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ELIMINATION OF THE FLUCTUATION OF THE INPUT TORQUE OF PLANAR MECHANISMS BY SPRING-CAM SYSTEM

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ABSTRACT

The paper proposes the use of a cam with a reciprocating or oscillating follower for the elimination of the fluctuation of the torque input to a planer mechanism. The cam is fixed to the input shaft, which is rotating at a uniform speed, and the follower is maintained in contact with the cam by means of a spring. The expression of the input torque, which is required to drive a planar mechanism at a constant input speed, with the output leads and inertia forces considered, is derived. The cam profile is designed such that the cam input torque counters the fluctuating part of the mechanism input torque. This condition is employed in deriving the cam design equation, which is a differential equation describing the follower displacement. A special numerical method is proposed for the solution of this equation. Also, a formula is derived for the contact force between the cam and follower. This formula is applied at different mechanism positions, after the solution of the cam design equation, to check the contact between the cam and follower. The proposed method is applied to a four-bar linkage by using a disc cam with an oscilating follower as a numerical example.

1. INTRODUCTION

The input torque, which is required to drive a mechanism at a constant input speed, in many cases, is fluctuating. Reduction of this fluctuation is usually recommended in order to reduce the variation in the input speed and the maximum torque to be delivered through the input shaft. In the bulk of the work devoted for this purpose the reduction was accomplished by either: (1) proper distribution of the link masses ([1]-[5]), or (2) attaching springs to some of the moving links ([6]-[9]).

Sherwood [1] introduced a method for determining the mass distribution of the coupler of a four-bar linkage which yields the minimum fluctuation in the input torque when the mechanism is moving against no output load. The same problem was considered by Sherwood and Hockey [2], but with the use

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of the principle of dynamically similar systems. This method was improved by Hockey [3], to avoid the unreal results which were obtained by its application. The improved method was applied by Hockey [4] to a four-bar linkage with output load. Elliott and Tesar [5] considered the planar four-link mechanisms: the four-bar, the slider-crank, the inverted slider-crank and the oscillating block mechanisms. For each of these mechanisms, they derived an expression to be used for determining the unknown link mass parameters by which the input torque at up to three of the mechanism positions can take predetermined values.

Cenova [6] and Skreiner [7] suggested the use of a spring attached to the input and output links respectively, to reduce the fluctuation of the input torque. Mathew and Tesar [8,9] established a method for synthesis of spring parameters to satisfy specified energy levels in planar mechanisms. The method was employed for the minimization of the fluctuation of the input torque.

In the present paper, it is shown that, complete elimination of the fluctuation of the input torque can be achieved by the use of a cam fixed to the mechanism input shaft and actuating a follower against a spring. Sections 3 to 6 deal with a reciprocating follower. The use of an oscillating follower is presented in section 7.

2. MECHANISM INPUT TORQUE

The torque required to drive a mechanism consists, in general, of two components, namely the torque T_o, which is necessary to overcome the loads acting on its output links, and the torque T_r, which is required to drive the mechanism against the inertia forces and couples acting on the moving links. The planar mechanism under consideration is assumed to have n links numbered such that the driving crank, which is rotating at a uniform speed ω, is link (1), whereas the numbers from n_1 to n denote the output links.

The first torque T_o may be calculated, without considering the forces acting on the links individually, by the application of the principle of energy conservation, since the friction at all joints is ignored and the mechanism is regarded as a conservative system. Thus, the power input by T_o is assumed to be equal to the output power exerted by the output links against the output loads. The expression defining the power delivered through an output link depends on the type of the load to be overcome by the link, i.e. whether the load is a force or a torque. For an output link moving against a force load, the power P_ρ is obtained from:

$$P_\rho = - S_\rho v_\rho \cos \Psi_\rho ,$$

where: ρ = link number ,

S_ρ = load force acting on the link ,

v_ρ = velocity of the point of the application of the load force on the link ,

Ψ_ρ = phase angle between F_ρ and v_ρ.

When the output link is rotating against a load torque, the power is given by:

$$P_\ell = -T_\ell \dot{\Psi}_\ell ,$$

where: T_ℓ = load torque acting on the link,

Ψ_ℓ = angular position of the link with respect to a fixed coordinate system.

If the links are numbered such that the numbers from n_1 to n_2-1 belong to the output links which overcome force loads, whereas the numbers from n_2 to n refer to the output links revolving against load torques, then the application of the principle of energy conservation gives:

$$T_o \omega = - \sum_{\ell=n_1}^{n_2-1} S_\ell \, v_\ell \cos \Psi_\ell - \sum_{\ell=n_2}^{n} T_\ell \dot{\Psi}_\ell . \tag{1}$$

The energy developed by the torque T_r appears as a change in the kinetic energy of the moving links of the mechanism, and therefore the power input by T_r is equal to the rate of change of the kinetic energy of the moving links. The driving crank may be excluded in the calculation of the kinetic energy, since it is rotating at a uniform speed about a fixed axis.

Thus,

$$T_r \omega = \sum_{\ell=2}^{n} \frac{d}{dt} [\frac{1}{2} m_\ell (\dot{x}_\ell^2 + \dot{y}_\ell^2) + \frac{1}{2} I_\ell \dot{\Psi}_\ell^2] , \tag{2}$$

where: m_ℓ = mass of link (ℓ),

x_ℓ, y_ℓ = coordinates of the mass centre of link (ℓ) with respect to a fixed coordinate system,

I_ℓ = mass moment of inertia of link (ℓ) about the axis perpendicular to the plane of motion through its mass centre,

t = time.

The input torque then is given by:

$$T_i = T_o + T_r \tag{3}$$

Assuming that the mechanism cycle corresponds to a complete revolution by the input crank, the mean driving torque T_m may be obtained by equating the energy input to mechanism by the driving crank with the work done by the output links during a complete revolution of the driving crank, since the kinetic energy of the moving links is not changed by a complete mechanism cycle. This gives:

$$T_m = \frac{1}{2\pi} \sum_{\ell=n_1}^{n} W_\ell , \tag{4}$$

where W_ℓ is the work done by the output link (ℓ) against its load during a complete mechanism cycle.

3. CAM INPUT TORQUE

The torque required to drive the cam depends on the follower type, i.e. whether reciprocating or oscillating. In this section and in the next three sections the follower is assumed to be reciprocating, whereas in section 7 the use of an oscillating follower is considered. The driving torque consists of the torque T_s, which is necessary to counter the effect of the spring force on the follower, and torque T_f, which is required to drive the moving masses (the follower and the equivalent mass of the spring) against their inertia forces. Both torques may be obtained by the application of energy conservation, as done in the foregoing section. This gives:

$$T_s \omega = Kz\dot{z} \ , \tag{5}$$

and

$$T_f \omega = (m_f + m_e)\ \ddot{z}\dot{z} \ , \tag{6}$$

where: K = spring stiffness,

z = follower displacement, measured from the position which produces no load in the spring, and increasing as the follower moves away from the cam centre,

m_f = follower mass,

m_e = equivalent mass of the spring (≈1/3 spring mass according to Rayleih's method [10]).

From eqs. (5) and (6), it follows that, the torque required to drive the cam is given by:

$$T_c = \frac{1}{\omega}\ [Kz\dot{z} + (m_f + m_e)\ \ddot{z}\dot{z}] \tag{7}$$

4. DESIGN EQUATION OF THE CAM

In order to obtain a uniform total torque, the cam profile is so designed that the torque T_c counters the fluctuating part of T_i. Thus:

$$T_c = T_m - T_i \ . \tag{8}$$

Substituting for T_c, this equation may be put in the form:

$$Kzz' + M\omega^2\ z'\ z'' = f'\ (\Gamma_1) \ , \tag{9}$$

where: $z' = \dfrac{dz}{d\Gamma_1} = \dfrac{\dot{z}}{\omega}$,

$z'' = \dfrac{d^2z}{d\Gamma_1^2} = \dfrac{\ddot{z}}{\omega^2}$,

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$$M = m_f + m_e ,$$

$$f'(\Psi_1) = T_m - T_i . \tag{10}$$

Now, for every position of the driving crank (Ψ_1), the output loads and the kinematic state (positions, velocities and accelerations) of the moving links can be determined, and accordingly $f'(\Psi_1)$ may be calculated. Thus, eq. (9) is a differential equation relating the dependent variable z to the independent variable Ψ_1. However, this equation may be simplified if its sides are integrated with respect to Ψ_1, for its variation from an arbitrary initial value, say $\Psi_1 = 0$, to a general value Ψ_1. This yields:

$$\frac{1}{2} K (z^2 - z_0^2) + \frac{1}{2} M\omega^2 (z'^2 - z_0'^2) = f(\Psi_1) , \tag{11}$$

where: z_0 , z_0' = values of z and z' at $\Psi_1 = 0$,

$$f(\Psi_1) = T_m \Psi_1 - \int_0^{\Psi_1} T_o \, d\Psi_1 - \frac{1}{2} \sum_{\ell=2}^{n} [m_\ell(\dot{x}_\ell^2 + \dot{y}_\ell^2) + I_\ell \dot{\Psi}_\ell^2] + E_0 , \tag{12}$$

where: E_0 = kinetic energy of links 2, 3, ..., n at $\Psi_1 = 0$. It is to be noticed that the solution of eq. (11) does not require the calculation of the accelerations $\ddot{x}_\ell$ and $\ddot{y}_\ell$, which are necessary for the solution of eq. (9).

5. SOLUTION OF THE DESIGN EQUATION

If z_0 and z_0' are arbitrary assumed, then eq. (11) becomes a quadratic first order differential equation in the dependent and independent variables z and Ψ_1 with known initial conditions, and accordingly it may be solved numerically. Obviously, it can not be insured that the solution obtained in this way satisfies the conditions imposed by the nature of the problem, which are:

i. the value of z at $\Psi_1 = 2\pi$ must be equal to z_0 ,

and ii. the value of z' at $\Psi_1 = 2\pi$ must be equal to z_0' .

However, from the definition of $f(\Psi_1)$, it can be seen that, it vanishes at $\Psi_1 = 2\pi$, and therefore eq. (11) guarantees that, if one of the foregoing conditions is fulfiled, the other is also fulfiled. Hence, only one of the unknowns z_0 or z_0' may be arbitrary assumed, while the other is left for the fulfilment of the conditions. Therefore, eq. (11) can not be solved by one of the known numerical methods of solution of the differential equations. The method proposed for solving eq. (11) is presented below.

Let the function period (Ψ_1 = 0 to 2π) be divided at (N-1) points into equal intervals each of length $\Delta\Psi_1$, and the values of $f(\Psi_1)$, z and z' at the ith point of these points be denoted by f_i, z_i, z_i' respectively. The constants $f_1, f_2, \dots, f_{N-1}$ can be calculated by the use of eqs. (1), (4) and (12), whereas the values of z and z' at the division points are to be determined by solving the equation. Application of eq. (11) at the division points gives:

$$\frac{1}{2} K (z_i^2 - z_0^2) + \frac{1}{2} M\omega^2 (z'^2_i - z'^2_0) = f_i \qquad (i=1,2,..., N-1). \qquad (13)$$

If the interval length $\Delta\Psi_1$ is sufficiently small, then the values of z' at the mid-points between the (i-1) th, ith and (i+1) th points may be considered equal to $(z_i - z_{i-1})/\Delta\Psi_1$ and $(z_{i+1} - z_i)/\Delta\Psi_1$ respectively. Also the mean of these values may be taken as z'_i, i.e.,

$$z'_i = \frac{z_{i+1} - z_{i-1}}{2\Delta\Psi_1} .$$

By applying the same principle z'_0 may be expressed as:

$$z'_0 = \frac{z_1 - z_{N-1}}{2\,\Delta\Psi_1} .$$

Substitution into eq. (13) yields:

$$\frac{1}{2} K (z_i^2 - z_0^2) + A[(z_{i+1} - z_{i-1})^2 - (z_1 - z_{N-1})^2] = f_i \qquad (i=1,2,...,N-1), \qquad (14)$$

where: $A = \dfrac{M\omega^2}{8(\Delta\Psi_1)^2}$.

In the present method z_0 is assumed, and therefore expression (14) represents (N-1) nonlinear equations in the unknowns z_1, z_2, ... and z_{N-1}. In order to solve these equations by Newton's Raphson iteration method they are put in homogeneous form:

$$F_i = 0 \qquad (i=1,2,...,N-1), \qquad (15)$$

where: $F_i = \frac{1}{2} K (z_i^2 - z_0^2) + A[(z_{i+1} - z_{i-1})^2 - (z_1 - z_{N-1})^2] - f_i$. (16)

The iteration formula of this method may be written as:

$$[B^{k+1}] = [B^k] - [D^k]^{-1} [C^k] , \qquad (17)$$

where: $[B^k] = [b_1^k , b_2^k , ... , b_{N-1}^k]^T$,

$[C^k] = [c_1^k , c_2^k , ... , c_{N-1}^k]^T$,

$[D^k]$ = an $(N-1)\times(N-1)$ matrix with elements $d_{11}^k, d_{12}^k, ..., d_{(N-1)(N-1)}^k$,

where: b_i^k = k th iteration value of z_i ,

c_i^k = the value of F_i at the k th iteration stage, i.e. when z_i is put equal to b_i^k,

d_{ij}^k = the value of $\dfrac{\partial F_i}{\partial z_j}$ at the k th iteration stage.

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In order to put $\partial F_i/\partial z_j$ in a form which can be easily programmed for the use of digital computer, F_i is expressed in the form:

$$F_i = U_i - V - f_i - \frac{1}{2} K z_0^2 \qquad (i=1,2,\ldots,N-1), \qquad (18)$$

where: $$U_i = A\,(z_{i+1} - z_{i-1})^2 + \frac{1}{2} K z_i^2, \qquad (19)$$

$$V = A\,(z_1 - z_{N-1})^2 . \qquad (20)$$

Thus, $\partial F_i/\partial z_j$ can be obtained from:

$$\frac{\partial F_i}{\partial z_j} = \frac{\partial U_i}{\partial z_j} - \frac{\partial V}{\partial z_j}, \qquad (21)$$

where:
$$\frac{\partial U_i}{\partial z_j} = -2\,A\,(z_{i+1} - z_{i-1}) \qquad (j=i-1),$$

$$= K z_i \qquad (j=1),$$

$$= 2\,A\,(z_{i+1} - z_{i-1}) \qquad (j=i+1),$$

$$= 0 \qquad \text{(otherwise)},$$

$$\frac{\partial V}{\partial z_j} = 2\,A\,(z_1 - z_{N-1}) \qquad (j=1),$$

$$= -2\,A\,(z_1 - z_{N-1}) \qquad (j=N-1),$$

$$= 0 \qquad \text{(otherwise)}.$$

Rapid convergence by Newton's Raphson method is achieved if the starting values $b_1^0, b_2^0, \ldots, b_{N-1}^0$ are sufficiently close to the exact values of $z_1, z_2 \ldots, z_{N-1}$, which satisfy the (N-1) equations. If M is small, reasonable starting values may be obtained by putting M = 0 in eq. (13). This yields:

$$(b_i^0)^2 = z_0^2 + \frac{2 f_i}{K} \qquad (i=1,2,\ldots,N-1). \qquad (22)$$

However, if M is not small, the equations may be solved by the same method, but in successive steps. This is accomplished by replacing M by a smaller value M_1, and solving the equations by the use of the starting values obtained from eq. (22). The resulting values of the variables are then taken as starting values for the next step in which M_1 is increased to a greater value M_2. In this way, the value considered for M is increased gradually until it reaches its actual value.

6. CONTACT BETWEEN CAM AND FOLLOWER

After determining the follower displacement z corresponding to the different positions of the driving crank, the contact force between the follower and cam is checked at the (N-1) division points, as well as at the end point of the mechanism cycle, which is referred to as point no. (N), to insure continuous contact between the follower and cam throughout the mechanism cycle. Since friction is neglected, the component of the contact force along the follower displacement may be expressed as:

$$F_{ci} = K\, z_i + M\omega^2\, z_i'' \qquad (i=1,2,\ldots,N). \qquad (23)$$

The values of z_i'' may be expressed, by the use of the values of z' at the mid-point between the (i-1)th, ith and (i+1)th division points, as:

$$z_i'' = \frac{z_{i+1} - 2z_i + z_{i-1}}{(\Delta \Psi_1)^2}.$$

This gives:

$$F_{ci} = (K-16A)\, z_i + 8A\,(z_{i-1}+z_{i+1}) \qquad (i=1,2,\ldots,N). \qquad (24)$$

Seperation between the follower and cam occurs if any of the values of F_{ci} is negative. This may be remedied by either: (i) increasing K, (ii) decreasing M, or (iii) increasing z_0.

7. OSCILLATING FOLLOWER

In many situations the use of an oscillating follower is preferred due to the smallness of its friction losses, especilly in the case under consideration, where the contact force may be relatively high and no output is taken from the follower. If the contact between the follower and cam is maintained by a torsional spring of stiffness K, the preceding equations are still applicable, but with z replaced by Θ, which is the angular displacement of the follower measured from the position which produces no force in the spring, and M replaced by I, which denotes the moment of inertia of the follower mass together with the equivalent mass of the spring about the axis of rotation of the follower.

If a helical spring, with stiffness K, is used (Fig. 1), in addition to the above replacements, the design equation (eq. (11)) is changed to:

$$\frac{1}{2}K\left\{[\ell-\sqrt{H_1-H_2\cos(\beta-\Theta)}]^2-[\ell-\sqrt{H_1-H_2\cos(\beta-\Theta_0)}]^2\right\}+\frac{1}{2}I\omega^2(\dot{\Theta}^2-\dot{\Theta}_0^2)=f(\Psi_1), \quad (25)$$

where: β = the angle between QD and QE, when the spring is unloaded,

$H_1 = \ell_1^2 + \ell_2^2$,

$H_2 = 2\ell_1\,\ell_2$,

$\ell = \sqrt{H_1 - H_2\cos\beta}$.

Accordingly, eq. (14) is changed to:

$$\frac{1}{2}K\left\{[\ell-\sqrt{H_1-H_2\cos(\beta-\Theta_i)}]^2 - [\ell-\sqrt{H_1-H_2\cos(\beta-\Theta_0)}]^2\right.$$
$$+ A\,[(\Theta_{i+1}-\Theta_{i-1})^2 - (\Theta_1-\Theta_{N-1})^2] = f_i \qquad (i=1,2,\ldots,N-1). \quad (26)$$

F_i and $\partial F_j/\partial\Theta_i$, which are used in the solution of the (N-1) equations represented by expression (26), are in this case obtained from:

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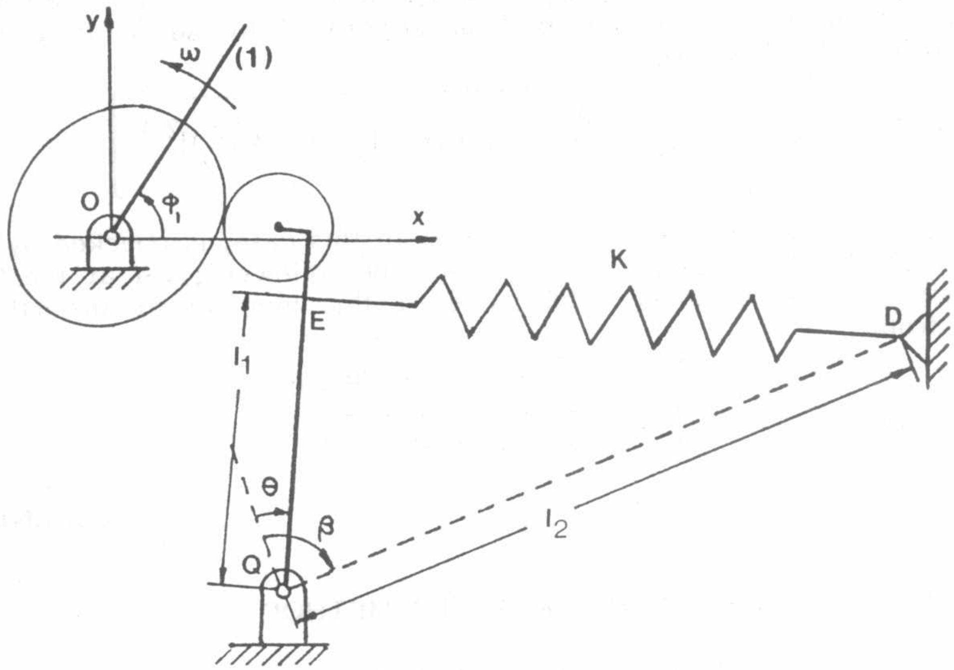

Fig.1: Oscillating follower with helical spring.

$$F_i = U_i - V - f_i - \frac{1}{2} K [l - \sqrt{H_1 - H_2 \cos(\beta - \Theta_0)}]^2 , \tag{27}$$

where: $$U_i = A (\Theta_{i+1} - \Theta_{i-1})^2 + \frac{1}{2} K [l - \sqrt{H_1 - H_2 \cos(\beta - \Theta_i)}]^2 , \tag{28}$$

$$V = A (\Theta_1 - \Theta_{N-1})^2 , \tag{29}$$

and $$\frac{\partial F_i}{\partial \Theta_j} = \frac{\partial U_i}{\partial \Theta_j} - \frac{\partial V}{\partial \Theta_j} , \tag{30}$$

where: $$\frac{\partial U_i}{\partial \Theta_j} = -2A (\Theta_{i+1} - \Theta_{i-1}) \qquad (j=i-1) ,$$

$$= \frac{K[l - \sqrt{H_1 - H_2 \cos(\beta - \Theta_i)}]\, H_2 \sin(\beta - \Theta_i)}{2\sqrt{H_1 - H_2 \cos(\beta - \Theta_i)}} \qquad (j=i) ,$$

$$= 2A (\Theta_{i+1} - \Theta_{i-1}) \qquad (j=i+1) ,$$

$$= 0 \qquad (\text{otherwise}) ,$$

$$\frac{\partial V}{\partial \Theta_j} = 2A (\Theta_1 - \Theta_{N-1}) \qquad (j=1) ,$$

$$= -2A (\Theta_1 - \Theta_{N-1}) \qquad (j=N-1) ,$$

$$= 0 \qquad (\text{otherwise}) .$$

The starting values of the variables, which are used f r the solution of the equations by Newton's Raphson method, are found by substituting for Λ, in eq. (26), by zero. This yields:

$$b_i^0 = \beta - \cos^{-1}\left[\frac{H_1}{H_2} - \frac{1}{H_2}\left\{f - \sqrt{\frac{2f_i}{K} + [f - \sqrt{H_1 - H_2 \cos(\beta - \Theta_0)}]^2}\right\}^2\right] . \qquad (31)$$

The contact between the cam and follower, in this case, is checked by calculating the moment of the contact force about the follower pivot at the N mechanism positions defined above. The value of this moment at the ith point is obtained from:

$$M_{ci} = \frac{K[f - \sqrt{H_1 - H_2 \cos(\beta - \Theta_i)}]\, H_2 \sin(\beta - \Theta_i)}{2\sqrt{H_1 - H_2 \cos(\beta - \Theta_i)}} + 8A(\Theta_{i-1} - 2\Theta_i + \Theta_{i+1}) \qquad (i=1,2,\ldots,N). \qquad (32)$$

8. EXTENSION OF THE PROPOSED METHOD

Although only planar mechanisms were considered above, the preceding equations can be applied to spatial mechanisms, if the expressions including the kinetic energy of the moving links (expressions (2) and (12)) are modified. Also, if the mechanism is driven by a device which produces variable torque T_v, such as an internal combustion engine, the spring-cam system may be used to counter the difference between the torque T_v and T_i, in order to maintain constant input speed. In this case, the above equations can still be applied but with the term T_m in eqs. (8) and (10) replaced by T_v, and the term $T_m \Phi_1$ in eq. (12) replaced by $\int_0^{\Phi_1} T_v \, d\Phi_1$.

9. EXAMPLE

The proposed method was applied to the four-bar linkage, which is represented diagramatically in Fig. 2. The mass centres of the coupler AB and the output link BC are at points G_2 and G_3 respectively. The space-fixed coordinate system is Oxy. The mechanism data was:

$a_1 = 0.09$ m	$a_2 = 0.24$ m	
$a_3 = 0.16$ m	$a_4 = 0.30$ m	
$p_2 = 0.12$ m	$m_2 = 1.2$ kg	$I_2 = 0.006$ kgm^2
$p_3 = 0.05$ m	$m_3 = 3.0$ kg	$I_3 = 0.024$ kgm^2
$\omega = 30$ rad/sec	$T_3 = -30\,\dot{\Phi}_3/\lvert\dot{\Phi}_3\rvert$ Nm.	

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The fluctuation of the input torque of this mechanism was eliminated by a cam operating an oscillating follower. The contact between the cam and follower was maintained by a helical spring. The particulars of the follower and spring (Fig. 1) were:

ℓ_1 = 0.06 m $\qquad$ ℓ_2 = 0.12 m $\qquad$ β = 90°

θ_0 = 27° $\qquad$ I = 0.0005 kgm^2 $\qquad$ K=20000 N/m.

The displacement diagram obtained by the proposed method is presented in Fig. 3.

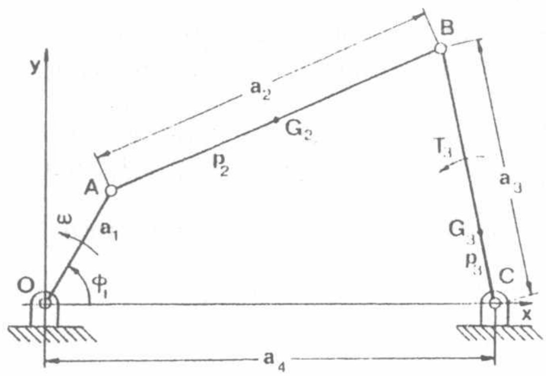

Fig.2: Example mechanism.

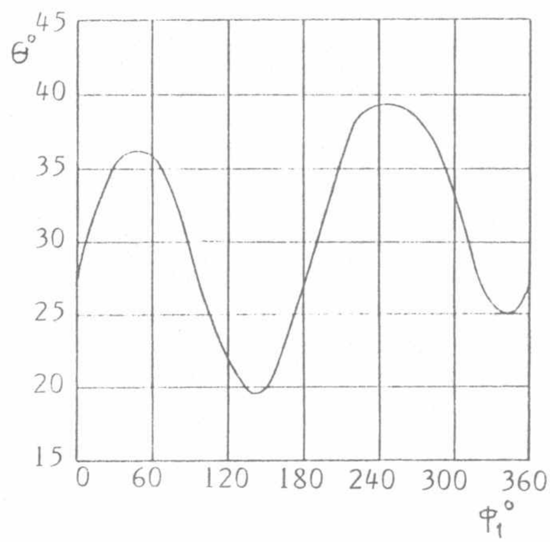

Fig.3: Displacement diagram.

10. CONCLUSIONS

1. The fluctuation of the input torque of a planar mechanism, with both output loads and inertia effects considered, can be completely eliminated by the use of a cam fixed to the input shaft.
2. By the application of the principle of energy conservation, the mechanism input torque, as well as the cam input torque, can be determined without calculating the reactions between the moving parts, since they are regarded as internal forces.
3. The method proposed for the solution of the cam design equation is straightforward and can be easily programmed for the use of digital computer.
4. The proposed method may be applied in the design stage, as well as for smoothing out the input torque of excisting mechanisms since it simply requires the addition of a spring-cam system without affecting the mechanism links or mobility.

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