



A PARAMETRIC STUDY OF THE SIX-BAR QUICK  
RETURN MECHANISM

M.G.SHEBL\*, Y.K.YOUNS\*\*

ABSTRACT

This paper deals with a computer aided design analysis of the six-bar quick return mechanism. The different dimensionless geometrical parameters and their effects on the kinematical characteristics of the mechanism are theoretically investigated. Compact design charts for velocity, acceleration and bearing reactions are presented and which provide the designer with optimal selections of link dimensions.

\*,\*\* Assistant Prof., Mechanical Design Dept., Faculty of Engineering and Technology, Helwan University, Cairo, Egypt.

## KEYWORDS

Quick return mechanism, computer aided design, kinematic, bearing reactions.

## INTRODUCTION

The six-bar quick return mechanism is a venerable one, having been used for many years in many applications. The most notable application being the shaper found in every mechanical shop. In most available text-books [1] and [2], the kinematical analysis of similar linked mechanisms is presented using the simple well known graphical techniques. These methods consider specific positions of the mechanism, leaving other positions for repeated laborious desk work.

In fact, modern developments in digital computers during the last few decades have changed basically these analytical schemes. The payoff comes by realizing that the complete analysis for all positions of the mechanisms can be obtained and stored in the hardware for further easy and quick use. In addition, plotted results for selected dimensional parameters provide overall insights for optimal design processes. The analysis of such mechanisms was studied by authors such as Elaraby [3]. In this paper a complete kinematical analysis for this mechanism is carried out. Input-output relationships are presented in dimensionless forms. In addition, the bearing reactions are analyzed.

## KINEMATICAL ANALYSIS

Fig.1 shows an outline sketch for the six-bar quick return mechanism. The dimensions of the different links and also each of angular and linear displacement of these links are clearly shown.

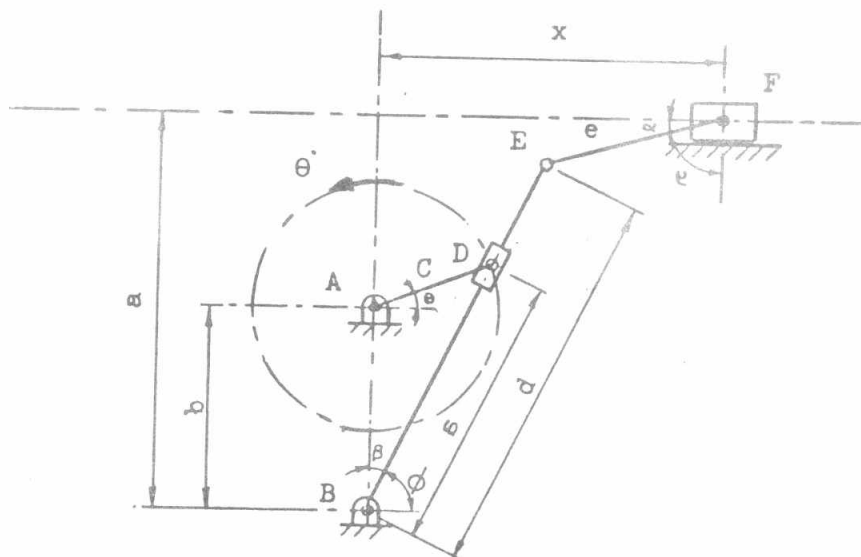


Fig.1 Outline Sketch for the Six-Bar Quick Return Mechanism.

All geometrical relations which express the constrained state of the mechanism may be written as follows:



$$S \cos \phi = c \cos \theta \quad (1)$$

$$\tan \phi = (b + c \sin \theta) / c \cos \theta \quad (2)$$

$$s^2 = b^2 + c^2 + 2bc \sin \theta \quad (3)$$

$$e \sin \mu = [e^2 - (a - d \sin \phi)^2]^{1/2} \quad (4)$$

$$x = d \cos \phi + e \sin \mu \quad (5)$$

In order to make the expressions (1) to (5) in a more compact form, the following dimensionless parameters are introduced,

$$\left. \begin{aligned} \lambda &= b/c \\ \rho &= s/c \\ k &= a/c \\ \xi &= d/a \\ \nu &= e/a \\ \sigma &= x/a \end{aligned} \right\} \quad (6)$$

Consequently the dimensionless form of eqns.(1) to (5) will be

$$\rho \cos \phi = \cos \theta \quad (7)$$

$$\tan \phi = \lambda \sec \theta + \tan \theta \quad (8)$$

$$\rho^2 = \lambda^2 + 2\lambda \sin \theta + 1 \quad (9)$$

$$\eta = [\nu^2 - (1 - \xi \sin \phi)^2]^{1/2} \quad (10)$$

$$\sigma = \xi \cos \phi + \eta \quad (11)$$

By differentiating with respect to time, we can obtain the kinematic relationships between the different links. The velocity of point F will be

$$\dot{\sigma} = \dot{x}/a = (d\sigma/d\phi) \dot{\phi} \quad (12)$$

We can get the expression  $d\sigma/d\phi$  from eqn.(11) such as

$$d\sigma/d\phi = -\xi [\sin \phi + (\xi \sin 2\phi - 2 \cos \phi) / (2\eta)] = f(\phi) \quad (13)$$

Now  $\dot{\phi}$  can be obtained by differentiation of the expression  $\phi$  which is founded in eqn.(8)

$$\dot{\phi} = \dot{\theta} (\lambda \sin \theta + 1) / (\lambda^2 + 2\lambda \sin \theta + 1) \quad (14)$$

Thus

$$\dot{\phi}/\dot{\theta} = (\lambda \sin \theta + 1) / \rho^2 = f(\theta) \quad (15)$$

Therefore, the output velocity represented at point F of the reciprocating slider in dimensionless form is

$$\dot{x}/a\dot{\theta} = f(\phi) f(\theta) \quad (16)$$

A similar procedure may be used to get the acceleration  $\ddot{\sigma}$  represented at the point F of the slider

$$\begin{aligned} \ddot{\sigma} &= \ddot{x}/a = d(\dot{x}/a)/dt \\ &= d[(1/a)(dx/d\phi)(d\phi/dt)]/dt \\ &= (1/a)[(dx/d\phi)(d^2\phi/dt^2) + (d\phi/dt)d(dx/d\phi)/dt] \end{aligned}$$



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$$\ddot{\sigma} = \ddot{\phi} \frac{d(x/a)}{d\phi} + \dot{\phi}^2 \frac{d^2(x/a)}{d\phi^2} \tag{17}$$

The expression  $\ddot{\phi}$  can be obtained by another differentiation for eqn. (15) with respect to the time

$$\ddot{\phi}/\dot{\theta}^2 = [\lambda(\lambda^2 - 1) \cos \theta] / \dot{\theta}^2 = g(\theta) \tag{18}$$

while  $d^2(x/a)/d\phi^2$  can be generated from eqn.(13) such as

$$\frac{d^2(x/a)}{d\phi^2} = -\xi \cos \phi - \frac{(\xi/\eta)(\xi \cos 2\phi + \sin \phi)}{-(\xi^2/4\eta^2)(2 \cos \phi - \xi \sin 2\phi)^2} \tag{19}$$

By substituting with eqns.(13),(14),(18) and (19) in eqn.(17) we obtain the dimensionless output acceleration of the output link

$$\begin{aligned} \ddot{x}/a\dot{\theta}^2 = & -\xi [\sin \phi + (\xi \sin 2\phi - 2 \cos \phi)/2\eta] g(\theta) \\ & - [\xi \cos \phi + \xi(\xi \cos 2\phi + \sin \phi)/\eta \\ & + \xi^2(2 \cos \phi - \xi \sin 2\phi)^2/4\eta^2] f^2(\theta) \end{aligned} \tag{20}$$

FORCE ANALYSIS

In order to deduce the bearing reactions of the system, the law of conservation of energy is applied assuming the joints to be frictionless. Figure 2 shows the system in which the force P is in opposite direction to the velocity  $x'$  of the slider 6 through the active stroke. Consequently the resisting force P will create the reactions  $F_{32}$  and  $F_{12}$  at the joints A and  $O_2$  of the crank respectively. Also, these reactions will cause a resisting torque equal and opposite to the driving torque T.

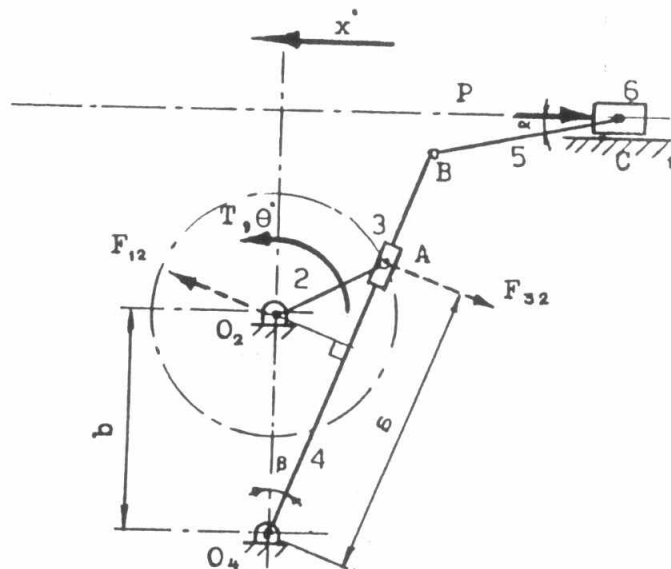


Fig.2 Outline of the forces acting on the mechanism.

According to the principle of the virtual work, we have

$$T \dot{\theta} = P \dot{x} \quad (21)$$

where  $T$  is the instantaneous torque driving the crank 2, eqn.(21) can be reduced to the form

$$T = P \dot{x} / \dot{\theta} \quad (22)$$

Referring to Fig.2, eqn.(22) is expressed by

$$F_{12} (S - b \cos \beta) = P \dot{x} / \dot{\theta} \quad (23)$$

By dividing both sides of eqn.(23) by  $a\dot{\theta}$  we get

$$F_{12} / P = k (\dot{x} / a\dot{\theta}) / (D - \lambda \cos \beta) \quad (24)$$

where  $k = a/c$  is given in eqn.(6), and  $\beta = 90^\circ - \phi$ .

The bearing reactions at the pivots  $O_2$  and  $O_4$ , beside the reaction at the slide way will be determined by considering the equilibrium of the whole system. Figure 3 shows the forces and reactions acting on the system if it is separated from frame 1 at the pairs located at  $O_2$ ,  $O_4$  and the sliding pair of slider 6.

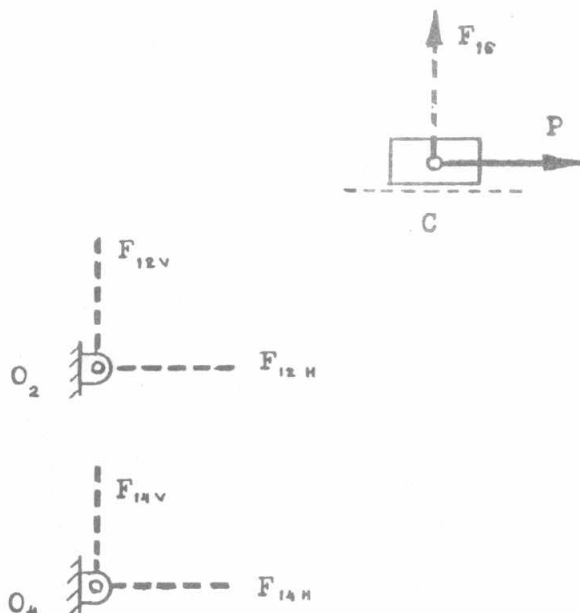


Fig.3 Forces and reactions acting on the mechanism if it is free at the pairs located at  $O_2$ ,  $O_4$  and  $c$ .

The bearing reactions  $F_{12} / P$  acting at the pivot  $O_2$  can be resolved into two components, one is horizontal and the other is vertical as

$$\left. \begin{aligned} F_{12H} &= F_{12} \cos \beta \\ F_{12V} &= F_{12} \sin \beta \end{aligned} \right\} \quad (25)$$

By considering the equilibrium of the horizontal forces, we can obtain the horizontal component of the pivot reaction at  $O_2$ .



$$F_{14H} = -(F_{12H} + P) \quad (26)$$

By dividing eqn.(26) by P we have

$$F_{14H}/P = -(F_{12H}/P + 1) \quad (27)$$

To determine the vertical component  $F_{14V}$ , we need first to find the vertical reaction at the slide way 1. Upon the geometrical conditions which will be mentioned later, the angle  $\alpha$  is always under the line of stroke and it is less than  $90^\circ$ . Then referring to the force polygon of the forces acting on the slider block 6, the slide way reaction  $F_w$  will be always acting upward as long as the resisting force P acting at the given direction, consequently

$$F_{16} = P \tan \alpha \quad (28)$$

The angle  $\alpha$  can be evaluated by considering the kinematics of the link 5, thus

$$\left. \begin{aligned} \sin \alpha &= (a - d \sin \phi)/e \\ \text{or } \sin \alpha &= (1 - \xi \sin \phi)/\nu \end{aligned} \right\} \quad (29)$$

Then by considering the equilibrium of the vertical forces, we obtain the vertical component of the pivot reaction at  $O_4$ ,

$$F_{14V} = -(F_{12V} + F_{16}) \quad (30)$$

By dividing eqn.(30) by P we have

$$F_{14V}/P = -(F_{12V}/P + \tan \alpha) \quad (31)$$

#### COMPUTER RESULTS AND DISCUSSION

A computer program is prepared with FORTRAN-IV language in order to apply the previous kinematical and force analysis. The analytical method is applied to mechanisms with different dimensionless parameters. The ratio  $\xi$  ( $d/a$ ) was taken with two values are 0.90 and 0.98. Different values for the ratio  $\nu$  ( $e/a$ ) 0.30 and 0.40 are applied at each value of  $\xi$ . Also different values for  $\lambda$  ( $b/c$ ) 1.5, 2.5, 5.0 and 10.0 are applied at each value of  $\nu$ .

In order to maintain the viability of results, the dimension of the mechanism must be within the geometry given by Fig.2. That means when the crank angle equals  $90^\circ$ , the following conditions must be satisfied:

$$b + c < d \quad (32)$$

$$\text{and } d < a \quad (33)$$

By dividing eqns.(32) and (33) by a and noting that  $b/a = \lambda/k$ , we obtain

$$\lambda/k + 1/k < \xi \quad (34)$$

$$\text{and } \xi < 1. \quad (35)$$



The values of the ratio  $k$  can be determined from eqn.(34) like that

$$k > (1 + \lambda) / \xi \quad (36)$$

The results shown in the following graphs are for mechanisms with values of  $\xi = 0.98$ ,  $\nu = 0.30$  and  $\lambda = 1.5, 2.5, 5.0$  and  $10.0$ . Since the results of the force analysis are directly affected by the value of  $k$ , the chosen values of  $k$  refer to the condition (36) will be 3, 4, 7 and 12 at the values of  $\lambda, 1.5, 2.5, 5.0$  and  $10$ .

The kinematical parameters  $x/a$ ,  $\dot{x}/a\dot{\theta}$  and  $\ddot{x}/a\dot{\theta}^2$  of the ram versus the angular displacement  $\theta$  of the crank are shown in the Figs. 4, 5 and 6 respectively. The given figures show the effect of the variations in the dimensionless geometrical parameters on the different kinematical parameters of the ram motion.

Hence, the results of the force analysis at the upper and lower pivots are shown in Figs. 7 and 8 respectively, while Fig. 9 shows also the values of the forces acting on the slide way of the ram. The results of these figures are given at different values of the crank displacement  $\theta$  corresponding to the active stroke only.

#### CONCLUSIONS

The kinematical and force analysis of the six-bar quick return mechanism are presented for different variations in the dimensionless geometrical parameters. The given charts can be considered as important tool in the design process of such mechanisms. The presented work shows the effect of the dimensionless parameter  $b/c$  on the maximum values for each of the kinematical parameters and the reaction values.

#### REFERENCES

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#### NOMENCLATURE

$F_{12}$	Bearing reaction at the upper pivot.
$F_{14}$	Bearing reaction at the lower pivot.
$P$	Resisting load at the ram.
$t$	Time.
$T$	Applied torque at the crank.
$x$	Ram displacement.
$\theta$	Angular displacement of the crank.

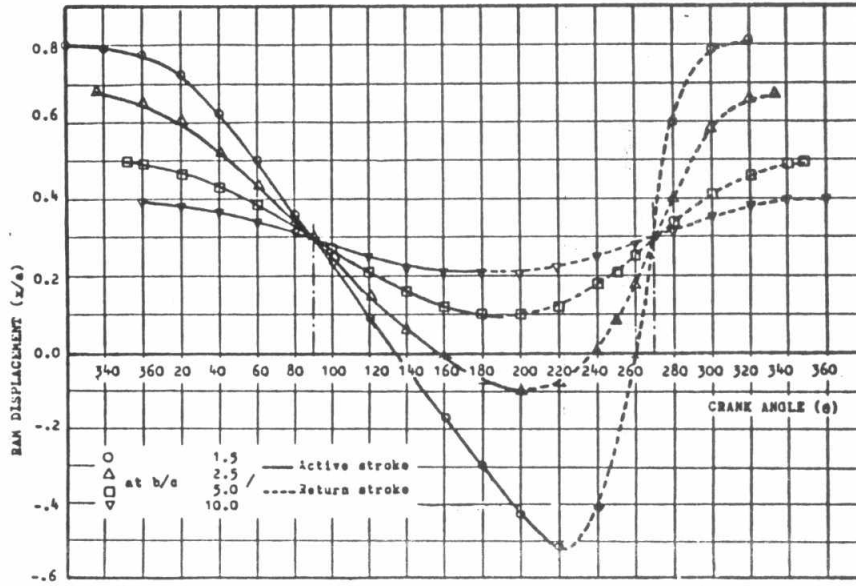


Fig.4 Ram displacement ( $x/a$ )

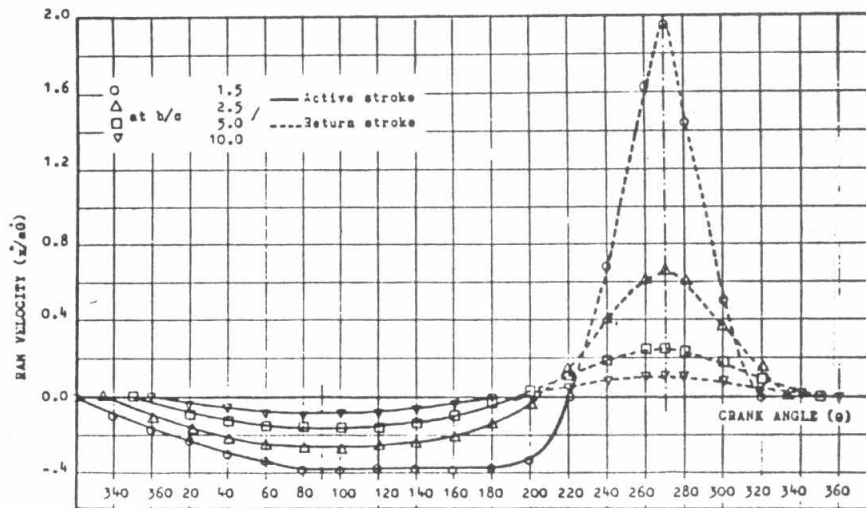


Fig.5 Ram velocity ( $\dot{x}/a\dot{\theta}$ )

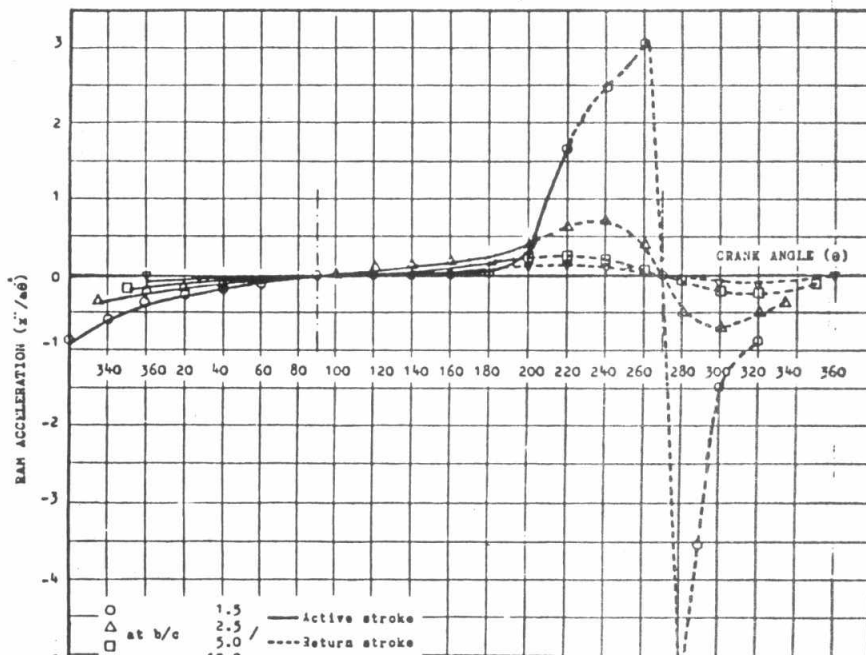


Fig.6 Ram acceleration ( $\ddot{x}/a\dot{\theta}^2$ )



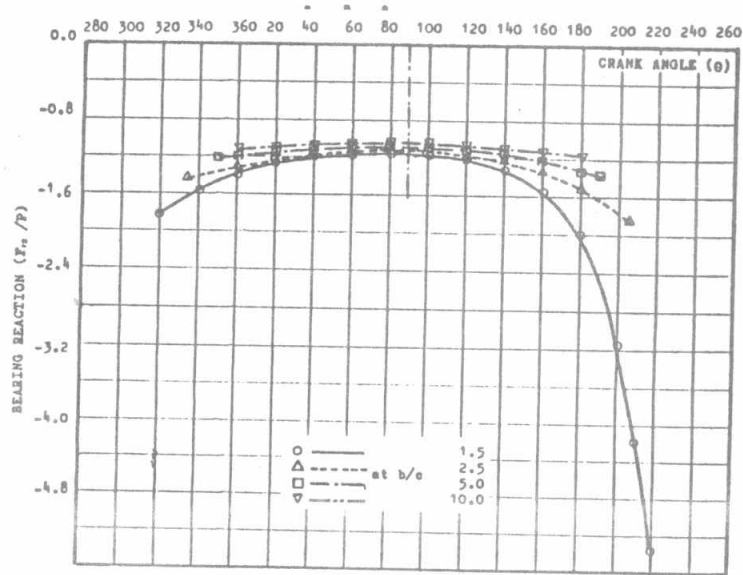


Fig.7 Bearing reaction ( $F_{12}/P$ ) at the upper pivot.

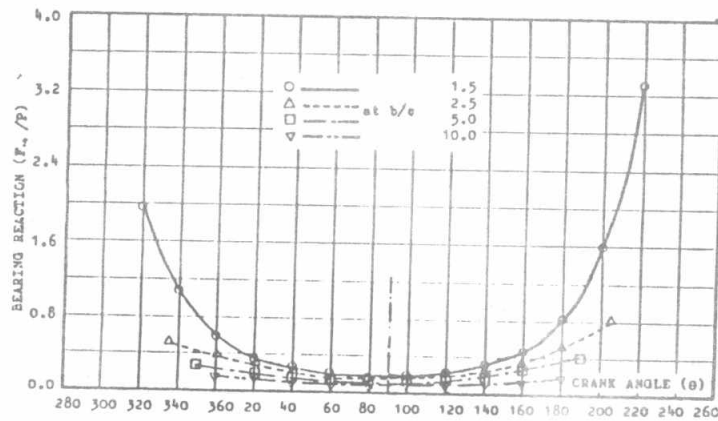


Fig.8 Bearing reaction ( $F_{13}/P$ ) at the lower pivot.

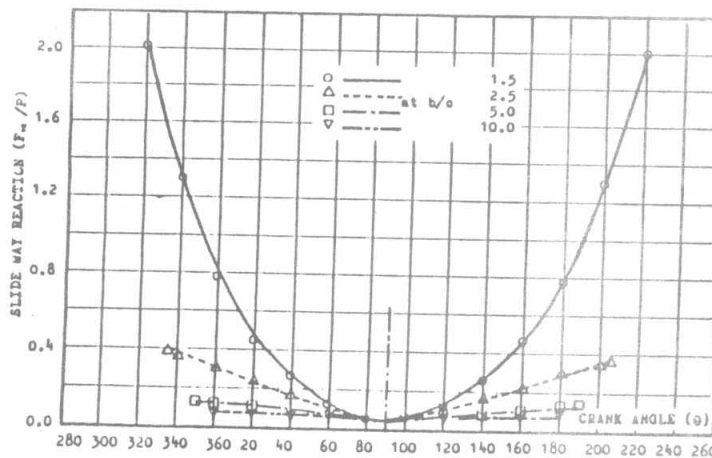


Fig.9 Slide way reaction ( $F_{16}/P$ )

