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ESTIMATION OF DYNAMIC COEFFICIENTS OF ELLIPTIC BEARINGS WITH VARIED ASPECT AND ELLIPTICITY RATIOS

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ABSTRACT

Computer solution of Reynold's equation applied to an elliptic bearing using numerical methods is employed to obtain the load carrying capacity of an elliptic bearing with horizontal and vertical axes. Numerical integration of pressure results in evaluating fluid-film forces. Numerical differentiation of fluid-film force components determines radial and tangential film stiffness. If the journal runs in equilibrium under some external load, and is displaced incrementally in direction of radial vector of the journal center, numerical solution of Reynold's equation, for each bearing arc, will result in dynamic pressure distribution. Numerical integration of dynamic pressure results in dynamic fluid-film forces. Numerical differentiation of dynamic forces results in tangential and radial damping coefficients. Equations of motion of a journal due to small oscillation of the journal center areused to determine the onset of instability in an elliptic bearing.

NOTATIONS

A	system dynamic coefficients $(\pi k C^3/2\mu L R^3 \omega_o)$
C, C_m	major and minimum bearing clearances.
d	offset distance in elliptic bearing.
e, e_1, e_2	nominal eccentricity, lower and upper bearing eccentricities
F_r, F_t	radial and tangential fluid film force components
$\bar{F}$	non-dimensiona force; $[F(C/R)^2/6\mu\omega RL]$
H	non-dimensional film thickness; (h/C)
k	stiffness of shaft in asymmetric bearings.
L	bearing length
M	mass of rotating parts (balanced)
P	non-dimensional film pressure; $[p\ (C/R)^2/6\mu\omega]$
R, R_j, R_b	nominal bearing radius, journal radius and actual bearing arc radius
s	angular speed ratio (ω_d/ω_o)

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x, y	coordinate system of journal bearing plane
X, Y	added film forces due to incremental displacement of journal center (ξ and η).
X_1, X_2	film extent in radians on lower and upper bearing arcs
Y_o	static load on journal.
z, Z	length coordinate; ($Z=z/L$)
δ	ellipticity ratio ($\delta = d/C$)
$\varepsilon, \varepsilon_1, \varepsilon_2$	nominal eccentricity ratio (e/C); actual eccentricity ratio w.r.t. lower bearing arc (e_1/C); and actual eccentricity ratio w.r.t. upper bearing arc (e_2/C).
ε_m	nominal eccentricity ratio referred to (C_m); (e/C_m)
$\dot{\varepsilon}, \varepsilon', \bar{\varepsilon}$	radial velocity of journal center; ($\dot{\varepsilon} = de/dt$; $\varepsilon' = \dot{\varepsilon}/\omega$; and $\bar{\varepsilon} = 2\dot{\varepsilon}/\omega$)
ϕ, ϕ_1, ϕ_2	attitude angles; (ϕ) referred to nominal bearing center, ϕ_1 is referred to lower bearing arc and ϕ_2 is referred to upper bearing arc.
$\dot{\phi}, \bar{\phi}$	angular whirl speeds; ($\dot{\phi} = d\phi/dt$; $\bar{\phi} = 2\dot{\phi}/\omega$)
θ	angular coordinate measured c.c.w. from the y-axis.
λ	bearing operating parameter[$\mu LR(R/C)^2/\pi = \lambda$]
μ	Viscosity of the lubricant
$\omega, \omega_0, \omega_d$	angular speed of the journal; system resonant speed; ($\omega_o = \sqrt{k/M}$); ω_d is whirl speed of shaft at the onset of instability.

INTRODUCTION

Non-circular bearings are proposed for precise running of machine tool spindles [1]. Elliptic bearings are but one kind of non-circular bearings. The journal is balanced in an elliptic bearing with fluid film forces generated in the converging spaces of the upper and the lower bearing arcs. Performance characterestics for an elliptic bearings with Reynold's boundary conditions ($p=dp/d\theta = 0$) are provided [2]. A solution for Reynold's equation in an elliptic bearing assuming half Sommerfeld model is obtained[3]. Both solutions show that for lightly loaded elliptic bearingswith small ellipticity ratios, the journal center may actually travel upward in the bearing under load. This is explained as being due to excessive fluid film pressure in the lower bearing arc.

The dynamic characterestics of fluid film bearings have a stronginfluence on the vibration of rotating machine. If the bearing is rigidly mounted, then the rotor has two whirl onset speeds one for translational whirl and one for conical whirl. In practice, only the lower of the two onset speeds can be reached since the bearing becomes unstable at all speeds above this critical value. Three methods of analysis have been developed for predicting the onset of instability in a self acting gas journal bearing :

1- the solution of the time dependent equations of motion for the rotor [4];
2- the solution of Reynold's equation using Galerkin method [6] ; and
3- the linearized theory for the dynamic behaviour of bearings [5].

6 Design data are only available for few bearing geometries. In theory, this information can be obtained from computer solution of Reynold's equation provided that the lubricant behaviour can be adequately defined during small dynamic motion at the onset of instability.

DYNAMICS OF THE SYSTEM

Fig. 1 shows a general configuration of an elliptic bearing. The external load vector $(-Y_o)$ is acting downward parallel to the y-axis. The general form of Reynold's equation for each bearing arc in normalized notation is;

$$\frac{\partial}{\partial\theta}\left(H_s^3 \frac{\partial P_s}{\partial \theta}\right) + \left(\frac{R_j}{L}\right)^2 \frac{\partial}{\partial Z}\left(H_s^3 \frac{\partial P_s}{\partial Z}\right) = \frac{\partial H_s}{\partial\theta} + \left\{ \varepsilon_s \bar{\phi}_s + \tfrac{1}{2}\left(\frac{C}{R_j}\right)\left[\bar{\varepsilon}_s \frac{\partial H_s}{\partial \theta} + \varepsilon_s \bar{\phi}_s H_s \right]\right\} \sin(\theta - \phi_s) + \left\{ \bar{\varepsilon}_s + \tfrac{1}{2}\left(\frac{C}{R_j}\right)\left[\bar{\varepsilon}_s H_s - \varepsilon_s \bar{\phi}_s\right]\right\} \cos(\theta - \phi_s) \quad ..(1)$$

$s \equiv 1$; for the lower lobe ; and
$s \equiv 2$; for the upper lobe.

Bearing geometrical parameters may be evaluated with the following relations [2];

$$\varepsilon_1 = e_1/C = [\varepsilon^2 + \delta^2 + 2\varepsilon\delta\cos\phi]^{\frac{1}{2}} \quad (2)$$

$$\varepsilon_2 = e_2/C = [\varepsilon^2 + \delta^2 - 2\varepsilon\delta\cos\phi]^{\frac{1}{2}} \quad (3)$$

$$\phi_1 = \sin^{-1}(\varepsilon \sin\phi / \varepsilon_1) \quad (4)$$

$$\phi_2 = \pi - \sin^{-1}(\varepsilon\sin\phi / \varepsilon_2) \quad (5)$$

$$H_1 = h_1/C = 1 + \varepsilon_1 \cos(\theta - \phi_1) \quad (6)$$

$$H_2 = h_2/C = 1 + \varepsilon_2 \cos(\theta - \phi_2) \quad (7)$$

Geometrical parameters related to major or minor clearance in the bearing may be calculated using the following:

$$C = C_m + d \quad ; \quad \delta_m = \delta/(1-\delta) \quad \text{and} \quad \varepsilon_m = \varepsilon/(1-\delta)$$

where; $\delta_m = d/C_m$ and $\varepsilon_m = e/C_m$

If the journal center moves inside the the bearing with radial velocity $(\dot{e})$ and angular whirl speed $\dot{\phi}$, there will be other whirling components with respect to each bearing arc;

$$\bar{\varepsilon}_1 = \bar{\varepsilon}\cos(\phi - \phi_1) - \varepsilon\bar{\phi}\sin(\phi - \phi_1) \quad (8)$$

$$\bar{\varepsilon}_2 = \bar{\varepsilon}\cos(\phi_2 - \phi) + \varepsilon\bar{\phi}\sin(\phi_2 - \phi) \quad (9)$$

$$\bar{\phi}_1 = \left(\frac{\bar{\varepsilon}}{\varepsilon_1}\right)\sin(\phi - \phi_1) + \left(\frac{\varepsilon}{\varepsilon_1}\right)\bar{\phi}\cos(\phi - \phi_1) \quad (10)$$

$$\bar{\phi}_2 = \left(\frac{\varepsilon}{\varepsilon_2}\right)\bar{\phi}\cos(\phi_2 - \phi) - \left(\frac{\bar{\varepsilon}}{\varepsilon_2}\right)\sin(\phi_2 - \phi) \quad (11)$$

. . .

The boundary conditions assuming half Sommerfeld model are:

$$P_1 = 0 \; ; \text{ at } \Theta = \tfrac{1}{2}\pi - \beta \; ; \text{ and at } \Theta \geqslant \pi + \phi_1 \; ; \text{ and}$$

$$P_2 = 0 \; ; \text{ at } \Theta = (3\pi/2) + \gamma \text{ and at } \Theta \geqslant \pi + \phi_1$$

β and γ are very small angles that occurred due to the displacement of the journal cneter from the horizontal x-axis of the bearing , fig. 1.

By examination of the right hand side of eqn. (1), whereas H_s is expressed with eqns. (6) or (7) and neglecting terms including (C/R) , being of order 10^{-3}, the r.h.s. may be approximately expressed as :

$$\text{r.h.s.} \cong - \varepsilon(1 - \bar{\phi}) \sin(\Theta - \phi) + \bar{\varepsilon} \cos(\Theta - \phi)$$

If $\bar{\phi} = 0$, fluid film force (radial and tangential) components will be expressed with;

$$F_r = \lambda\omega f_r(\varepsilon , \bar{\varepsilon}) \quad ; \quad \text{and} \quad F_t = \lambda \omega f_t(\varepsilon , \bar{\varepsilon})$$

If $\bar{\phi} \neq 0$, fluid film force components may be expressed by :

$$F_r = \lambda \omega (1 - \bar{\phi}) \; f_r(\varepsilon , \bar{\varepsilon}') ; \text{ and } F_t = \lambda\omega(1 - \bar{\phi}) \; f_t(\varepsilon , \bar{\varepsilon}')$$

where; $\bar{\varepsilon}' = \bar{\varepsilon}/(1 - \bar{\phi})$

Both expressions of fluid film force components are of the same form if $(1 - \bar{\phi})$ is considered as scale factor of time.

A spring analogy has been proposed for linearized systems [8] and [9]. Stiffness and damping coefficients of fluid film are obtained from small displacement and velocity perturbation [9]. Stiffness coefficient K may be defined as;

$$K_{ij} = - \Delta F_i / \Delta x_j ; \text{ where: } x_i \equiv x \text{ and } x_j \equiv y$$

A damping coefficient may also be defined as;

$$C_{ij} = - \Delta F_i / \Delta \dot{x}_j ; \text{ and } \dot{x}_j \text{ is the perturbation velocity} \qquad \ldots(11\text{-a})$$

in direction of x or y. Although instability in a bearing is a two dimensional motion, analysis of linearized systems [8] and [9] assumes same criterion for resonance as that for the one dimensional analogy of a mass on a spring and dashpot.

If a journal runs in equilibrium under some external load and is displaced incrementally by ξ and η to another position, fig. 1, some additional fluid film force components (X and Y) will be superimposed. According to Pinkus [9], the added force components are:

$$X = \lambda\left(- \frac{\omega \xi f_r}{\varepsilon C} - \frac{2 f_t}{\varepsilon C} + \omega \frac{\eta}{C} \frac{\partial f_t}{\partial \varepsilon} + \frac{\dot{\eta}}{C} \frac{\partial f_t}{\partial \varepsilon'}\right) \qquad (12)$$

$$Y = \lambda\left(-\frac{\omega \xi f_t}{\varepsilon C} + \frac{2 \xi f_r}{\varepsilon C} - \omega \frac{\eta}{C} \frac{\partial f_r}{\partial \varepsilon} - \frac{\eta}{C} \frac{\partial f_r}{\partial \varepsilon'} \right) \qquad (12\text{-a})$$

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where; the fluid film force components are defined as:

$$f_r = \pi\,(C/R)^2\, F_r / \mu \omega R L = 6\,\pi\,\bar{F}_r\ ;$$

$$f_t = \pi\,(C/R)^2\, F_t / \mu \omega R L = 6\,\pi\,\bar{F}_t\ ;$$

$$\partial f_r/\partial \varepsilon' = 12\,\pi\,\partial \bar{F}_r / \partial \bar{\varepsilon}\ ; \text{ and}$$

$$\partial f_t/\partial \varepsilon' = 12\,\pi\,\partial \bar{F}_t / \partial \bar{\varepsilon}'$$

The added film forces are responsible for extra deflection of shaft (x & y) and for setting the whole system in acceleration;

$$2X = M(\ddot{x} + \ddot{\xi}) = -k\,x$$

$$2Y = M(\ddot{y} + \ddot{\eta}) = -k\,y \qquad (13)$$

ONSET OF INSTABILITY

Threshold of instability in journal bearing is obtained by solving equations of motion of a journal in a bearing for small oscillations about a position of equilibrium that corresponds to some external load (Y_o). Assuming that when whirling sets in, the displacement vector is;

$$x = x_o \exp(\nu\tau); \text{ where } \tau = \omega_o\, t \text{ and } \nu = g_d + i\,\omega_d \qquad (14)$$

g_d is the growth factor and ω_d is the oscillating factor. Solution of eqns. (12),(13) and (14) results in eigen value problem, of which the characterestic equation may be written. At onset of instability, $g_d = 0$ and by introducing the following notation, eqns for the onset of instability may be obtained;

$$\zeta = A\,\nu^2/(1 + \nu^2)$$

$$\frac{\zeta}{s} = \left\{-2\left(f_t \frac{\partial f_r}{\partial \varepsilon} - f_r \frac{\partial f_t}{\partial \varepsilon}\right) - \left(f_r \frac{\partial f_r}{\partial \varepsilon'} + f_t \frac{\partial f_t}{\partial \varepsilon'}\right)\right\} \Big/ \left(2 f_t + \varepsilon \frac{\partial f_r}{\partial \varepsilon'}\right) \qquad (15)$$

$$\left(\frac{\nu}{s}\right)^2 = \frac{\left\{-\left(\frac{\zeta}{s}\right)^2 - \left(f_r + \varepsilon\frac{\partial f_r}{\partial \varepsilon}\right)\left(\frac{\zeta}{s}\right) - \left(f_r \frac{\partial f_r}{\partial \varepsilon} + f_t \frac{\partial f_t}{\partial \varepsilon}\right)\right\}}{2\left[f_t \frac{\partial f_r}{\partial \varepsilon'} - f_r \frac{\partial f_t}{\partial \varepsilon'}\right]} \qquad (16)$$

$$s = \left\{ A\left(\frac{\nu}{s}\right)^2 \pm \sqrt{\left[A\left(\frac{\nu}{s}\right)^2\right]^2 - 4\left(\frac{\zeta}{s}\right)^2\left(\frac{\nu}{s}\right)^2} \right\} \Big/ 2\left(\frac{\zeta}{s}\right)\left(\frac{\nu}{s}\right)^2 \qquad (17)$$

DYNAMIC COEFFICIENTS

Computer aided numerical solution of eqn.(1) for an elliptic bearing with different ellipticity ratios (δ = 0.25, 0.5 and 0.75) and for different aspect ratios (L/D = 1.5, 1 and 0.5) is carried out using iteration procedure with relaxation to speed up convergence [3]. Results are shown in fig. 2 for the attitude angle ϕ and the resultant fluid film force $\bar{F}$. The resultant may be resolved in radial and tangential directions;

$$\bar{F}_r = -\bar{F}\cos\phi \quad \text{and} \quad \bar{F}_t = \bar{F}\sin\phi \tag{18}$$

The radial force component is positive if directed outwards from the bearing centre against the bearing surface. It may be seen that if the attitude angle is greater than 90^o, the radial force is positive, whereas it will become negative; (i.e. directed towards the bearing centre) , if the attitude angle is less than 90^o. Numerical differentiation using Lagrange's interpolation polynomial for four consecutive points [10] is used to evaluate force gradients; $\partial\bar{F}_r/\partial\varepsilon_m$ and $\partial\bar{F}_t/\partial\varepsilon_m$. Fig. 3 and fig. 4 are for the film force gradients . It may be seen that the gradient of the radial force is either positive, zero or negative, since the radial force changes from positive to negative with change of angle ϕ.

Damping coefficients, $\partial\bar{F}_r/\partial\bar{\varepsilon}$ and $\partial F_t/\partial\bar{\varepsilon}$ are evaluated by assuming that the journal center travels with a velocity of ± 0.01 in the radial direction ($\bar{\varepsilon} = \pm 0.01$). Radial and angular velocity components, being calculated by eqns. (8),(9),(10) and (11) are substituted in eqn.(1). Eqn(1) is then solved numerically, and the resulting pressure distributions are integrated to yield radial and tangential force components in the perturbed state. Damping coefficients are calculated according to eqn [11-a]. Damping factors calculated for positive radial velocity are the same as calculated for negative radial velocity of the journal center. Figs. 5 and 6 are for gradients $\partial\bar{F}_r/\partial\bar{\varepsilon}$ and $\partial\bar{F}_t/\partial\bar{\varepsilon}$ against eccentricity ratio. Figs. 2,3,4,5 and 6 together with eqns.(18) are used to provide the necessary dynamic coefficients to be substituted in eqns (15), (16) and (17) to obtain the onset speed ratio. An example for calculation is carried out for a shaft system with (A = 1) and (L/D = 1), but for different ellipticity ratios, and is shown in table 1.

TABLE 1. (L/D = 1) (A = 1)

δ	ε_m	f_r	f_t	$\frac{\partial f_r}{\partial \varepsilon_m}$	$\frac{\partial f_t}{\partial \varepsilon_m}$	$\frac{\partial f_r}{\partial \varepsilon'}$	$\frac{\partial f_t}{\partial \varepsilon'}$	$\frac{\zeta}{B}$	$(\frac{\gamma}{B})^2$	B
¼	0.2	0.339	0.727	1.51	4.77	14.7	7.16	-2.02	-0.212	1.945
	0.4	0.473	2.433	-1.34	11.34	8.67	3.02	0.704	-0.689	2.1
	0.8	-8.37	12.41	-73.0	56.0	37.0	-33	21.0	-0.165	2.45
½	0.2	0.115	1.093	0.302	5.485	18.9	17.6	-3.49	-0.152	2.42
	0.4	0.134	2.56	-0.25	9.35	16.9	15.7	-3.26	-0.342	1.56
	0.8	-0.86	8.135	-7.56	22.3	14.4	3.02	2.61	-0.72	1.38
¾	0.2	-0.35	2.196	-1.17	9.42	20.1	37.1	-9.08	-0.367	1.59
	0.4	-0.67	4.45	-1.58	12.3	20.0	35.3	-8.64	-0.476	1.39
	0.8	-1.9	10.75	-3.85	21.4	22.1	28.8	-6.78	-0.526	1.31

DISCUSSION AND CONCLUSION

Elliptic bearings have some pecularities of their own. For ordinary cylindrical journal bearing, the attitude angle is always less than 90°, while it is sometimes greater than 90° for elliptic bearings. Therefore, the radial film stiffness defined as the gradient of the radial force component, may change both in magnitude and sign (positive to negative) as the attitude angle changes from values greater than 90° to values less than 90°. The tangential component is always positive and increasingwith increase of eccentricity ratio. Therefore, the tangential film stiffness is always positive. It may be seen from fgs. 3 and 4 that film stiffness depends on ellipticity ratios. Its rate of increase decreases, generally,with increase of ellipticity ratio.

It may be difficult to give physical explanation of values obtained. These values may be simply called system coefficients without resorting to the analogy of spring-dashpot-mass system. It is known, however, that in journal bearings when the external load is removed, the journal whirls about the new equilibrium position rather than perform linear oscillation about this new position.

It may be seen from table1for calculation of onset speed that; generally the speed ratio (s) is lower than that would occur in ordinay cylindrical journal bearing. Elliptic bearings do not seem to pocess particular advantage over cylindrical journal bearing, if hydrodynamic instability is considered.

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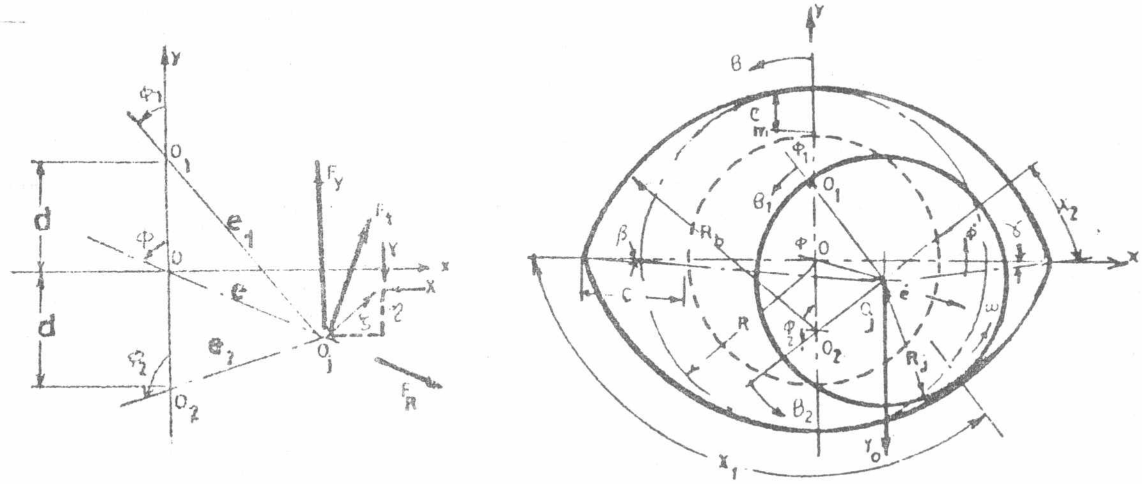

Fig. 1: Configuration of an elliptic bearing

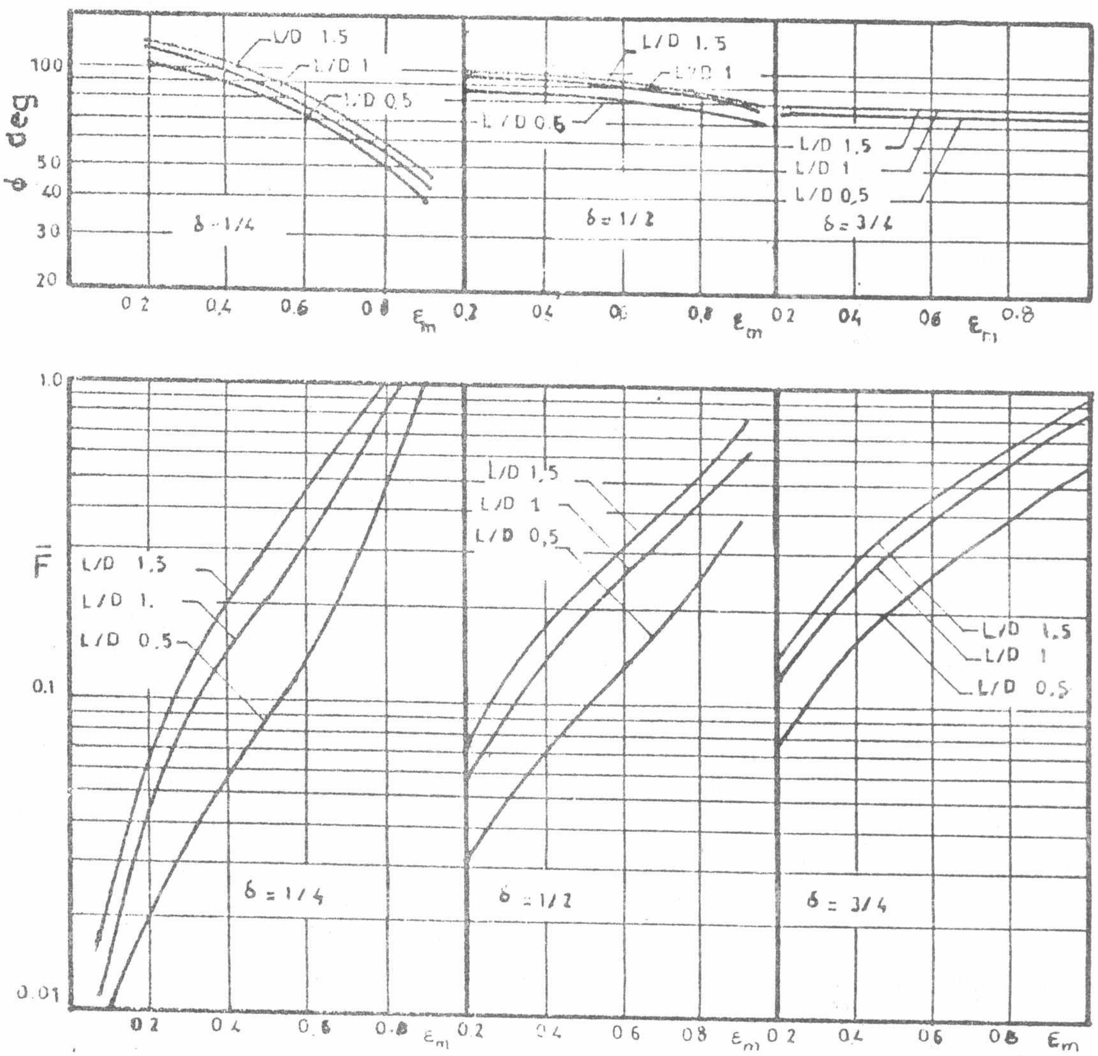

Fig.2:Fluid film force and attitude angle in elliptic bearing

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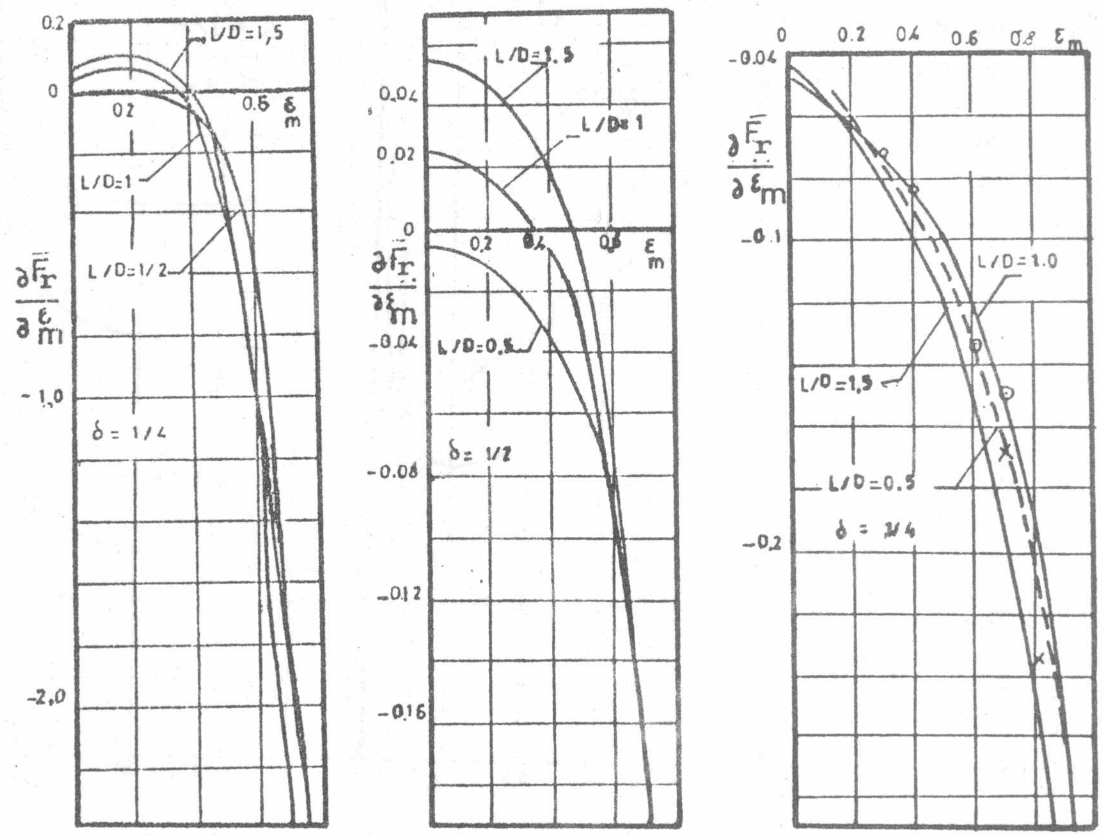

Fig.3:Rate of change of radial force component with eccentricity

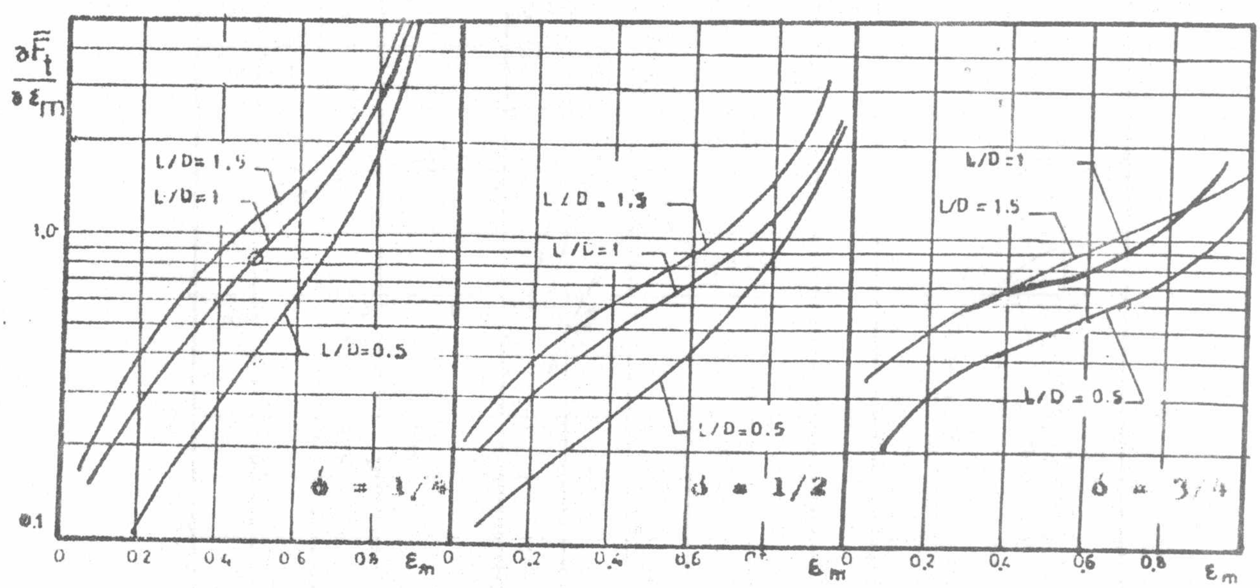

Fig.4:Rate of change of tangential force component with eccentricity

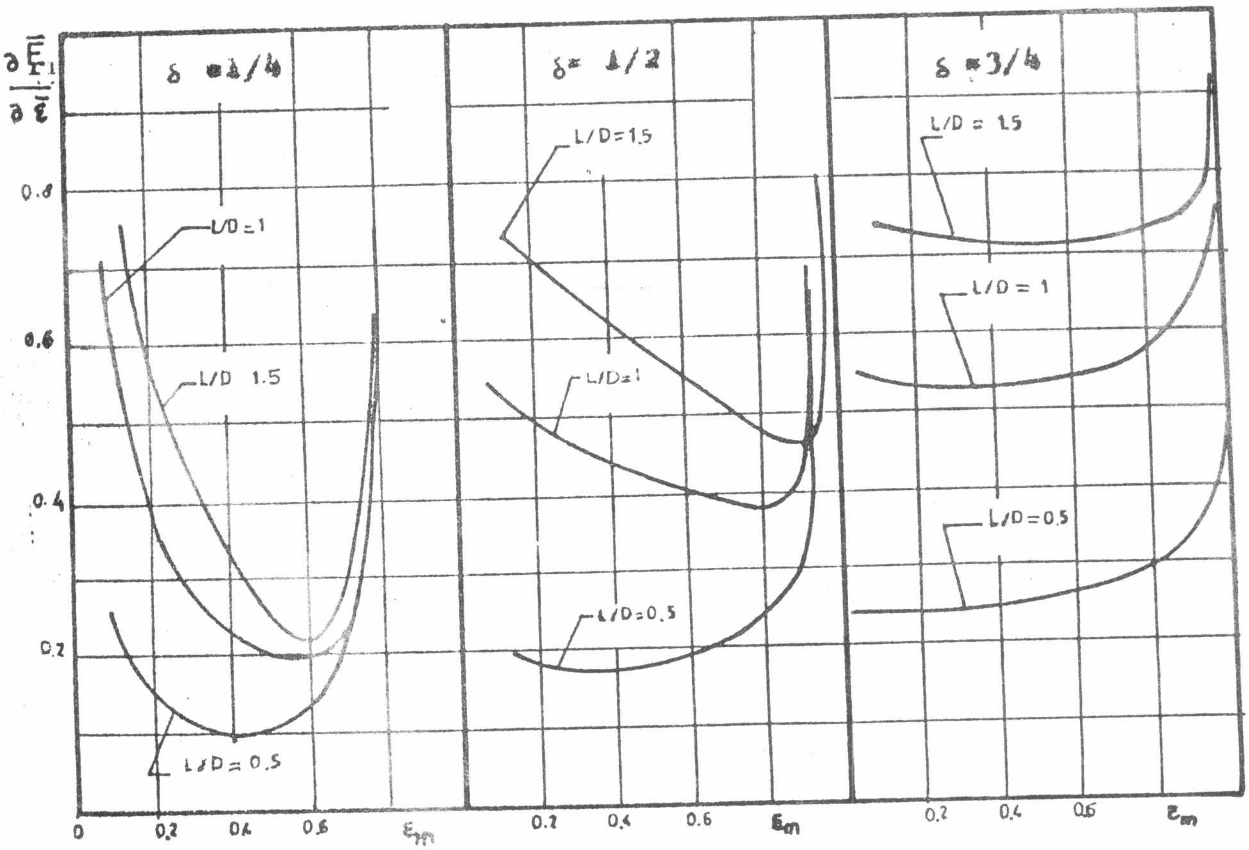

Fig. 5: Radial damping coefficient with eccentricity ratio

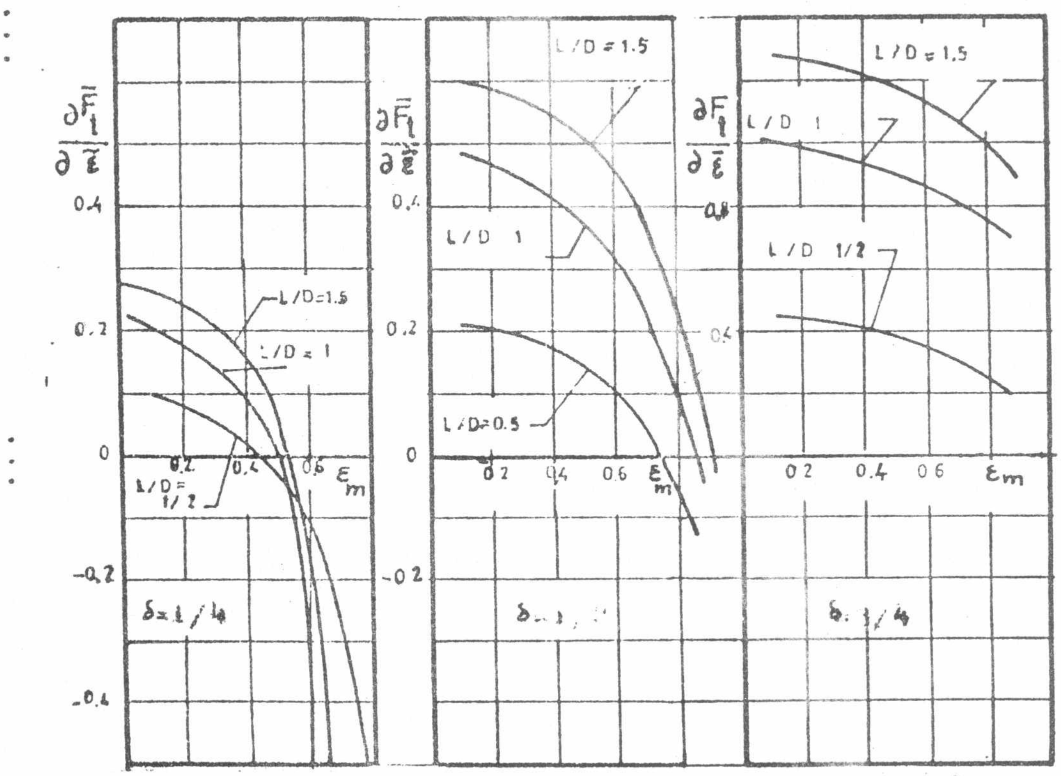

Fig.6: Tangential damping coefficient with eccentricity ratio