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AXISYMMETRIC VIBRATION OF STEPPED ANNULAR PLATES

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ABSTRACT

Free axisymmetric vibrations of annular stepped plates have been studied on the basis of the classical theory of plates. Applying the uniform plate solution to each zone of different thickness; considering the continuity and boundary conditions, yields a system of eight equations. Frequency determinants of such plates, with different step radii, are derived for various combinations of boundary conditions. Consequently the values of the resonant frequencies of the first two symmetric modes are calculated. Different graphs are included to facilitate the use of this material for design purpose.

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KEYWORDS

Transvers vibration,annular plates,stepped circular plates,boundary conditions,natural frequency,symmetric modes.

INTRODUCTION

The vibration analysis of circular plates with uniform thickness is a classical problem which can be found in literature. In a recent papers,authers like Kutter [1],Kirkhope [2] and others have studied the vibration of circular plates with variable thickness. Some of these plates which are of interest to engineers are the annular plates. The vibration of these plates with uniform thickness has been treated by Vogel [3] and Raju [4] , while the vibration of annular plates with variable thickness was studied by authers such as Soni [5] and Irie [6].

The previous two papers [7] and [8] were performed to obtain the modal analysis of stepped circular plates with different boundary conditions and stepped plates with elastically built-in boundary condition respectively.

In the present work, the modal analysis of symmetric vibration of stepped annular plates is developed to describe the effect of the step radius on the natural frequencies of these plates. The results are obtained for plates with different combinations of boundary conditions. Fortunantely the solution procedure gave good results.

PROBLEM FORMULATION

In this work, the vibration is assumed to be governed by small deflection , thin-plate theory , in which the effect of rotary inertia and shear deformation are ignored. The differential equation govering the transverse vibration of the circular plates in polar coordinate I8I is

$$\nabla^4 w(r) - \beta^4 w(r) = 0 \qquad (1)$$

where

$$\beta^4 = 12(1 - \nu^2)\rho\omega^2/Eh^2 \qquad (2)$$

The solution of eqn.(1) is

$$W(r) = A\, J_o(\beta r) + B\, Y_o(\beta r) + C\, I_o(\beta r) + F\, K_o(\beta r) \qquad (3)$$

The values of the constants A,B,C,F and β are determined from the boundary conditions of the plates.

Figure 1 shows the cross section of two particular forms of annular plates with stepped thickness. The plate may be described by two zones I and II in which each one represents annular plate with uniform thickness. The common radius r_1 is called the step radius.

For the two annular plates I and II , the solutions are given by eqn. (3) respectively

$$W_1(r) = A_1 J_0(\beta_1 r) + B_1 Y_0(\beta_1 r) + C_1 I_0(\beta_1 r) + F_1 K_0(\beta_1 r) \tag{4}$$

$$W_2(r) = A_2 J_0(\beta_2 r) + B_2 Y_0(\beta_2 r) + C_2 I_0(\beta_2 r) + F_2 K_0(\beta_2 r) \tag{5}$$

where

$$\beta_1^4 = 12(1 - \nu^2)\,\rho\,\omega^2 / E h_1^2 \tag{6}$$

$$\beta_2^4 = 12(1 - \nu^2)\,\rho\,\omega^2 / E h_2^2 \tag{7}$$

(a)

(b)

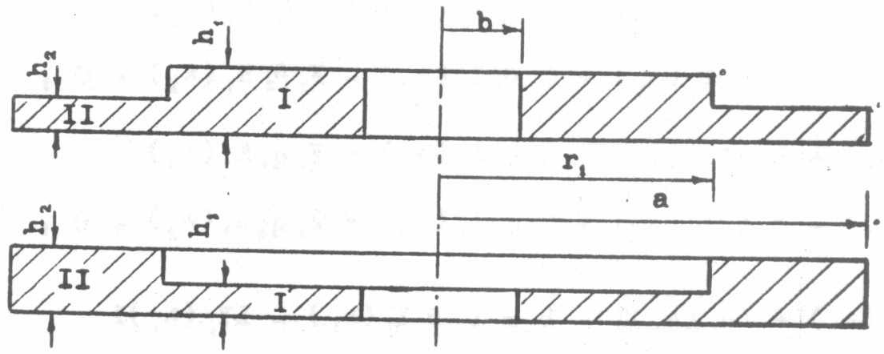


Fig.1 Cross section of two types of stepped annular plates.

a- plate with central region raised.
b- plate with central region unraised.

The different coefficients of eqns.(4) and (5) must be chosen to satisfy the continuity conditions at the step radius r_1 and the boundary conditions at r=a.

CONTINUITY CONDITIONS

The following are the four continuity conditions for each of deflection, slope, shear force and bending moment respectively

$$W_1(r_1) = W_2(r_1) \tag{8}$$

$$dW_1(r_1)/dr = dW_2(r_1)/dr \tag{9}$$

$$Q_1(r_1) = Q_2(r_1) \tag{10}$$

$$M_{r1}(r_1) = M_{r2}(r_1) \tag{11}$$

Where Q and M_r are the expressions for shear force and bending moment respectively, [9]

The expressions of W_1 and W_2 will have the following forms at $r=r_1$.

$$W_1(r_1) = A_1 J_0(\lambda_1) + B_1 Y_0(\lambda_1) + C_1 I_0(\lambda_1) + F_1 K_0(\lambda_1) \tag{12}$$

$W_2(r_1) = A_2 J_0(\lambda_2) + B_2 Y_0(\lambda_2)$ ...

Consequently the condition (8) can be expressed as follows:

$$A_1 J_0(\lambda_1) + B_1 Y_0(\lambda_1) + C_1 I_0(\lambda_1) + F_1 K_0(\lambda_1)$$
$$-A_2 J_0(\lambda_2) - B_2 Y_0(\lambda_2) - C_2 I_0(\lambda_2) - F_2 K_0(\lambda_2) = 0 \qquad (14)$$

Refering to the differentiation of the Bessel's functions [10],the conditions (9),(10) and (11) are given respectively as:

$$A_1 \beta_1 J_1(\lambda_1) + B_1 \beta_1 Y_1(\lambda_1) - C_1 \beta_1 I_1(\lambda_1) + F_1 \beta_1 K_1(\lambda_1)$$
$$-A_2 \beta_2 J_1(\lambda_2) - B_2 \beta_2 Y_1(\lambda_2) + C_2 \beta_2 I_1(\lambda_2) - F_2 \beta_2 K_1(\lambda_2) = 0 , \qquad (15)$$

$$A_1 q_1 J_1(\lambda_1) + B_1 q_1 Y_1(\lambda_1) + C_1 q_1 I_1(\lambda_1) - F_1 q_1 K_1(\lambda_1)$$
$$-A_2 q_2 J_1(\lambda_2) - B_2 q_2 Y_1(\lambda_2) - C_2 q_2 I_1(\lambda_2) + F_2 q_2 K_1(\lambda_2) = 0 , \qquad (16)$$

$$A_1 m_1 [-\beta_1 J_0(\lambda_1) + L J_1(\lambda_1)] - B_1 m_1 [\beta_1 Y_0(\lambda_1) - L Y_1(\lambda_1)]$$
$$+C_1 m_1 [\beta_1 I_0(\lambda_1) - L I_1(\lambda_1)] + F_1 m_1 [\beta_1 K_0(\lambda_1) + L K_1(\lambda_1)]$$
$$-A_2 m_2 [-\beta_2 J_0(\lambda_2) + L J_1(\lambda_2)] - B_2 m_2 [-\beta_2 Y_0(\lambda_2) + L Y_1(\lambda_2)]$$
$$-C_2 m_2 [\beta_2 I_0(\lambda_2) - L I_1(\lambda_2)] - F_2 m_2 [\beta_2 K_0(\lambda_2) + L K_1(\lambda_2)] = 0 \qquad (17)$$

where

$$\left.\begin{array}{lll} \lambda_1 = \beta_1 r_1 , & & \lambda_2 = \beta_2 r_1 \\ q_1 = \beta_1^3 h_1^3 , & & q_2 = \beta_2^3 h_2^3 \\ m_1 = \beta_1 h_1^3 , & & m_2 = \beta_2 h_2^3 \\ & L = (1 - \nu)/r_1 & \end{array}\right\} \qquad (18)$$

BOUNDARY CONDITIONS

The boundaries of the annular plate either at r=a or r=b may be clamped, simply supported or free. By using the expressions of the deflection described in eqns.(12) and (13) and their derivatives we can obtain the following forms of the boundary conditions for each case.

Clamped Supported Conditions

a- $W = 0$, or

$$A_i J_0(\lambda_i) + B_i Y_0(\lambda_i) + C_i I_0(\lambda_i) + F_i K_0(\lambda_i) = 0 \qquad (19)$$

b- $dW/dr = 0$, or

$$-A_i J_1(\lambda_i) - B_i Y_1(\lambda_i) + C_i I_1(\lambda_i) - F_i K_1(\lambda_i) = 0 \qquad (20)$$

6

Simply Supported Conditions

a- W = 0 , see eqn.(19)

b- M = 0 , or

$$A_i[-\beta_i J_0(\lambda_i) + PJ_1(\lambda_i)] + B_i[-\beta_i Y_0(\lambda_i) + PY_1(\lambda_i)]$$
$$+C_i[\ \beta_i I_0(\lambda_i) - PI_1(\lambda_i)] + F_i[\ \beta_i K_0(\lambda_i) + PK_1(\lambda_i)] = 0 \qquad (21)$$

Free Supported Conditions

a- Q = 0 , or

$$A_i J_1(\lambda_i) + B_i Y_1(\lambda_i) + C_i I_1(\lambda_i) - F_i K_1(\lambda_i) = 0 \qquad (22)$$

b- M = 0 , see eqn.(21)

where

$$\left.\begin{aligned} &\lambda_i = \beta_i r , \\ &i = \begin{cases} 1 & \text{for the inner boundary} \\ 2 & \text{for the outer boundary ,} \end{cases} \\ &r = \begin{cases} b & \text{for } i = 1 \\ a & \text{for } i = 2 , \end{cases} \\ &P = (1 - \nu)/a \end{aligned}\right\} \qquad (23)$$

Hence,it is clear that for an annular plate,there are four continuity conditions and four boundary conditions. This system of equations can be arranged in a matrix form in order to obtain the frequency determinant as follows:

$$[Y]\ [C] = [0] \qquad (24)$$

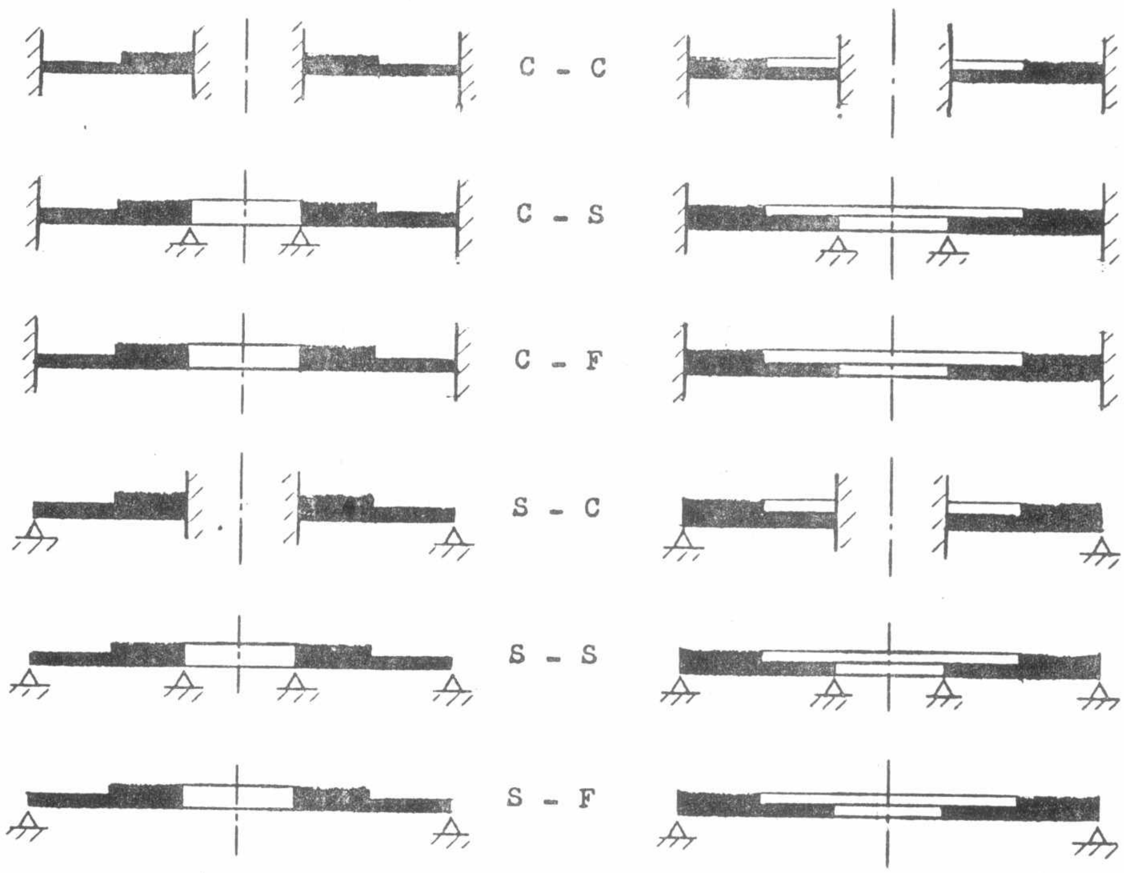
where

[Y] is 8x8 square matrix
[C] is the column matrix containing the coefficients described before in eqns.(4) and (5).

The natural frequencies of the symmetric modes of such plates can be obtained by equating the determinant of the matrix [Y] by zero.

COMPUTED RESULTS AND DISCUSSIONS

The numerical computation for the natural frequencies are obtained for annular plates with different combinations of the boundary conditions. Figure 2 shows such plates in which the inner radius b=45 mm and the outer radius a=150 mm. In the case of plates with central region raised,the thicknesses h_1 and h_2 are 1.0 and 0.75 mm respectively,while in the case of plates with central region unraised,h_1 and h_2 are 0.75 and 1.0 mm respectively.



a- Annular plates with central region raised.

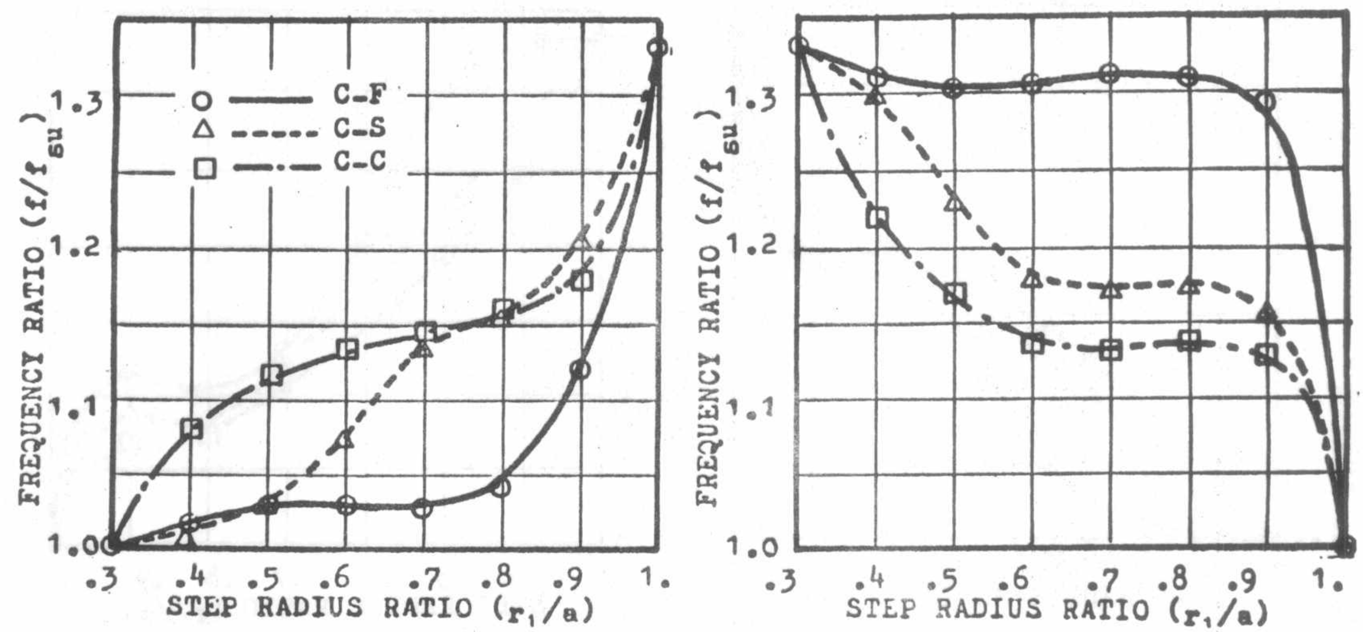
b- Annular plates with central region unraised.

Fig.2 Stepped annular plates with different boundary conditions.

These models are made of Pirespex with Young's modulus of 262 x10^{7} N.m^{-2}, Poisson's ratio ν = 0.38 and a density of 1.237 x10^{3} Kg.m^{-3}.

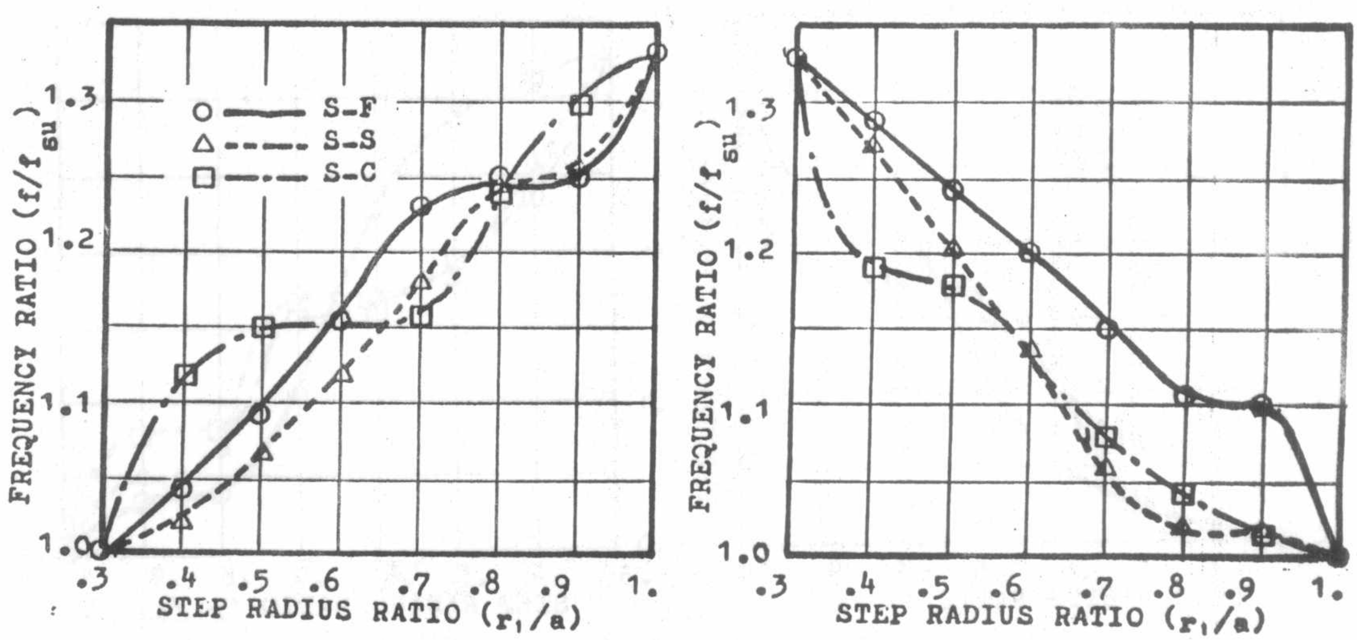
The results obtained by one of the computer methods are expressed in dimensionless form, therefore there are eight step radius ratios r_1/a vary from 0.3 to 1.0 which are corresponding to the step radii r_1 = 45, 60,75,90,105,120,135 and 150 mm. Also the natural frequencies, f are expressed in dimensionless form f/f_{su}, where f_{su} represents the natural frequency of uniform annular plate with the smallest thickness 0.75 mm either in the case of central region raised or unraised.

Figure 3 shows the results of natural frequencies of C-F, C-S and C-C stepped annular plates, while Fig. 4 shows the results for S-F, S-S and S-C stepped annular plates. The two figures show the effect of the variation of step radius on the natural frequencies of the different plates at the first symmetric mode 00. Similarly, Figs. 5 and 6 show the same relationships at the second symmetric mode 01.



(a) (b)

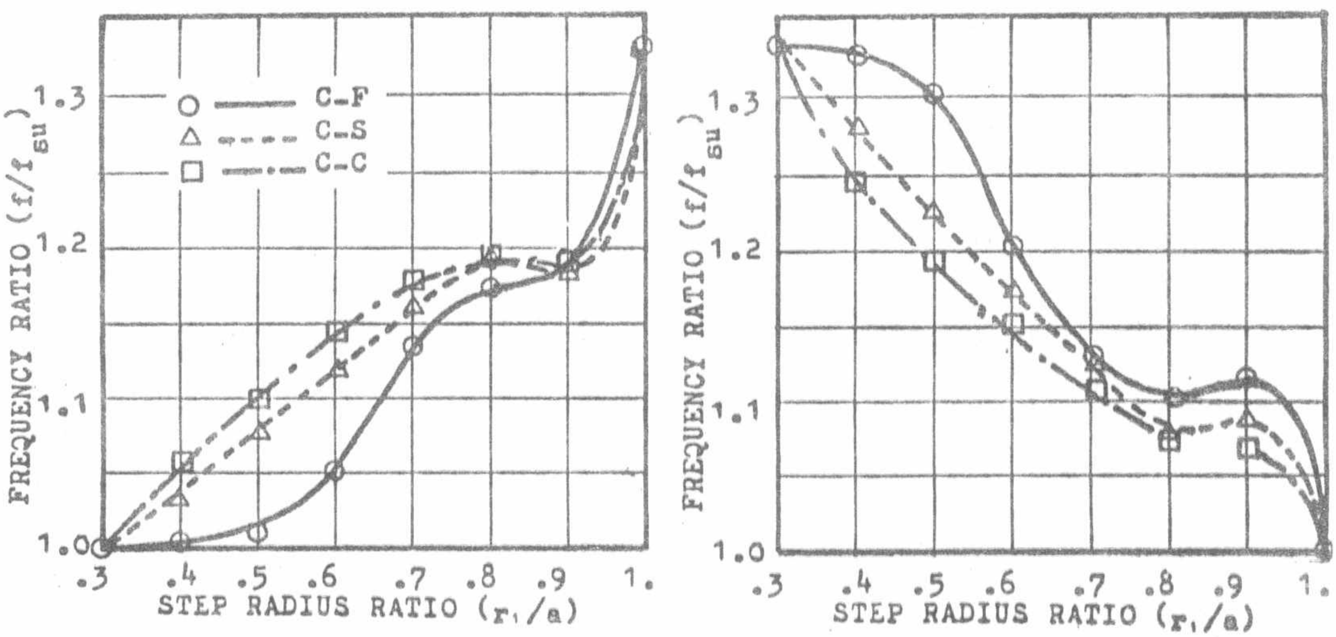
Fig.3 Natural frequencies of C-F, C-S and C-C stepped annular plates at the first symmetric mode 00.



(a) (b)

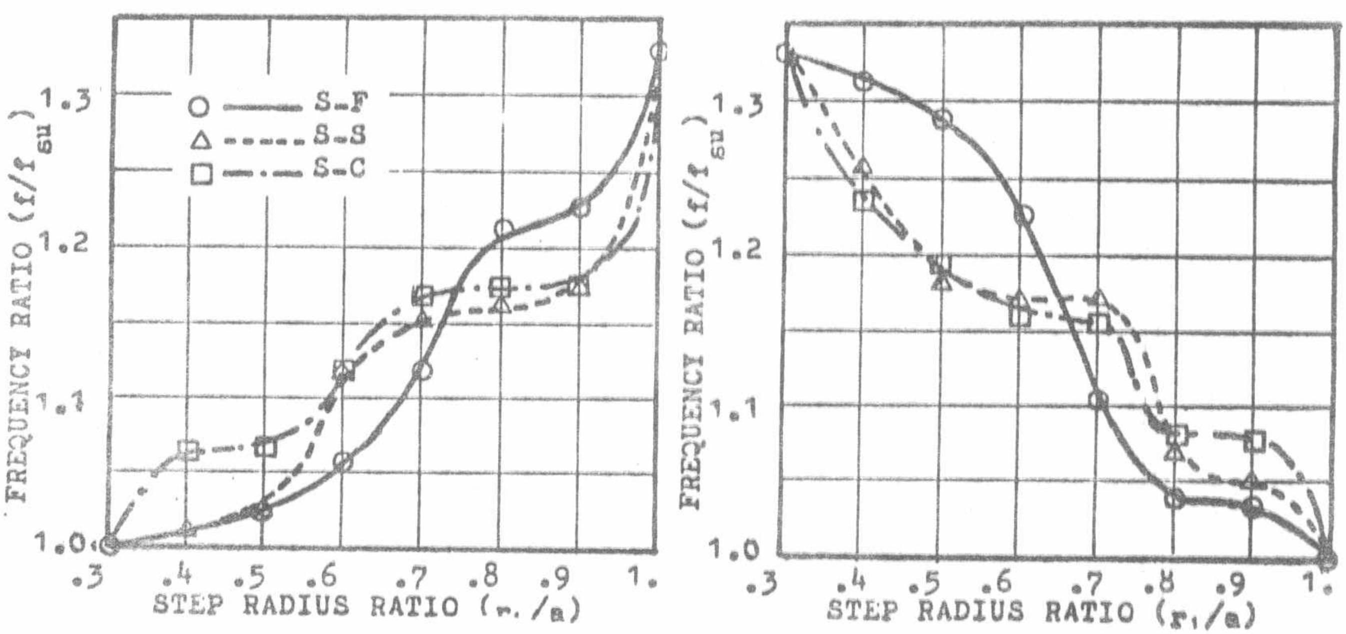
Fig.4 Natural frequencies of S-F, S-S and S-C stepped annular plates at the first symmetric mode 00.

a- plates with central region raised.
b- plates with central region unraised.



(a) (b)

Fig.5 Natural frequencies of C-F, C-S and C-C stepped annular plates at the second symmetric mode 01.



(a) (b)

Fig.6 Natural frequencies of S-F, S-S and S-C stepped annular plates at the second symmetric mode 01.

a- plates with central region raised.
b- plates with central region unraised.

In the case of central region raised, when the step radius r, equal to the inner radius b, the plate will be with uniform thickness of 0.75 mm while when the step radius equal to the outer radius a, the plate will be with uniform thickness of 1.0 mm. Consequently the vice-versa will be in the case of central region unraised. Table 1 shows the frequency results for these annular plates of uniform thicknesses and its comparison with those obtained by Raju [4]. The results were convenient and useful for the application of this process in a large scale.

Table 1. Natural Frequencies of Uniform Thickness Annular Plates.

Type of Boundaries	Mode mn	Natural Frequencies (Hz) by			
		Raju [4]		Given Method	
		h=0.75 mm	h=1.00 mm	h=0.75 mm	h=1.00 mm
C - F	00	27.45	36.60	27.00	36.00
	01	124.50	166.00	122.00	163.00
C - S	00	81.07	108.10	80.00	107.00
	01	250.50	334.00	246.00	329.00
C - C	00	109.26	145.69	107.20	144.12
	01	301.05	401.40	296.00	395.00
S - F	00	11.17	14.90	11.00	14.60
	01	89.10	118.80	87.00	117.00
S - S	00	50.77	67.80	50.00	67.00
	01	197.02	262.70	194.00	260.00
S - C	00	71.92	95.90	71.00	95.00
	01	240.82	321.10	237.50	317.50

CONCLUSIONS

The analysis of the axisymmetric vibrations of stepped annular plates are presented for plates with different combinations of boundary conditions. It is seen that in the case of plates with central region raised, the value of the natural frequency usually increases as the step radius increases. In the case of plates with central region unraised the vice-versa will happened excpet that there are some of the adjacent step radii at which the corresponding frequencies can be considered equal . Also the given charts show the effect of the boundary conditions on the results from the design point of view.

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NOMENCLATURE

a	Outer radius of the annular plate
E	Modulus of elasticity.
b	Inner radius of the annular plate.
f	Natural frequency.
h	Plate thickness.
I_0, I_1	Modified Bessel functions of the first kind of order 0,and 1.
J_0, J_1	Bessel functions of the first kind of order 0 and 1.
K_0, K_1	Modified Bessel functions of the second kind of order 0 and 1.
m	No. of nodal diameters.
M	Radial bending moment.
n	No. of nodal circles.
Q	Transverse shear force.
r	Plate radius.
W	Plate deflection.
Y_0, Y_1	Modified Bessel functions of the second kind of order 0 and 1.
β	Eigenvalue of the frequency equation.
ρ	Density of the plate.
ν	Poisson's ratio.
θ	Angular displacement.
ω	Natural frequency.