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A SIMPLE METHOD FOR DETERMINATION OF THE FRACTURE-MECHANICS GEOMETRIC PARAMETER K_I

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ABSTRACT

The method discussed in this paper describes a three parameter method of analysis for accurate detrmination of the stress intensity factor (SIF) K_I using the crack tip stress field information given by photoelasticity. The sole objective is, in fact, to determaine a specific parameter or set of parameters which when used with the experimental results provide, the best match between the experimental and analytical results. A new three parameter method is using information from only one fringe loop is introduced which followes the method developed by Fattah, [1] for determination of mixed mode stress intensity factors (K_I and K_{II}).

The three parameter used in the present method are, $\sigma_{ox} = \alpha K/\sqrt{2\pi a}$, which acts parallel to the direction of the crack extension, the stress intensity factor (K_I), and a parameter β which is a term added to the Westergaard stress function to provide one more degree of freedom. A new simple formula for β determination has been introduced in this analysis to take care of fare field stresses, finite dimension, crack tilt angle and curvature effects.

A method of solution based on measurements r_m, θ_m and r at $\theta = \pi/2$ from only one fringe loop is given.

A systematic experimental study on finite CCT, SEN and DEN plates has been carried out for various value of λ (=2a/W) with fringe data collected from near the crack tip. The results obtained for Dobeckote-505 show good agreement with the analtical results.

INTRODUCTION

Wells and Post [2] and Post [3] were the first to study the stress distribution for a static as well as a dynamic crack. Irwin [4] first showed a method for determining the opening mode stress intensity factor K_I. Irwin's method is only applicable if the shearing mode stress intensity factor K_{II}=0.

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Bradley and Kobayashi [5] have mdified Irwin's approach and use data from two points (r_1,θ) and (r_2,θ) along a line intersecting two different fringe orders. Schroedl and Smith [6] employ a differencing technique identical to that used by Bradley and Kobayashi except that θ is set equal to 90 deg. Etheridge and Dally [7] introduced a third parameter into the analysis by modifying the Westergaard stress function to more closely account for stress field variation near the crack tip. Etheridge, Dally and Kobayashi [8] improved upon the three parameter method, and present a four parameter method for determining the dynamic stress intensity factors. Fattah, Rajaiah and Bose [9] developed a new four parameter method for determining the opening mode stress intensity factor K_I.

STATIC CRACK ANALYSIS

The static stress field for tension loading of the crack following the Westergaad approach is

$$\sigma_y = R_e Z + y I_m Z'$$

$$\sigma_x = R_e Z - y I_m Z'$$

$$\tau_{xy} = - y R_e Z' \tag{1}$$

where the stress function, Z, and its derivative, Z'. are functions of the complex variable $Z= r e^{i\theta}$

Following Irwin [10] and factoring out $K/\sqrt{2\pi z}$ the stress field in the vicinity of the crack tip may be approximated by selecting Westergaad type stress function of the form

$$Z_{(z)}= \frac{K}{\sqrt{2\pi z}} \left[1 + \beta (z/a)\right] \tag{2}$$

For the crack loaded in the opening mode, the stress field in the neighbourhood of the crack tip is given [9], as

$$\sigma_x = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(1-\sin\frac{\theta}{2} \sin\frac{3\theta}{2}\right)$$

$$+ \cos\frac{\theta}{2} \left(1+\sin^2\frac{\theta}{2}\right) \beta (r/a) + \alpha\sqrt{r/a} \tag{3}$$

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(1+\sin\frac{\theta}{2} \sin\frac{3\theta}{2}\right)$$

$$+ \cos\frac{\theta}{2}\ (1-\sin^2\frac{\theta}{2})\ \beta\,(r/a) \qquad (4)$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}}\ \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} - \beta(r/a)\ \cos\frac{\theta}{2} \qquad (5)$$

where K is the mode-I stress intensity factor and r and θ are polar co-ordinates centered at the crack tip.

From these stresses, the maximum in-plane, shear stress may be comuted from

$$\tau_{max} = \left[\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2 \right]^{1/2} \qquad (6)$$

and is found to take the form

$$\tau_{max} = \frac{K_I}{2\sqrt{2\pi r_m}} \left[\sin^2\theta_m\ (1-2\beta\ (\frac{r_m}{a})\ \cos\theta_m + \beta^2\ (\frac{r_m}{a})^2 \right.$$

$$- 2\alpha\sqrt{r_m/a}\ \ \sin\theta_m\ (\ \sin\frac{3\theta_m}{2} - \beta\ (\frac{r_m}{a})\ \sin\frac{\theta_m}{2}$$

$$\left. + \alpha^2 (\frac{r_m}{a}) \right]^{1/2} \qquad (7)$$

The description of a fringe loop observed in photoelastic tests is given by Eqn. (5) where the parameters K, a, α and β control the characteristic shape of the fringe loop. The stress optic law states that

$$\tau_{max} = \frac{N\ f\sigma}{2h} \qquad (8)$$

where N is the isochromatic fringe order, $f\sigma$ is the material fringe value, h is the model thickness.

Equating Eqns. (5) and (6) and simplifying, gives the stress int ensity factor K_I as

$$K_I = (\frac{N\ f\sigma}{h})\ \sqrt{2\ r_m} \left[\sin^2\theta_m (1-2\beta\, b_m\ \cos\theta_m + \beta^2\ b_m^2\ ) \right.$$

$$-2\alpha\sqrt{b}\ \ \sin\theta\ (\ \ \ 3\theta_m\ \ \ \ \theta_m\ \ \ ^2\ \ ]^{-1/2}$$

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where $b_m = r_m/a$

COMPUTATION OF THE FITTING PARAMETERS α AND β

In order to obtain α and β values, measurements of r and θ on the typical isochromatic loop along a line perpendicular to the crack tip ($\theta = \pi/2$), and the angle θ_m for which the isochromatic reaches the largest radius r_m, ($\partial\tau/\partial\theta = 0$) can easily executed, using only one isochromatic fringe loop (photoelastic fringe order N) Fig. 1

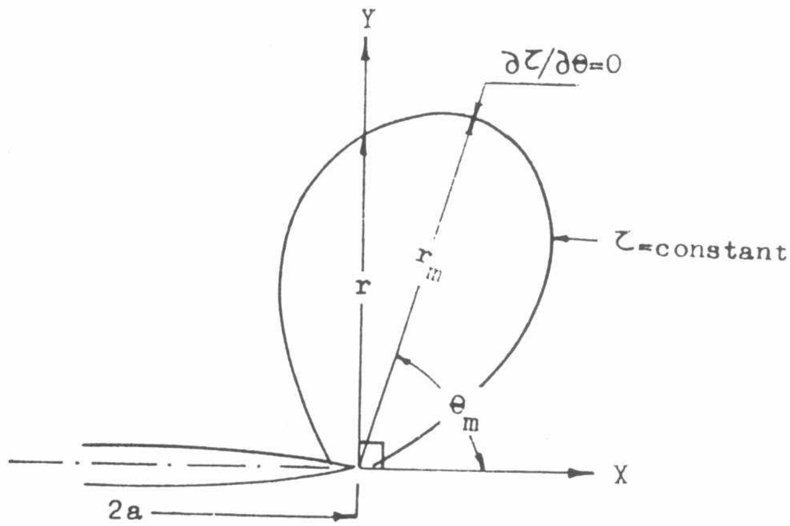

Fig. 1. Isochromatic fringe loop intersect the lines at $\theta = \pi/2$ and where $\partial\tau/\partial\theta = 0$ at r_m and $\theta = \theta_m$

Equation (7) is valid for any point (r, θ) on the fringe, however,it is convenient to employ (r_m, θ_m) and (r, at $\theta = \pi/2$) in the equation since measurements of r_m and r will be more accurate than for points of other values.

Substituting the geometric parameters (r_m, θ_m) and (r, $\theta = \pi/2$) into Eqn. (7), two equations can be written, give the maximum shear stress as

$$\tau^2_{max} = \frac{1}{r_m}\Big[\sin^2\theta_m\,(1 - 2\,\beta\, b_m \cos\theta_m + \beta^2\, b_m^2) - 2\alpha\sqrt{b_m}\,\sin\theta_m\,\big(\sin\frac{3\theta_m}{2} - \beta\, b_m \sin\frac{\theta_m}{2}\big) + \alpha^2\, b_m\Big] \qquad (10)$$

$$\tau^2_{max} = \frac{1}{r}\Big[(1 + \beta^2 b^2) - 2\alpha\sqrt{b/2}\;(1 - \beta\, b) + \alpha^2\, b\Big] \qquad (11)$$

By equqting Eqns. (10) and (11) and simplifying, it is possible to retrieve fitting parameter α as

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$$\alpha = \frac{\left[\sin^2\theta_m \left(1 - 2\beta\, b_m \cos\theta_m + \beta^2 b_m^2\right]\right. - \left[(1+\beta^2 b^2)/b\right]}{\left[2\sin\theta_m \left(\sin\frac{3\theta_m}{2} - \beta\, b_m \sin\frac{\theta_m}{2}\right)/\sqrt{b_m}\right] - \left[\sqrt{2}\,(1-\beta b)/\sqrt{b}\right]} \qquad (12)$$

where $b_m = r_m/a$, r_m the largest radius of the fringe loop at $\theta = \theta_m$, $b = r/a$ and r the radius of the fringe loop at $\theta = \pi/2$.

The fitting parameter β is important in that it must be carefully selected to achieve a good fit in selecting the correct value of α of which dose significantly influence the value of K_I (SIF).

From a careful examination of the isochromatic fringe loops at the crack tip and the behaviour of different mathematical functions of use to the the present case, and after a study of several trial function, a functional representation for β was chosen as

$$\beta = \frac{(r_m/a)\ \lambda\ f_N \left|\cos\theta_m\right|^{(3r_m+\rho)/a}}{\left[\sin\frac{\theta_m}{2}\cos\frac{\theta_m}{2}\ \cos\frac{3\theta_m}{2}\right]} \qquad (13)$$

where $f_N = (N_2/N_1)^{2\left[1-(N_1/N_2)\right]} . (N_3/N_2)^{2\left[1-(N_2/N_3)\right]} \ldots\ldots$

$$\ldots\ (N_{n+1}/N_n)^{2\left[1-(N_n/N_{n+1})\right]}$$

$\left|\cos\theta_m\right|$ = modulus of $\cos\theta_m$, $\lambda = 2a/W$ (2a the crack lengths, W the width of the plate) and ρ, the crack tip radius.

Using the values of α and β and the measured values of r_m and θ_m on a given fringe loop (locus of constant τ_{max}) which is furthest removed from the crack tip ($\partial\tau_{max}/\partial\theta = 0$), the stress intensity factor K_I can be determined from Eqn. (9).

EXPERIMENTAL PROCEDURE

Experimental work was carried out on finite rectangular plates of width W = 150 mm and length L = 300 mm made of Dobeckot-505 [9], having central crack, (CCT), single edge notch (SEN) and double edge notch (DEN), subjected to uniform tensile stress, using transmission technique. Tests were run for different crack length to the width ratio (= 2a/W) with applied stress, being kept constant througthout. The isochromatic fringe pattern were photographed as shown in Figs. 2., 3. and 4. Measurements of the coordinates (r_m, θ_m and r at $\theta = \pi/2$) for only one fringe loop near the crack tip, were recorded after suitable enlargement.

Two types of cracks were introduced for the experimental work, first, is the natural crack of $\rho = 0$ and second is the artificial crack of $\rho = 0.3$ mm.

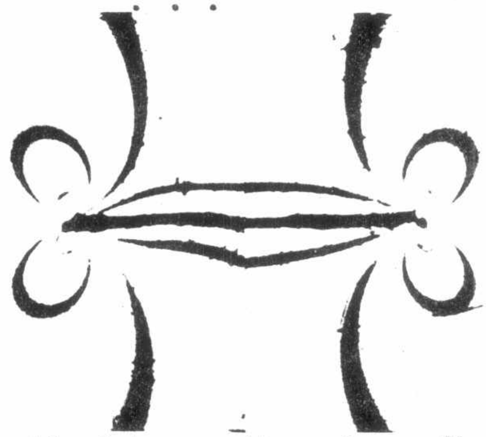

Fig. 2. Isochromatic fringe pattern for a finite plate with a central crack submitted to uniaxial tension.

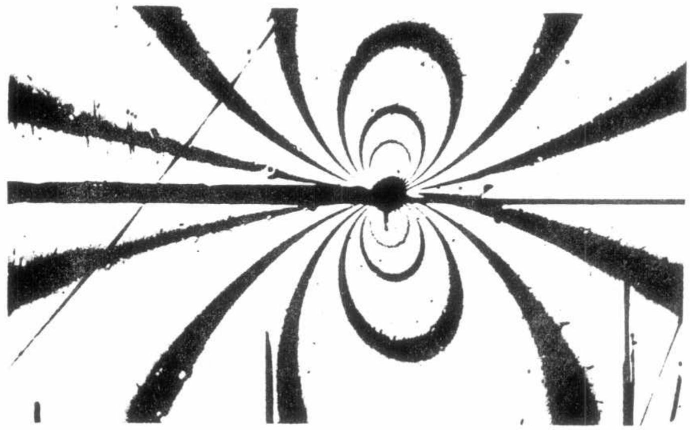

Fig. 3. Isochromatic fringe loop for a finite plate having single edge notch (SEN).

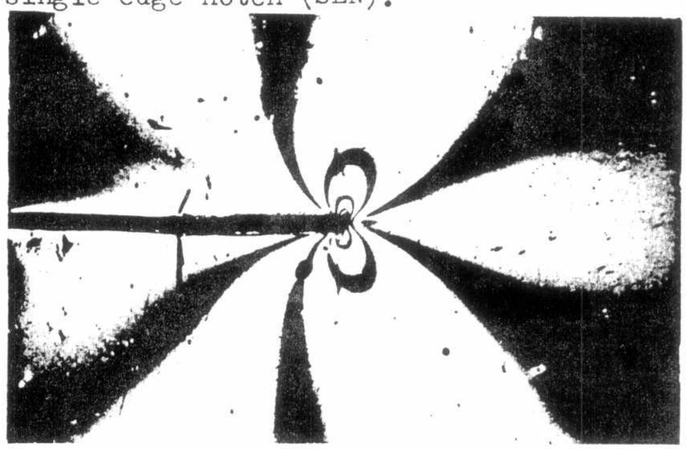

Fig. 4. Isochromatic fringe loop for a finite plate having double edge notch (DEN).

RESULTS AND DISCUSSION

The results from previously described method of analysis using experimental data from only one fringe loop information is compared with the analytical results for finite plates having central crack (CCT), as shown in Fig. 5. Similar trends are also observed for SEN and for DEN specimens as shown in Figs. 6 and 7. with artificial and natural crack problems.

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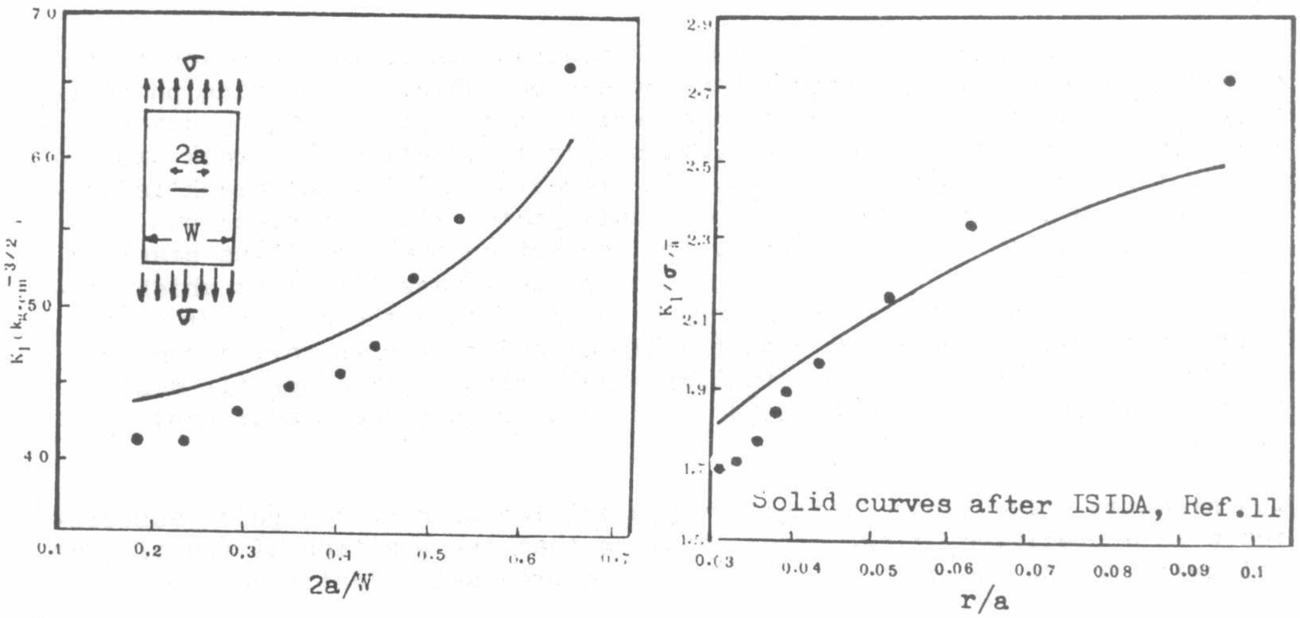

Fig. 5. Comparison of experimental results with analytical results for CCT specimens.

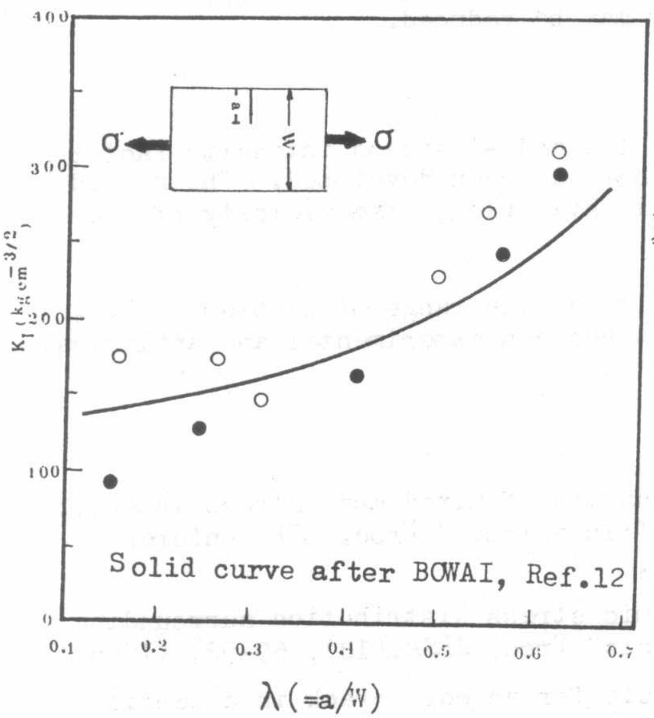

Fig. 6. Comparison of experimental results with analytical results for SEN specimens.

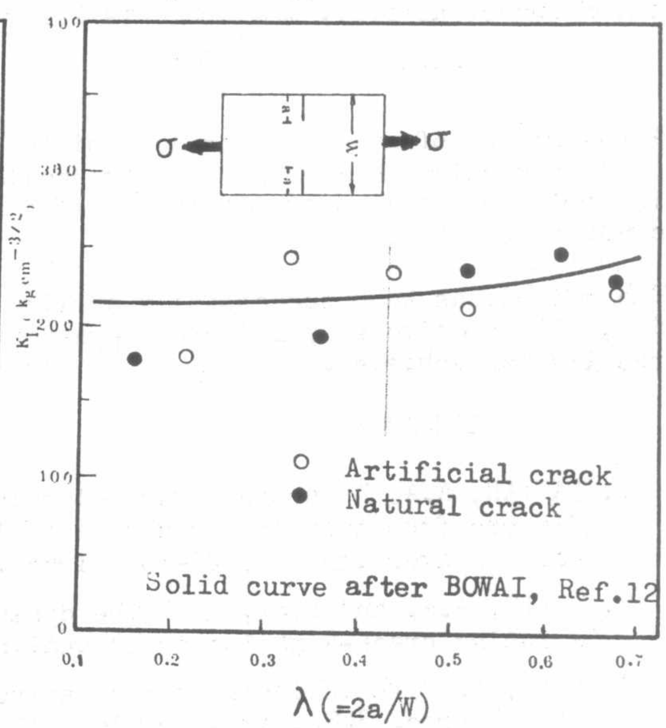

Fig. 7. Comparison of experimental results with analytical results for DEN specimens.

The use of the experimental data, yield the accurate values of stress intensity factor K_I for the corresponding λ values. These SIF values always in good agrement with the corresponding analytical results [11-12], in the range of $\lambda (= 2a/W)$ from 0.1 to 0.6 The data required for verification was obtained from only one isochromatic fringe loop available around the crack tip at specified locations identified by θ_m, r_m and r at θ = 90 degrees.

Since the method gives good results with accurate data, the accuracy of the method is depend upon the accuracy which can be achieved by making photoelastic measurements on selected fringe loop in the vicinity of the crack tip. Measurement error arrives due to difficulties in locating the crack tip and in identifying the fringe location. In preparing models, the crack is usually machined into the plate of photoelastic material. The crack has a finite width in th range of (0.3-1.0 mm) and often the tip of the crack is not sharp (artificial crack). Location of the crack tip point which repesents the origin of the r, θ co-ordinate is usually the main source of measurements error occurring with the higher order fringes very close to the crack tip. Efforts to manufacture models with sharp crack tip designated as natural crack is suggested in order to reduce the measurements error.

Isochromatic fringes have a finite width and location of the point associated with a specified fringe order requires that the position of the minimum intensity across the width of the fringe be precisely identified.

Photographic techniques where large exposure times are used with high contrast film reduce the width of the fringe and increasing the sharpness of the fringes, however, measurements error may be reduced.

CONCLUSIONS

A new method of analysis for determining the mode-I stress intensity factor (K_I) from only one isochromatic fringe loop has been developed. The method is applicable when only one fringe loop is availble in the vicinity of the crack tip.

The method has been successfully applied to a wide range of problems (CCT, SEN, DEN, specimens), and good agreement between experimental and analytical results was achieved.

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