1 # ON THE APPLICATION OF OSCILLOGYRO IN INERTIAL NAVIGATION

2 # SYSTEMS

By

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4 ## ABSTRACT

The trend of development of gyroscopes used in Inertial Navigation and Guidance systems, led to the adaptation of vibrating gyros to fulfil the basic requirements on such systems specially for military applications in aircraft and guided missiles. The oscillogyro is a new type of vibrating gyroscopes suitable for stabilizing inertial platforms. It is characterized by its simplicity, reliability and low cost of production.

In this work, the application of oscillogyro in inertial navigation systems (INS) is introduced. After a short description of the oscillogyro as a two axes rate or displacement gyro, the mass- unbalance error is analyzed. The propagation of such error into an INS with single-axis vertical indicating platform, is investigated. The results of analysis show quite satisfactory performance and promising future for the application of oscillogyro in a wide range of INS and guidance systems.

1. INTRODUCTION

Inertial navigation and guidance systems are nowadays widely used in military applications for aircraft navigation and missile guidance. Gyroscopic sensors represent the heart of such systems so that higher demands on their performance characteristics impose severe requirements on gyro accuracy.

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Attempts to improve the accuracy of conventional (spinning wheel) gyros have been mainly through the introduction of refined means of suspension, however, this led to the increase of production cost of gyros.

Recently, intensive efforts have been paid towards the adaptation of vibrating gyros to fulfil the basic requirements of inertial navigation (INS) and guidance systems. Oscillogyro belongs to this last category of vibrating gyros.

The oscillogyro was first introduced by WHALLEY [1] in 1967. It is characterized by its simplicity, reliability ruggedness ,and low cost of production. It may be employed both as a two-axis-rate or displacement gyroscope .

The oscillogyro is shown schematically in Fig.1. Its sensitive mass is in the form an oscillobar (1) joined at its mass centre to a gimbal (2) through an elastic hinge (3) which allows the vibration of the oscillobar w.r.t. the gimbal. The gimbal is driven by a motor (5) at a high accurately controlled speed about its spin axis (4). When the casing (6) is turned about an axis perpendicular to the spin axis, the originating dynamic forces cause the oscillobar to vibrate about the elastic hinge axis through an angle θ. The resulting vibration provides a measure of the applied turn

Two pairs of pick-offs are used to measure the proximity of the oscillobar to a reference plane on the instrument.

During the rotation of the oscillobar, one pair of the pick offs will measure the displacement of the gyro about one axis and a quarter of revolution later the other pair will measure the displacement about the perpendicular axis.

The general equation of motion of a perfectly balanced oscillogyro as well as its response to steady rate of turn under different tuning and

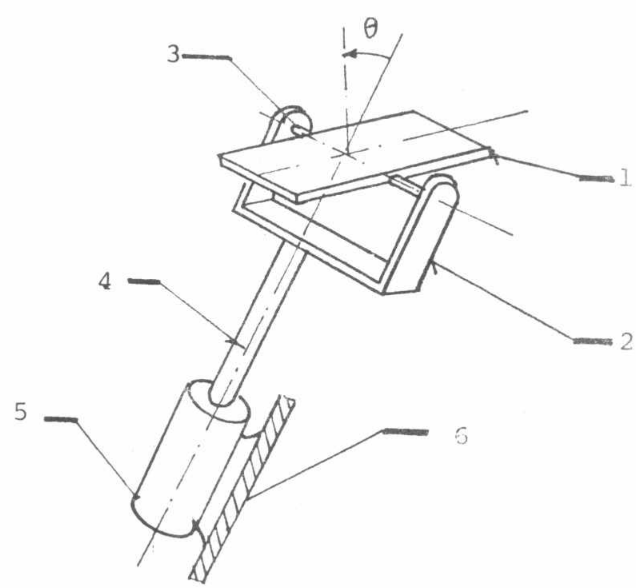

Fig. (1): The Oscillogyro.

damping conditions are presented by Holgate and Maunder [2].

In this work the effect of mass-unbalance on the oscillogyro performance has been investigated. The general equation of motion of an unbalanced oscillogyro are derived. On basis of a linearized equation of motion, formulee expressing oscillogyro error in both displacement and rate configurations are derived. Results concerning a case study are presented.

The propagation of oscillogyro unbalance error in an vertical indicating INS is analyized showing remarkable advantages of application of oscillogyros in the domain of space-stabilized INS.

2. EQUATIONS OF MOTION OF AN OSCILLOGYRO WITH MASS UNBALANCE:

The sensitive mass of gyro having mass-unbalance is shown in Fig. (2-a). The oscillobar is supported at O, by means of the elastic hinge. The center of mass CM of the oscillobar is, however, not coinciding with the center of support O. The elastic hinge allows the deflection of the oscillobar about its axis OY' through an angle θ w.r.t. the gimbal. The gimbal rotates at a speed $\dot{\psi}$ about the spin axis OZ.

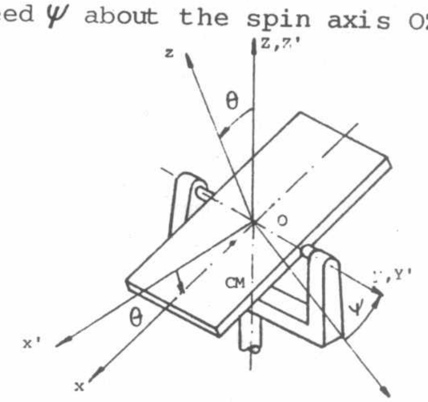

Fig.(2-a) Unbalanced Oscillogyro (O.G.)

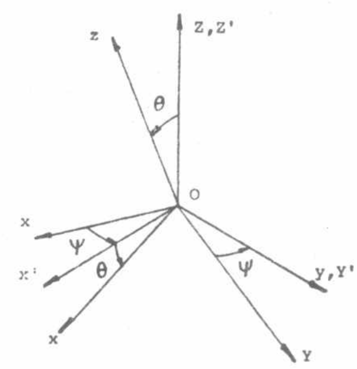

Fig.(2-b) System of axes

Systems of axes:

The systems of reference axes are selected as shown in Fig. (2-b):

* OXYZ is a system of axes attached to the casing of the O.G. It follows the applied turn φ about the OY axis. OY is perpendicular to the spin axis OZ.
* OX'Y'Z' is a system of axes attached to the gimbal. It is derived from OXYZ by a rotation about the spin axis OZ through the spin angle .

The system of axes Oxyz is attached to the oscillobar. Its axes coincide with principal axes of inertia of the oscillobar. It is derived from OX'Y'Z' system by a rotation through the angle θ about the hinge axis Oy=OY'.

The equations of motion of the oscillobar is derived from the angular momentum equation for a system of particles about a point O [3];

$$\frac{d\vec{h}_o}{dt} = \vec{T}_o - \vec{r}_{co} \times m\, \vec{a}_o \qquad (1)$$

where ; $\vec{h}_o$ is the relative angular momentum vector of the oscillobar about point O;

$\vec{T}_o$ is the resultant of all external torques acting on the oscillobar about point O;

m is the total mass of the oscillobar ;

$\vec{r}_{co}$ is the position vector of the oscillobar center of mass w.r.t. point O as origin, $\vec{r}_{co} = (x_c, y_c, z_c)$.

$\vec{a}_o$ is the acceleration vector of point O which is the same as the acceleration of the vehicle to which the gyro is mounted ; $\vec{a}_o = (a_x, a_y, a_z)$.

The response of the oscillobar is in fact described by its equation of motion about the hinge axis Oy i.e., the scalar component of eqn (1) along Oy. This equation reduces to ;

$$\begin{aligned} & B\ddot{\theta} + c\dot{\theta} + k\theta + (C-A)(\dot{\psi}^2 - \dot{\varphi}^2 \sin^2\psi)\sin\theta\cos\theta = \\ & T_2' - B\ddot{\varphi}\cos\psi + [B + (C-A)\cos 2\theta]\dot{\psi}\dot{\varphi}\sin\psi \\ & - m z_c(a_X\cos\psi\cos\theta + a_Y\sin\psi\cos\theta - a_Z\sin\theta) \\ & + m x_c(a_X\cos\psi\sin\theta + a_Y\sin\psi\sin\theta + a_Z\cos\theta); \qquad (2) \end{aligned}$$

Where A,B and C are the mass moments of inertia of the oscillobar about the axes O_x, O_y and O_z respectively ;

T_2' is the control torque about O_y ;

a_X, a_Y and a_Z are the components of vehicle acceleration in the directions of OX,OY and OZ axes respectively ;

k is the elastic hinge stiffness coefficient ;

c is the coefficient of viscous damping associated with the friction moment about O_y.

x_c, y_c and z_c are the coordinates of the oscillobar mass center w.r.t. the Oxyz axes.

Linearized Equation of Motion:

Equation (2) may be linearized if the order of magnitude of the following parameters are considered:

* the input angular rate $\dot{\varphi}$ is very small compared with the spin rate i.e., $\dot{\varphi}^2 \ll \dot{\psi}^2$;
* θ is small ($\theta \ll 1$ radian), i.e. $\sin\theta \doteq \theta$, $\cos\theta \doteq 1$;
* the inertia of the driving motor is sufficiently large that $\dot{\psi} = \text{const.} = \omega$

Equation (2) then reduces to ;

$$B\ddot{\theta} + c\dot{\theta} + \left[k + (C-A)\omega^2\right]\theta = T_2' - B\ddot{\varphi}\cos\omega t +$$
$$+ (B+C-A)\dot{\varphi}\,\omega\sin\omega t - m z_c\left[a_X\cos\omega t + a_Y\sin\omega t - a_Z\theta\right]$$
$$+ m x_c\left[(a_X\cos\omega t + a_Y\sin\omega t)\theta + a_Z\right] \tag{3}$$

Equation (3) is the linearized equation of motion of the oscillogyro with mass unbalance. It differs from the linearized equation derived by Holgate and Maunder [3] in the last two terms which represent the effect of mass unbalance.

EFFECT OF MASS UNBALANCE ON TUNED OSCILLOGYRO OPERATION:

The investigation of the effect of mass-unbalance on the oscillogyro operation is carried out hereafter under the basic assumption that the mass center is shifted along the longitudinal axis of the oscillobar , i.e., $z_c = y_c = 0$ while $x_c \neq 0$. This assumption is justified by the requirement that the mass of the oscillobar is concentrated near its longitudinal axis Ox. This requirement is imposed in order to attain the nominal performance of the oscillogyro [4] . If, moreover zero angular input ($\varphi(t)=0$) and zero torque input ($T_2^{\prime}$ - 0) are considered , the lineaized eqn (3) - under the effect of mass-unbalance alone-reduces to :

$$\ddot{\theta} + 2\zeta p\,\dot{\theta} + \left[\frac{k}{B} + \left(\frac{C-A}{B}\right)\omega^2 - \frac{m x_c}{B}\left(a_X \cos\omega t + a_Y \sin\omega t\right)\right]\theta = \frac{m\,x_c}{B}\,a_Z \tag{4}$$

Where $\zeta = \frac{c}{2Bp}$, $p^2 = \left(\frac{k}{B} + \frac{C-A}{B}\,\omega^2\right)$

The order of magnitude of the individual terms of the third coefficient on the L.H.S. of eqn (4) have been considered on case studies. It was found that :

$$(k/B) = O(10^4)\ s^{-2} \quad , \quad \left(\frac{C-A}{B}\right)\omega^2 = O\ (10^6)\ s^{-2}$$

$$\left(\frac{m\,x_c}{B}\right) a_X = O(10^{-2})s^{-2} \quad , \quad \left(\frac{m\,x_c}{B}\right)a_Y = O(10^{-2})s^{-2}$$

Therefore the term $\frac{m\,x_c}{B}$ $(a_X \cos\omega t + a_Y \sin\omega t)$ may be neglected compared with $\frac{k}{B}$ and $\frac{C-A}{B}\,\omega^2$ and Eqn (4) reduces to

$$\ddot{\theta} + 2\zeta p\,\dot{\theta} + p^2\,\theta = \frac{m\,x_c}{B}\,a_Z \tag{5}$$

3.1. Effect of a Constant Vehicle Acceleration :

If the component of vehicle acceleration a_Z is constant i.e, $a_Z = a_Z^{\prime}$ = const.; the solution of Eqn (5) for zero initial conditions is given by :

$$\theta(t) = \theta_c - \theta_i\, e^{-\zeta pt} \cos(p_d t - \alpha) \tag{6}$$

Where $\theta_c = \frac{m\,x_c}{B\,p^2}\,a_Z^{\prime}$, $\theta_i = \frac{m\,x_c}{B\,p^2}\,\frac{1}{\sqrt{1-\zeta^2}}\,a_Z^{\prime}$

$$\alpha = \tan^{-1}\left(\zeta/\sqrt{1-\zeta^2}\right) \quad ; \quad p_d = p\sqrt{1-\zeta^2}$$

The solution cosists of two components : a damped oscillotory component of initial amplitude θ_i, superimposed on a constant component θ_c. The constant component is eliminated by the type of the pick-off used. Therefore the only

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component measured by the pick off is :

$$\theta_{ms} = -\theta_i\, e^{-\zeta pt} \cos(p_d t - \alpha) \qquad (7)$$

Moreover, in view of the fact that the damping factor is enough small ($\zeta = 10^{-4} \div 10^{-3}$) and that the oscillogyro is tuned ($p = \omega$) [4] , the measured component θ_{ms} reduces to

$$\theta_{ms} = \frac{m\, x_c}{B\omega^2}\, a_Z'\, e^{-\zeta\omega t} \cos\omega t \qquad (7')$$

3.2. Gyro Error In Displacement Measuring Configuration :

The comparison between the measured component of the response expressed by Eqn (7'), and the response of the oscillogyro to an applied turn, φ (t), which hich is given by Eqn (8) [3] ;

$$\theta = -\varphi(t) \cos\omega t \quad ; \qquad (8)$$

it may be seen that the oscillogyro indicates an error in angular displacement, given by :

$$\varphi_{er} = \frac{m\, x_c}{B\omega^2}\, a_Z'\, e^{-\zeta\omega t} \qquad (9)$$

3.3. Gyro Error In Rate Measuring Configuration :

The response of a tuned oscillogyro to an input rate of turn $\Omega = d\varphi/dt$ is given by [3] ;

$$\theta = -\left(\frac{B+C-A}{B}\right) \frac{\Omega}{2\zeta\omega} \cos\omega t \qquad (10)$$

Comparing the above response, Eqn (10) , to the measured response of the oscillogyro due to mass unbalance, which is given by Eqn (7') , it may be seen that the oscillogyro indicates an erroneous rate of turn, given by :

$$\Omega_{er} = \frac{2\zeta\, m\, x_c}{(B + C - A)}\, a_Z'\, e^{-\zeta\omega t} \qquad (11)$$

From eqns (9) and (11), it may be readily seen that the error of a tuned oscillogyro, due to a constant vehicle acceleration, in both the displacement and the rate measuring configuration decrease exponentially with time. Practically the error vanishes after a transient period. This transient period may be measured by the classical decay time T_d given by :

$$T_d = \frac{1}{\xi\omega} \ln 20 \qquad (12)$$

3.4. Case Studies:

The effect of mass-unbalance on oscillogyro performance has been investigated for three practical oscillogyros denoted O.G.1., O.G.2. and O.G.3. The construction parameters of O.G.1, O.G.2 and O.G.3 are given in Table 1. The damping factor ξ for these gyros is : $\xi = 0.1 \times 10^{-3}$ (In Displacement Measuring Configuration);
$\xi = 1.3 \times 10^{-3}$ (In Rate Measuring Configuration)

	$A \times 10^6$	$B \times 10^6$	$C \times 10^6$	ω	m	k	$x_c \times 10^5$
Gyro	$kg\,m^2$	$kg\,m^2$	$kg\,m^2$	$rad\,s^{-1}$	kg	N.m/rad	m
O.G.1	6.32	69.76	65.76	438.57	0.0331	0.446	0.30
O.G.2	7.64	38.87	38.98	296.5	0.0331	0.486	0.30
O.G.3	10.10	38.69	32.81	220.9	0.0331	0.780	0.30

Table 1: Construction Parameters of Oscillogyro Under Investigations

A typical response curve of an oscillogyro to constant vehicle acceleration is shown in Fig. 3. On this figure , the envelope of the oscillatory response is represented. The response consists of a damped oscillatory component of initial amplitude $\theta_i = 0.232 \times 10^{-6}$ rad , superimposed on a steady state constant component.

The predicted values of the oscillogyros errors due to mass-unbalance in both the displacement and the rate measuring configurations - see eqns 9 and 11 - decrease exponentially with time. The maximum values of the predicted errors for O.G.1, O.G.2 and O.G.3 are given by Tables 2 and 3.

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a'_Z	(φ_{er})max. rad. x 10^9		
	O.G.1	O.G.2	O.G.3
a'_Z = g	7.713	300.80	516.6
a'_Z =4g	30.85	1203.4	2066.0

Table.2: Maximum values of error in displacement measuring configuration.

a'_Z	(Ω_{er})max (rad.s^{-1}) x10^9		
	O.G.1.	O.G.2	O.G.3
a_Z=g	4.482	125.39	170.35
a'_Z=4g	17.93	501.5	681.4

Table 3: Maximum values of errors in rate measuring configuration.

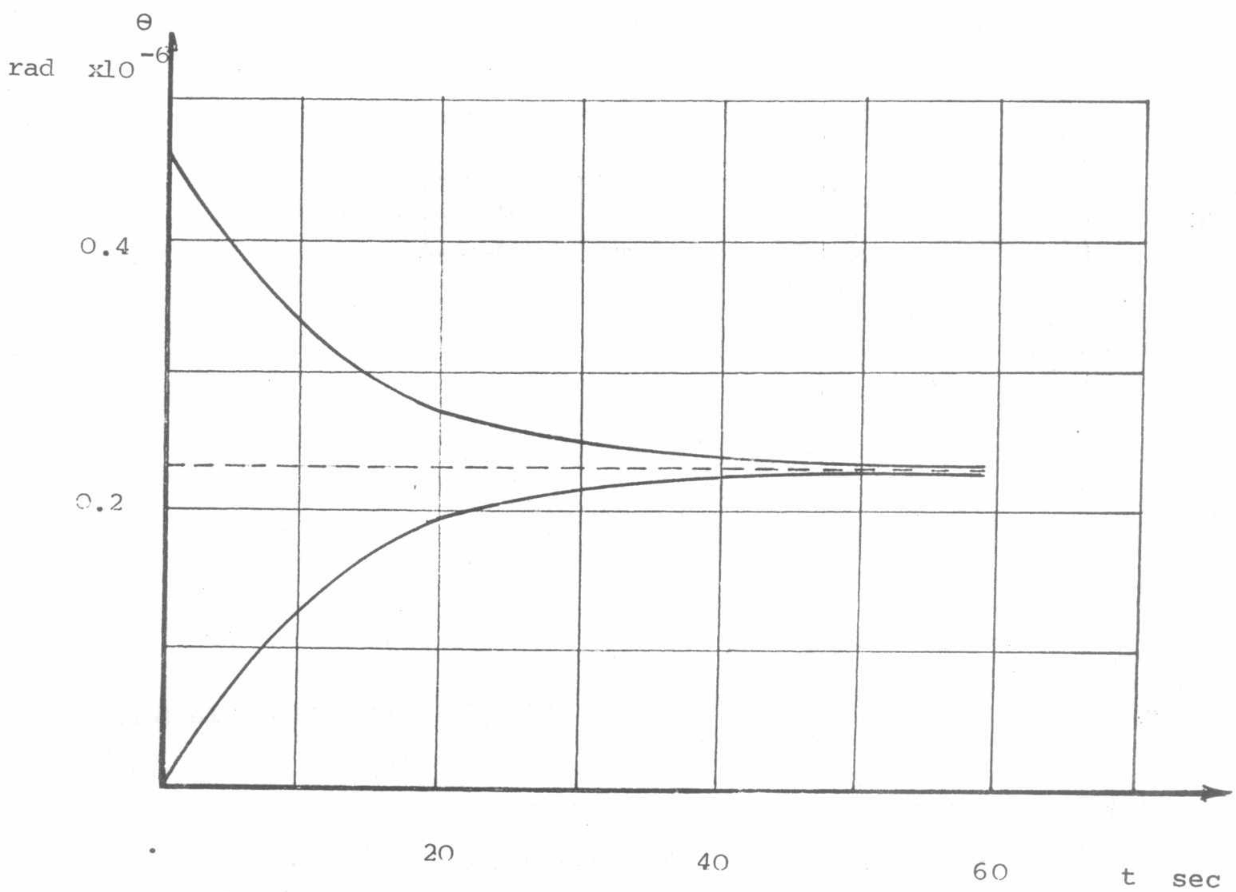

Fig. (3) Envelope of the oscillogyro response (O.G.1, a_Z=2g , $\xi = 0.1 \times 10^{-3}$)

4. PROPAGATION OF OSCILLOGYRO ERROR IN INS

INS may be looked as a black box carried by the vehicle. The only inputs are accelerations including vehicle acceleration and gravity acceleration. Outputs of INS are position and velocity of the vehicle with respect to the earth. Gyroscopes represent main elements in space-stabilized INS as they provide stable platforms, as well as in strop-down INS they determine the vehicle angular position with respect to the inertial space. Thus in performing error analysis of INS, we must consider that gyroscope error can propagate into the system producing displacement error.

For a case study, let us consider an INS single - axis vertical indicating platform using oscillogyro. With gyro drift rate (δ_g) as shown in Fig. (4).

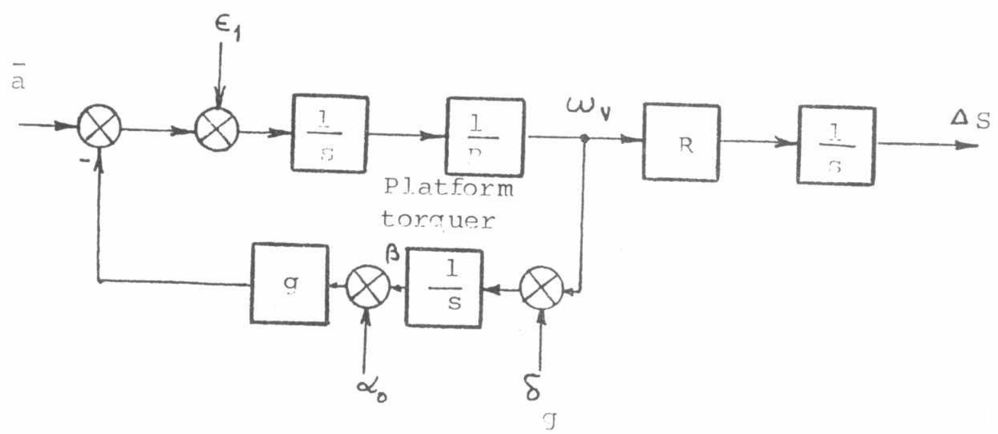

Fig. (4): Block diagram of single-axis vertical indicating platform

Where:

ϵ_1 ... accelerometer error

α_o ... initial platform allignement error

g ... acceleration of gravity

β ... platform deflection

V_H ... vehicle velocity

ω_v ... vehicle angular velocity

ΔS ... position error of INS

R ... Earth radius

Here , the output of accelerometer is integrated, then devided by (R) to provide a signal $\omega_{vehicle} = \frac{V_H}{R}$. Such signal is employed to torque the gyro - stabilized platform such that the gravity component of acceleration input to the accelerometer is maintained at zero [5].

Such a loop represents single - axis Schuller loop , while the oscillogyro drift rate affects the platform torquer as a source of error expressed by:

$$\frac{\Delta s}{\delta g} = \frac{R}{s(1 + \frac{R}{g} s^2)}$$

Noting φ_{er} ... gyro drift angle $\varphi_{er}(s) = \frac{\delta g}{s}$ then;

$$\frac{\Delta s}{\varphi_{er}} = \frac{g}{s^2 + \Omega^2} \quad \text{where } \Omega = \sqrt{\frac{g}{R}} = 1.2508 \times 10^{-3}\ s^{-1}$$

From previous analysis of unbalance error of oscillogyro, φ_{er} is periodic with frequency $\omega \gg \Omega$ (table 1); Thus the INS platform servo torquer attenuates the oscillogyro drift error by an attenuation factor given by:

$$A = \left| \frac{\Delta s}{\varphi_{er}} \right| \doteq \frac{g}{\omega^2}$$

this attenuation factor varies between 5.1×10^{-5} and 2.01×10^{-4} for the oscillogyro under investigation.

Such attenuation will secure a displacement error (Δs) with a negligible amplitude and zero mean value. This conclusion represents a quite important result when compared with conventional gyro applications in which (φ_{er}) increases linearly with time in the presence of mass-unbalance. This gives promising future for the employement of oscillogyro in the field of INS and guidance of military aircrafts and missiles.

CONCLUSIONS

The analysis performed in this work shows that the unbalanced oscillogyro is sensitive only to the component of vehicle acceleration along the spin axis (a_Z). The oscillogyro error associated with a constant vehicle acceleration (a_Z) decreases exponentially with time. The order of magnitude of error due to vehicle acceleration are within the range of permissible errors imposed by the demands of gyro accuracy in inertial navigation.

The study of propagation of oscillogyro mass-unbalance error in an inertial navigation system shows vary satisfactory results.

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