENCES

1-Theodorsen,T and Garrick,E :General potential theory of arbitrary wing sections,NACA Rep.No.452,1933.

2-Korn and Korn : Mathematical hand book for scientists and engineers. McGraw-Hill Book,1986.

3-Study,E:Vorlesongen uber ausgewahlte gegenstande der geometrie. Teubner,Leipzig,Berlin 1913.

4-Filchakov,P.F.:Approximate methods in conformal mapping. Naooka Domka,Kiev,1964.

5-Rektorys,K.:Handbook of applied mathematics. SNTL , Praha 1968.

6-Filchakova: Conformal mapping of domains of special type. Naooka Domka,Kiev,1972.

7-Nozicka,J.:Analogy methods in flow. Academia Praha 1970.

8-Abdelhamid,O.E.:Electromodelling of conformal mapping of external domains. VZLU,Praha,1974.

9-Abdelhamid,O.E.:Aerodynamic damping in pitch of oscillating airplane.fuselage. Ninth international congress for statistics and computer science. 31 March-5 April,1984.

10-Abdelhamid,O.E.: An extension to the slender body theory.Ph.D. thesis. VAAZ,Berno,Czechoslovakia,1975.