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CREEP BUCKLING OF CYLINDERICAL THIN PANELS

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ABSTRACT

A method of analysis is presented for circular cylinderical panels under external loads and steady creep deformations. The analysis are valid for small and moderately large displacements. Initial imperfections are included. The case of simply supported cylinderical panels under external axial compressive load is investigated in detail. The buckling and post buckling behavior of cylinderical panels is described first, the analysis is based on Von Karman Donnell general equations for cylinderical shells.

Using the Norton's secondary creep law, the deformation rate and creep buckling were described in order to predict the critical creep time of cylinderical panels under steady creep conditions.

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1. INTRODUCTION

In the design process, the engineer seeks the simplest possible mathematical model which will adequately predict the main features of structural response. Although the complete creep behavior up to the point of structure may be analysed at least in principle by combination of suitable constitutive laws and modern computing techniques. Such an approach is likely to be too costly and time consuming to have a direct effect on the majority of design decisions. There is therefore, a need for a simple procedure which may be used to assess the rupture life of any structure. A simple procedure is explored here to analyse circular cylinderical shells under external loads and steady deformation. The initial state beings at small deformation region in which loads are constant across the area of the shell. The analysis based on Von Karman Donnell general equations of equilibrium for cylinderical shells using the Norton's secondary creep law. The advantages and limitations of this procedure presented here for analysing the shells under steady creep deformations is explored by combinations of the results due to its application with results due to several approaches by other authors.

2. CREEP LAW

The creep behavior of the material is taken to be steady creep as described by the generalized Norton's Law 1.

$$\acute{e}^{c}_{ij} = 3B\sigma_e^{m-1} S_{ij} /2 \qquad \ldots 1.$$

where $\acute{e}^{c}_{ij}$ is the creep strain tensor.

The constants B and m are temprature-dependent matrial parameters.

S_{ij} stress deviator tensor.

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk} \qquad \ldots 2.$$

σ_e effective stress.

$$\sigma_e^2 = \frac{3}{2} S_{ij} S_{ij} \qquad \ldots 3.$$

δ_{ij} is the Kronecker's delta and (') denotes differentiations with respect to time.

3. ASSUMPTIONS

In order tosimplify the subsequent analysis, it is assumed that:

— The effective stress is taken to be constant through the thickness.

— The analysis is made for a material which elastic strains rates are neglected thus $\dot{\varepsilon}^c_{ij}$ given by (1) is the total strain rate.

These assumptions were also assumed in (1&2).

Equation 1 yields the following stress-strain rate relations.

$$\dot{\varepsilon}^c_x = B\,\sigma_e^{m-1}\,(\sigma_x - \sigma_{y/2})$$

$$\dot{\varepsilon}^c_y = B\,\sigma_e^{m-1}\,(\sigma_y - \sigma_{x/2})$$

$$\dot{\gamma}^c_{xy} = 2\,\dot{\varepsilon}^c_{xy} = 3B\,\sigma_e^{m-1}\,\tau_{xy}$$

and

$$\sigma_e^2 = \sigma_x^2 + \sigma_y^2 - \sigma_x\,\sigma_y + 3\,\tau_{xy}^2 \qquad \ldots 4.$$

Equation 4 can be inverted in the following form:

$$\sigma_x = \frac{\sigma_e^{1-m}}{3B}\,(\dot{\varepsilon}^c_x + \dot{\varepsilon}^c_{y/2})$$

$$\sigma_y = \frac{\sigma_e^{1-m}}{3B}\,(\dot{\varepsilon}^c_y + \dot{\varepsilon}^c_{x/2})$$

$$\tau_{xy} = \frac{2\,\sigma_e^{1-m}}{3B}\,\dot{\varepsilon}^c_{xy} \qquad \ldots 5.$$

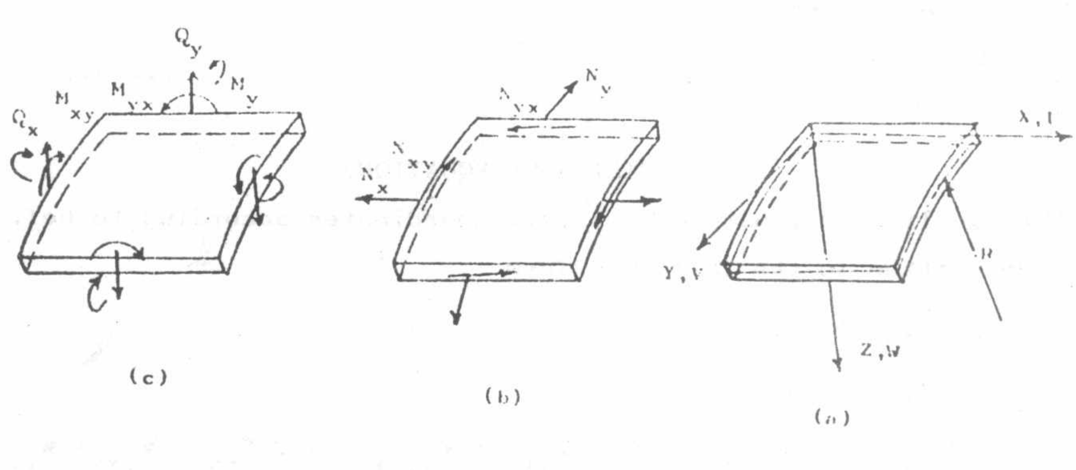

Fig. 1 Cylinderical panel-displacement and forces

4. MEMBRANE FORCES AND BENDING MOMENTS

The relation between forces and bending moments and stress are:

$$N_x = \int_{-h/2}^{h/2} \sigma_x \, .dz \qquad M_y = \int_{-h/2}^{h/2} \sigma_x \, .z \, .dz$$

$$N_y = \int_{-h/2}^{h/2} \sigma_y \, .dz \qquad M_y = \int_{-h/2}^{h/2} \sigma_y \, .z \, .dz$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xy} . dz \qquad M_{xy} = -\int_{-h/2}^{h/2} \tau_{xy} . z \, .dz \qquad \ldots 6.$$

The strain at distance z from the middle surfaces and the curvature due to bending moment can be written in the form:

$$\varepsilon_x = \varepsilon_{ox} - Z \, W_{,xx} \qquad K_x = - W_{,xx}$$

$$\varepsilon_y = \varepsilon_{oy} - Z \, W_{,yy} \qquad K_y = - W_{,yy}$$

$$\varepsilon_{xy} = \varepsilon_{oxy} - Z \, W_{,xy} \qquad K_{xy} = - W_{,xy} \qquad \ldots 7.$$

where symbol o refers to the middle surface

and $(\;)_{,x} = \partial(\;)/\partial x$ and so on .

Making one of the differentiation with respect to time for the equation 7 and then substitute into equation (5) . The bending moments in equation 6 can be written in the form of curvature rate as following:

$$M_x = - \xi \, \sigma_e^{1-m} \left(\dot{W}_{,xx} + \frac{1}{2} \dot{W}_{,yy} \right)$$

$$M_y = - \xi \, \sigma_e^{1-m} \left(\dot{W}_{,yy} + \frac{1}{2} \dot{W}_{,xx} \right)$$

$$M_{xy} = \frac{1}{2} \xi \, \sigma_e^{1-m} \; \dot{W}_{,xy} \qquad \ldots 8.$$

$$\text{where } \xi = \frac{h^3}{9B} \qquad \ldots 9.$$

5. EQUILIBRIUM EQUATIONS

The equilibrium equations in x,y,z coordinates according to Ref. 3 can be written in the following form:

$$N_{x,x} + N_{xy,y} = o$$

$$N_{y,y} + N_{xy,x} = o$$

$$M_{x,xx} + M_{y,yy} - 2 M_{xy,xy} + N_x \left(W_{,xx} + W_{i,xx} \right) + 2 N_{xy} \left(W_{,xy} + W_{i,xy} \right)$$

$$+ N_y (W_{,xx} + W_{i,xx} + \frac{1}{R}) = o \quad \dots 10.$$

where symbol i refers to the initial imperefections.

In quasi statical creep process the equilibrium equations are valid, and the definitions of bending moment in terms of the rate of curvature due to creep in equation 8 must satisfy also equation 10 .

6. GOVERNING CREEP EQUATION

For small deflection analysis the third equilibrium equation 10 in Z-direction is independent of the two equilibrium equations in x and y directions. Making use of equation 8 , equation 10 gives:

$$- \xi \sigma_e^{1-m} (\dot{W}_{,xxxx} + 2\dot{W}_{,xxyy} + \dot{W}_{,yyyy}) + N_x (W_{,xx} + W_{i,xx})$$
$$+ N_y (W_{,yy} + W_{i,yy} + \frac{1}{R}) + 2N_{xy} (W_{,xy} + W_{i,xy}) = o \quad \dots 11.$$

Equation 11 is the governing equation of creep analysis for small deflection region which means constant applied internal stresses.

7. COMPARISON WITH VIGGO 2

In case of axially compressed plate subjected to load N_{ox} equation 11 becomes

$$2\dot{W}_{,xxxx} + 4\dot{W}_{,xxyy} + 2\dot{W}_{,yyyy} - \frac{18 \sigma_e^{m-1}}{H^3} N_{ox} B (W_{,xx} + W_{i,xx}) = 0 \quad \dots 12$$

In ref. 2 the governing equation for creep analysis of a plate analysed by the perturbation procedure is

$$\frac{m+3}{2m} W_{,1111} + 4W_{,1122} + 2W_{,2222} - \frac{18 \sigma_e^{m-1}}{H^3} N_{o11} B W_{,11} = 0 \quad \dots 13$$

The last two equations are nearly the same that indicates the advantages and limitation of the analysis presented here.

8. COMPARISON WITH HOFF 4
(small deflection regions)

In this section we introduce the advantages and limitation of the procedure presented here for analysing the shells under steady creep. for this reason we make a comparison between using equation 11 for solving the creep buckling of circular cylinderical shell and the method presented by Hoff.4 for the same case.

Assuming the total deflection due to creep in the form

$W = f_1 (t) \sin \pi x/\lambda + f_o^{`} (t)$ ... a.

where λ is the half wave length of the axi - symmetric deformation.

At the beginning of the loading at time t=0 the axisymmetric deformation of the middle surface of the wall from exact cylinder shape is given by

$W_o = f_o \cos \pi x/\lambda$ ... b.

Substitution into the governing equation 11 by the above displacement function gives

$- \left\{ \sigma_e^{1-m} \quad \frac{\pi^4}{\lambda^4} \dot{f} \cos \pi x/\lambda - N_x \; f \; \frac{\pi^2}{\lambda^2} \cos \pi x/\lambda = 0 \right.$ ... c.

Where f is the total displacement at given instant

$f = f_1 + f_o$

N_x is the applied middle surface load that has been assumed to be constant during small deflection regions, it is given by

$N_x = - p_x H$ ... d.

Where p_x is the applied compressive stress at the boundary edges, H is the wall thickness of the cylinder.

The effective stress σ_e can be written according to equation 4 in the form

$\sigma_e^2 = p_x^2$ ... e.

Substitution with d and e into c and making use of Galerkin procedure one obtains for creep exponent m = 3 , the creep deformation rate in the form of

$\dot{f} / f = \frac{9}{2} \frac{\lambda^2}{H^2} B \; p^3$ ... f.

CRITICAL WAVE LENGTH:

The critical wave length (the most dangerous one) is the one for which the creep deformatin increases most rapidly. As we will see later the creep deformation increases most rapidly for the modes coincide with the critical modes of elastic deformation and buckling one.

For cylinderical shells one can calculate the critical wave length for the elastic buckling behavior by one of the well known methods of elastic buckling of cylinderical shells, it is

$\lambda_{cr} = \frac{\pi}{\sqrt{3}} (R.H)^{1/2}$ ... g.

$= 1.81 \; (R.H)^{1/2}$

The maximum creep rate is given by

In the analysis in Hoff.4 the actual wall of the shells was imagined to be replaced by an equivalent sandwich wall in which two faces, each of a thickness(h), are held apart a distance(d) by a core which is rigid in shear but unable to carry normal stresses. The equation 9 in ref. 4 gives

$$h = \frac{H}{2}$$

$$d = 0.528\ H \qquad \text{... i.}$$

H is thr thickness of the actual shell.

The creep rate in ref. 4 is given by

$$f_{1max} / f_1 = \frac{3}{2} B\ p^3\ \frac{R}{d} = 2.84\ \frac{R}{H}\ Bp^3 \qquad \text{... j.}$$

The critical wave length is given by

$$\lambda_{cr} = \frac{\pi}{\sqrt{2}}\ (R.d)^{1/2} = 1.614\ (R.H)^{1/2} \qquad \text{... K.}$$

Comparing the result in ref. 4 equations j,k and our results equations g,h the difference is only 5.3% where the simple method presented here is more powerful.

9. CONCLUSION

The creep buckling of cylinderical panels subjected to axial compressive load has been examined by the method presented here. The creep life time increases, if the initial imperefections are decreased or if we allow the deformation in the up ward direction only by means of an internal pressure. The creep deformation always follows the deflection shape of the deformation that has the lowest elastic energy levels.

However, the analysis used here turns out to be valid only for small deflection region ,Nevertheless, the majority of the creep critical time occurs during small deflection regions and the method presented here has value in its simplicity and direct application.

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