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SQUEAL NOISE IN DISC BRAKES: A THEORETICAL MODEL

M. EL-SHERBINY*

F. SALEM**

ABSTRACT

The paper examines in detail four separate models of disc brake squeal. The mathematics of each model are produced together with the underlying mechanism of squeal. The four models are compared one with another to demonstrate their essential features. Examinations of these models showed that two simplified models can be applied to the disc brake system. These are the cantilever-disc and pin disc models. A double pin-disc model is developed and some stability results are reported

NOMENCLATURE

a	Coefficient defined in equn. 20
C	Equivalent damping
c	Damping coefficient
F	Normal force
I	Inertia
K	Equivalent stiffness
k	Spring stiffness
L	Pin length, half the friction pad length
M	Equivalent mass
m	Mass
N	Normal force or reaction
N	Static preload or reaction
q	Vibration displacement
r	Pivot-tip radius

*Professor and **Associate Professor Dept. of Mech. Design and Production Faculty of Engineering, Cairo University.

u	Linear displacement
W	Linear displacement
X	General displacement
x	Linear displacement
α	Coefficient equn. 20
β	Coefficient equn. 20
θ	Relative angular position
μ	Coefficient of friction
ϕ	Torsional (angular) vibration coordinate

Subscripts

a	Axial direction as shown in fig.7
m	Mass
D	Disc
C	Cantilever
P	Pin
r	Rotational
t	Translational
o	Static
1,2	Pads numbers, coefficients numbers

Superscripts

.	First time derivative
..	Second time derivative

INTRODUCTION

Brakes usually suffer from unwanted vibration which can cause Caliper chatter and high brake noise. These vibrations can occur at any frequency, but squeal is defined as the vibrations within the range 1-5 KHz, and is self excited.

In an early work (1) a simplified model for drum brake squeal is presented. The present paper is therefore devoted to disc brake squeal. In this context Fosberry and Holubecki (2,3) published the first concerted attack on disc brake squeal in 1955. Their work was mainly experimental and rather limited to the frequencies and mode shapes of a squealing disc brake. Although no guaranteed cures were reported, they claimed that the insertion of a shim between the piston and the back plate gives a considerable promise in alleviation of squeal.

The earlist theories of squeal are reviewed in some detail by North in reference (4). These theories are based on a simple elastic rubbing system as shown in Fig.1. In this system the friction is assumed to decrease with increasing the velocity and therefore the resulting oscillation is in the direction of the friction force rather than being transverse to it, as usually found in practice. Stability analysis of this system has been published earlier (5).

In the following four more realistic models are reviewed before the present model is described.

1. DOUBLE CANTILEVER MODEL

Spurr (6) attempted new ideas for squeal models by considering the double cantilever model in fig.2. The mechanism of this model was explained qualitatively rather than quantitatively. Experimental work made by the same author showed that such a system did create stick-slip motion. The model was shown to excite transverse vibration of the disc and was the first to indicate the dependence of squeal on actual value of μ rather on the slope of the μ-velocity relationship.

2. CANTILEVER-DISC MODEL

The first analytical model of disc brake squeal is due to Jarvis (7). In this model a cantilever was clamped in a large horizontally-pivoted block and the tip was loaded against a disc as shown in fig.3 by means of dead weight. The cantilever tip is replaced by a mass m_c whose motion (u) is linear and perpendicular to the neutral axis of the cantilever. The mass m_c is constrained by stiffness k_c and damping c_m such that the the frequency and damping of mass m are the same as for the tip.

The direction of motion of m_c is at an angle θ to the disc surface. The disc motion ω however, is presented by a mass m_D free to move in a direction parallel to the disc axis. The associated stiffness k_{mD} and damping c_{mD} are chosen to give the same natural frequency and damping as the equivalent disc mode.

The two mating parts are constrained to remain in contact by balanced reactions N and friction force $F = \mu N$. The equations of motions are therefore given by

$$m_c \ddot{x} + K_m u + C_m \dot{u} = N(\sin\theta - \mu \cos\theta) \qquad (1)$$

$$m_D \ddot{w} + K_{mD} w + C_{mD} \dot{w} = - N \qquad (2)$$

$$w = u \sin\theta \qquad (3)$$

These equations can be reduced to an equation of the form

$$M\ddot{q} + C\dot{q} + Kq = 0 \qquad (4)$$

where

$$M = 1 - \frac{m_D}{2m} \sin 2\theta (\mu - \tan\theta)$$

$$C = \frac{c_m}{m} - \frac{m_D}{2m} \sin 2\theta (\mu - \tan\theta) \frac{c_{mD}}{m_D}$$

$$K = \frac{k_m}{m} - \frac{m_D}{2m} \sin 2\theta (\mu - \tan\theta) \frac{k_{mD}}{m_D}$$

The condition of instability is therefore given by

$$C/M < 0 \qquad (5)$$

According to reference 6 the value of M was always found to be positive and therefore equation 5 reduces to

$$1 > \frac{m_D}{2m} \sin 2\theta (\mu - \tan\theta) > \frac{c_m}{c_{m_D}} \frac{m_D}{m} \qquad (6)$$

and $\tan^{-1} \mu > \theta > 0$

Equn 6 indicates that instability can occur if θ falls within the range 0 to $\tan^{-1} \mu$ for proper values of (m_D/m) and (c_m/c_{mD}). Prevention of squeal can therefore be achieved by proper selection of μ , (m_D/m) and (c_m/c_{mD}).

The agreement of the experimental stability and theoretical findings was rather poor and therefore other models were proposed.

3. PIN-DISC MODEL

This model (8) consists of a disc and a pin fixed to the end of a shaft and loaded in contact with the disc as shown in figure 4. Consequently the pin was given two degrees of freedom. These are the translational motion x along the axis of the shaft but at an angle θ to the surface of the disc and a torsional freedom ϕ about an axis orthogonal to both the pin and the shaft axis. For the torsional vibration mode one can write the following equations

$$m_D \ddot{y} + c_D \dot{y} + k_D y = - N(t) \qquad (7)$$

$$I_p \ddot{\phi} + c_p \dot{\phi} + k_p \phi = N(t).L \sin(\theta+\phi) - \mu N(t) L \cos(\theta+\phi) \qquad (8)$$

6 $$y = \theta\ L \sin(\theta + \phi) \qquad (9)$$

For small values of ϕ the above equations reduce to

$$\ddot{\phi} + \frac{2B\ m_D\ S}{I + m_D\ AS}\dot{\phi}^2 + \frac{c_p + c_D\ AS}{I + m_D\ AS}\phi + \frac{k_p + k_D\ AS}{I + m_D\ AS}\phi = 0 \qquad (10)$$

where

$A = L \sin(\theta + \phi)$

$B = L \cos(\theta + \phi)$

$S = L(\sin(\theta+\phi) - \mu \cos(\theta+\phi))$

$T = L(\cos(\theta+\phi) - \mu \sin(\theta+\phi))$

Further developments of this model are given in refs. 9 and 10. In ref. 9 a multidegree of freedom is assumed and in ref. 10 a new system arrangement (as shown in fig.5) is proposed. In both cases however the condition of instability remains the same

$$\tan^{-1} \mu > \theta > 0$$

4. EIGHT DEGREES OF FREEDOM MODEL

This model has been described in refs. 9 and 10. It consists of a disc of thickness t sandwiched between layers of friction lining of stiffness k. The disc has a mass (m) and inertia(I) and vibrates with two degrees of freedom y and θ. It is restrained by translational stiffness k_t and rotational stiffness k_r.

The condition of instability accordingly was shown to be

$$K' = \frac{16\ m\ I\ f\ \mu t}{(I - m\ L^2/3)^2} = \theta \qquad (11)$$

where L is half the friction-pad length

f $= \mu N_o$ where N_o is the static preload, between the pad and the disc.

COMPARISONS OF THEORIES

Theories 1 and 2 assumed linear motion of the cantilever tip so that, with the kinematic constraint, a linear single degree of freedom equation is obtained.

Theory 3, however, is based on the non linear motion of the pin tip to create energy input to the system. It was demonstrated, however, that for a 2.5 cm pin the rotation is typical less than 10^{-3} radians for inclination angles θ of greater than 5°. It seems therefore that the non-linearity is very mild and prediction based on self-excited vibrations must take into account all sources of damping in the system.

Examination of the theoretical results with various experimental findings (11) indicate that the last two theories are the most widely accepted in disc-brake squeal.

A PROPOSED MODEL

Considering the disc brake assembly given in fig.6, it is possible to represent the system by the dynamic model shown in fig.7. This model represent further development to the pin-disc model (8), and can accommodate varying friction on both sides of the disc as well as independently varying mass, inertia, inclination, and stiffness for the two friction pads. The system is undamped and assumes that the pins remain in contact with the disc at all times and the displacement amplitudes of the self induced oscillations are small. It also assumes that the displacement mode of each pin parallel to the disc surface is uncoupled from the other modes and is inherently stable.

The system as shown in fig.7 allows three translational motions along the coordinates x_1 , x_2 and x_D in addition to two rotational motions in the coordinates ϕ_1 and ϕ_2. Now the equations of motions can be set-up as follows

$$m_1 \ddot{x}_1 + k_{a_1} x_1 = - F_1 \tag{12}$$

$$I_1 \ddot{\phi}_1 + k_{t_1} \phi_1 = \mu_1 F_1 r_1 \cos\theta_1 + F_1 r_1 \sin\theta_1 \tag{13}$$

$$m_2 \ddot{x}_2 + k_{a_2} x_2 = F_2 \tag{14}$$

$$I_2 \ddot{\phi}_2 + k_{t_2} \phi_2 = - \mu_2 F_2 r_2 \cos\theta_2 - F_2 r_2 \sin\theta_2 \tag{15}$$

$$m_D \ddot{x}_D + k_D x_D = F_1 - F_2 \tag{16}$$

The two necessary constraints to maintain contacts also yield

$$x_D = x_1 - r_1 \phi_1 \sin\theta_1 \tag{17}$$

$$x_D = x_2 - r_2 \phi_2 \sin\theta_2 \tag{18}$$

Eliminating x_1 and x_2 and F_1 and F_2 the above equations reduces to

$$[M_{ij}]\ \{X_i\} + [K_{ij}]\ \{X_i\} = \{0\}\ ,\ i,i = 1,2,3 \tag{19}$$

where all the elements of the M and K matrices are given in table 1.

Assuming a solution of the form $X = X_o\ e^{\lambda t}$ where X_o is the eigen-vector of any particular eigenvalue λ, the eigenvalues are obtained from the equation

$$a_o\ \lambda^6 + a_1\ \lambda^4 + a_2\ \lambda^2 + a_3 = 0 \tag{20}$$

where

$$a_o = \beta_1\ \beta_2\ m_D + \beta_1\ I_2\ m_2 + \beta_2\ I_1\ m_1$$

$$a_1 = \beta_1 \beta_2\ K_D + (\alpha_1\ \beta_2 + \alpha_2 \beta_1) m_D + (\alpha_2\ m_1 + \beta_2\ K_{a_1})\ I_1 + (\alpha_1\ m_2 + \beta_1\ K_{a_2}) I_2 + \beta_1\ m_2\ K_{t_2} + \beta_2\ m_1\ K_{t_1}$$

$$a_2 = \alpha_1\ \alpha_2\ m_D + (\alpha_1\ \beta_2 + \alpha_2 \beta_1) K_D + (\alpha_2\ m_1 + \beta_2\ K_{a_1})\ K_{t_1} + (\alpha_1\ m_2 + \beta_1\ K_{a_2})\ K_{t_2} + \alpha_1\ I_2\ K_{a_2} + \alpha_2\ I_1\ K_{a_1}$$

$$a_3 = \alpha_1\ \alpha_2\ K_D + \alpha_1\ K_{a_2}\ K_{t_2} + \alpha_2\ K_{a_1}\ K_{t_1}$$

$$\beta_1 = I_1 + m_1\ r_1^2\ \sin\theta_1\ \cos\theta_1\ (\mu_1 + \tan\theta_1)$$

$$\beta_2 = I_2 + m_2\ r_2^2\ \sin\theta_2\ \cos\theta_2\ (\mu_2 + \tan\theta_2)$$

$$\alpha_1 = K_{t_1} + r_1^2\ K_{a_1}\ \sin\theta_1\ \cos\theta_1\ (\mu_1 + \tan\theta_1)$$

$$\alpha_2 = K_{t_2} + r_2^2\ K_{a_2}\ \sin\theta_2\ \cos\theta_2\ (\mu_2 + \tan\theta_2)$$

The conditions of stability (12) are:

1) All the coefficients of equation (20) must be positive, i.e. $a_o > o$, $a_1 > o$, $a_2 > o$ and $a_3 > o$ (21)

2) $a_1^2 - 3a_o\ a_2 > o$

$a_1^2\ a_2 - 4a_o\ a_2^2 + 3a_o\ a_1\ a_3 > o$ (22)

$a_1^2\ a_2^2 - 4a_o\ a_2^3 + 18a_o\ a_1\ a_2\ \ldots\ 27\ a^2\ a^2 + 4\ a^3$

i	j	M_{ij}	X_i	K_{ij}	X_i	F_i
1	1	$m_1\ r_1\ \cos\theta_1(\mu_1 + \tan\theta_1)$		$r_1\ K_{b_1}\ \cos\theta_1\ (\mu_1 + \tan\theta_1)$		
	2	$I_1 + m_1\ r_1^2\ \sin\theta_1\ \cos\theta_1(\mu_1+\tan\theta_1)$	$\ddot{x}_D$	$K_{t_1} + r_1^2\ K_{b_1}\ \sin\theta_1\ \cos\theta_1\ (\mu_1 + \tan\theta_1)$	x_D	0
	3	0		0		
2	1	$m_d + m_1 + m_2$		$K_d + K_{b_1} + K_{b_2}$		
	2	$m_1\ r_1\ \sin\theta_1$	$\ddot{\phi}_1$	$r_1\ K_{b_1}\ \sin\theta_1$	ϕ_1	0
	3	$m_2\ r_2\ \sin\theta_2$		$r_2\ K_{b_2}\ \sin\theta_2$		
3	1	$m_2\ r_2\ \cos\theta_2(\mu_2 + \tan\theta_2)$		$r_2\ K_{b_2}\ \cos\theta_2\ (\mu_2 + \tan\theta_2)$		
	2	0	$\ddot{\phi}_2$	0	ϕ_2	0
	3	$I_2 + m_2\ r_2^2\ \sin\theta_2\ \cos\theta_2(\mu_2+\tan\theta_2)$		$K_{t_2} + r_2^2\ K_{b_2}\ \sin\theta_2\ \cos\theta_2(\mu_2 +\tan\theta_2)$		

Table 1 Elements of mass, stiffness , acceleration and displacement matrices.

. . .

Equations 21 and 22 are solved for a specific system having $m_1 = m_2 = 100$ grams, $k_{t_1} = k_{t_2} = 800$ N.m/rad, $m_D = 0.12$ kg, $k_{a_1} = k_{a_2} = 3x10^5$ N/m , $I_1 = I_2 = 9x10^{-6}$ kg m^2. Obviously the domain of stable oscillation depends on the system data as well as on other physical parameters such as the coefficient of friction.

SAMPLE RESULTS

Fig. 8 shows the instability domains as determined for specific values of friction ($\mu_1 = \mu_2 = 0.7$) and pin tilt(θ_2=10,0 and -10^o). It is shown that the instability region narrows as the pin tilts are on opposite sides. As the coefficient of friction is reduced the instability region are also reduced (see for example fig.9). In both cases however the widest unstable θ_1 angle range is observed at values of disc stiffness close to .25x10^8 N/m.

Taking the above mentioned disc stiffness $K_d = .25x10^8$ into consideration one can get the $\theta_1 - \theta_2$ domain of instability for a wide range of friction coefficients. Obviously the higher the friction is the larger the domain of instable oscillation. This is clearly demonstrated in fig.10 for $0.1<\mu<0.6$. A noteworthy however is that setting the pin tilt angles($\theta = \frac{1}{2}\tan^{-1} -\mu$) equal to half the negative friction angle results in a maximum range of θ for the other pin instability.

Due to the wide range of system data one can generate more instability domains in various parameter planes, but these will be of little importance since they will be aplicable only to the present system. In practice one have to solve these equations again for any specific practical system. To this end the model yields results which qualitatively agrees with one's feeling.An experimental set-up is being designed to further verify the validity of this model and this will be reported in future publication.

CONCLUSIONS

Theories and models describing the squeal behaviour of disc brakes are briefly reviewed and some cross examinations are outlined. A double pin-disc model is proposed and the governing equations are produced. Sample results indicating the essential stability features are included. The model in its present form can handle any realistic undamped disc brake system.

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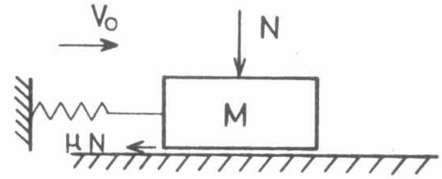

FIG. 1 Simple elastic rubbing system

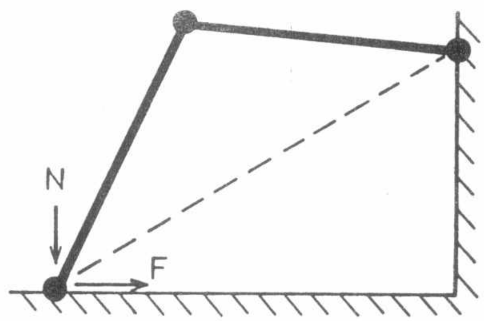

FIG. 2 Double cantilever system

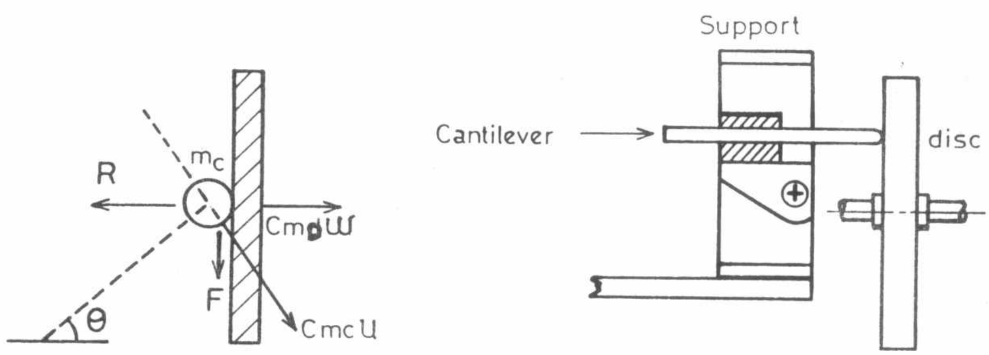

FIG. 3 Disc-Cantilever model

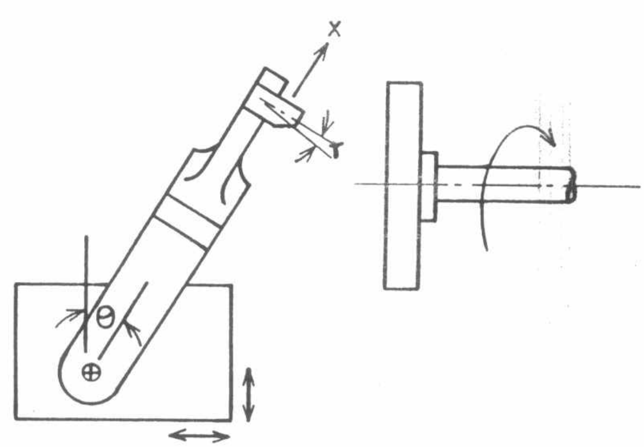

FIG. 4 Pin-disc model (8)

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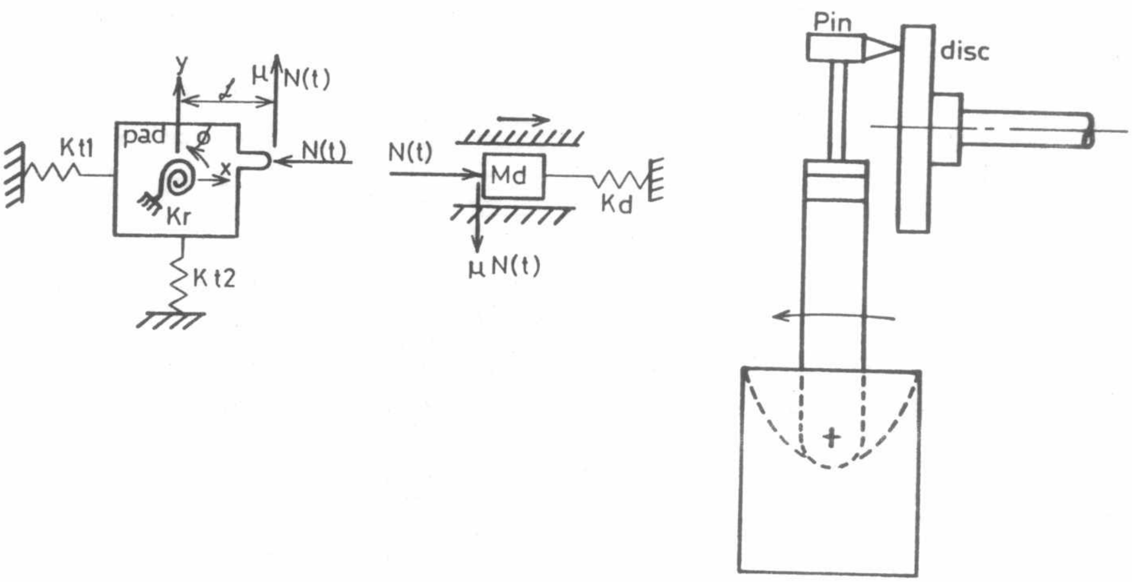

FIG. 5 Four degree of freedom pin-disc model

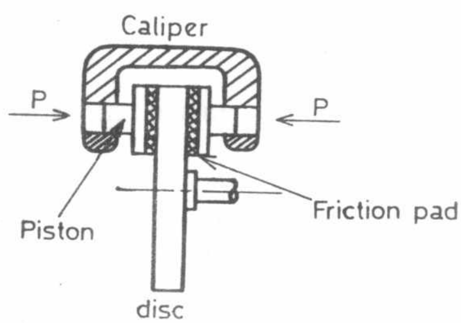

FIG.6 Basic features of a real disc brake assembly

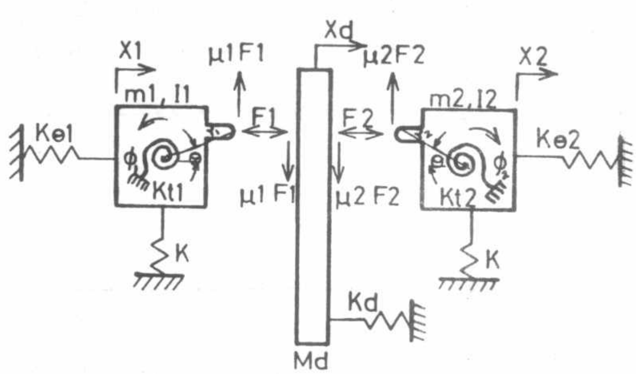

FIG. 7 Dynamic model of disc brake assembly

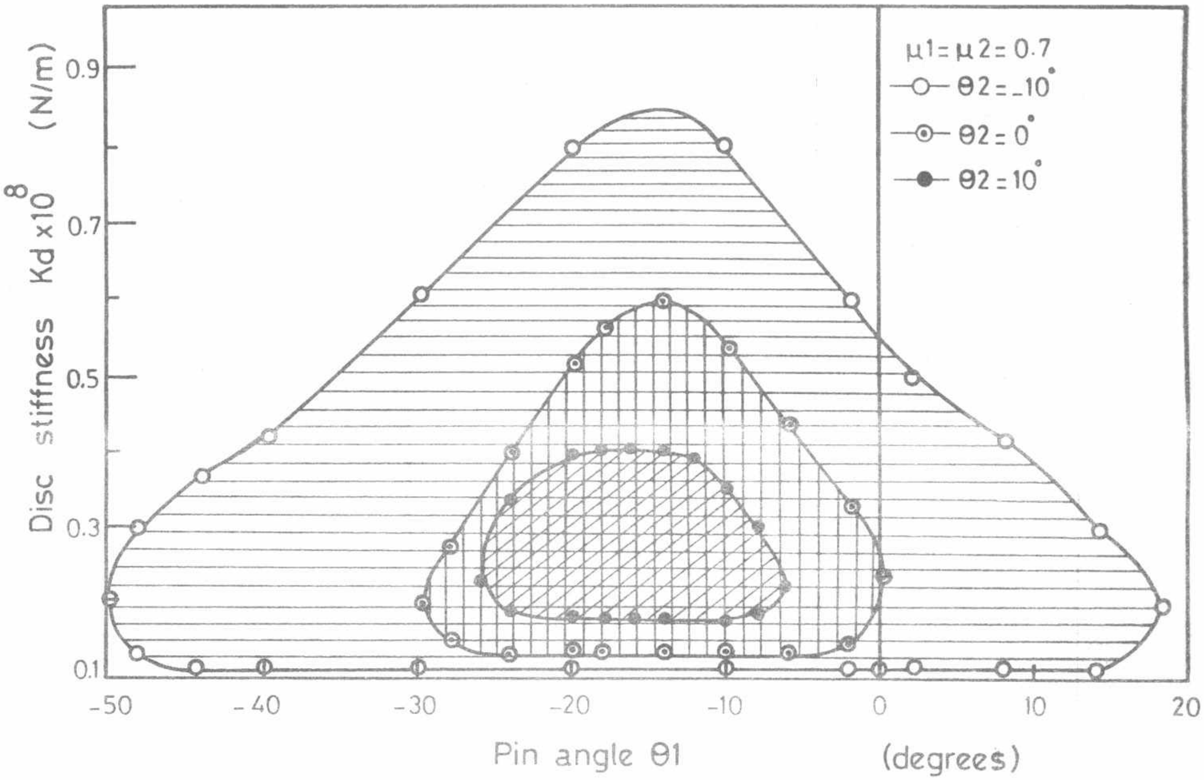

FIG. 8 Instability regions of the disc brake at high friction coefficients

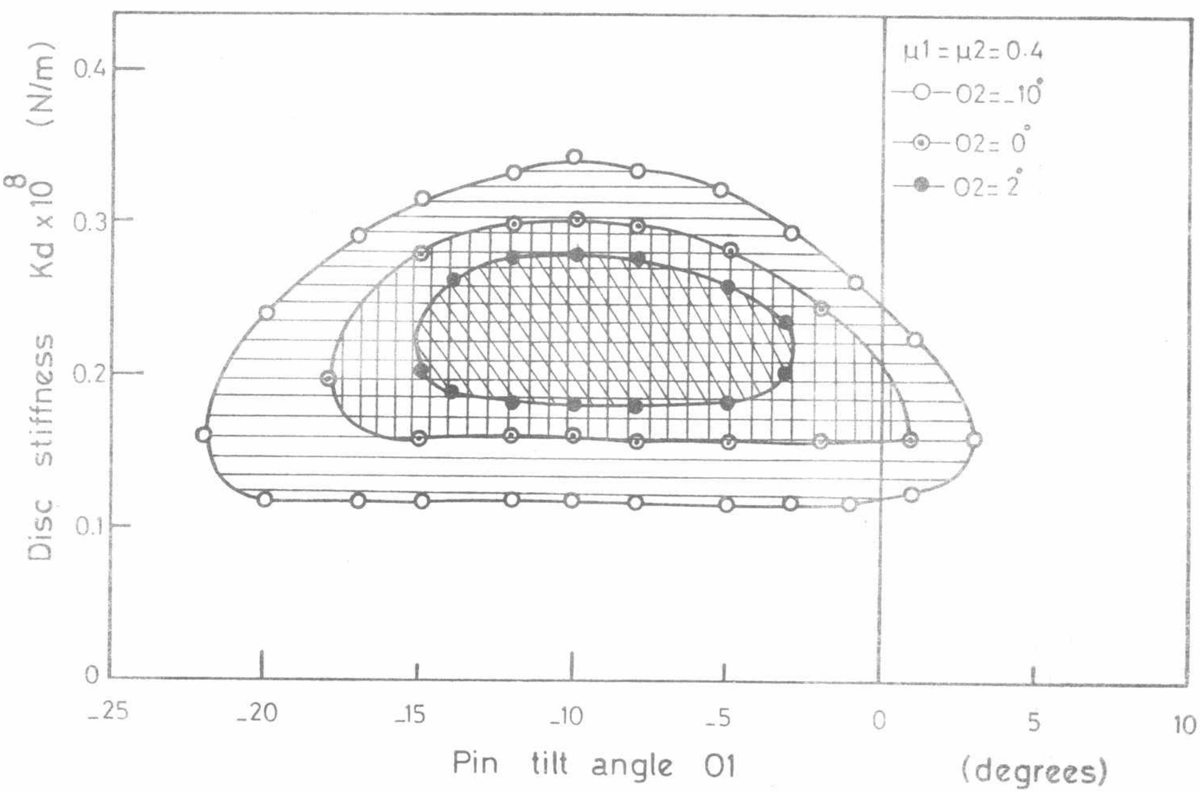

FIG. 9 Instability regions for the disc brake at moderate friction coefficients

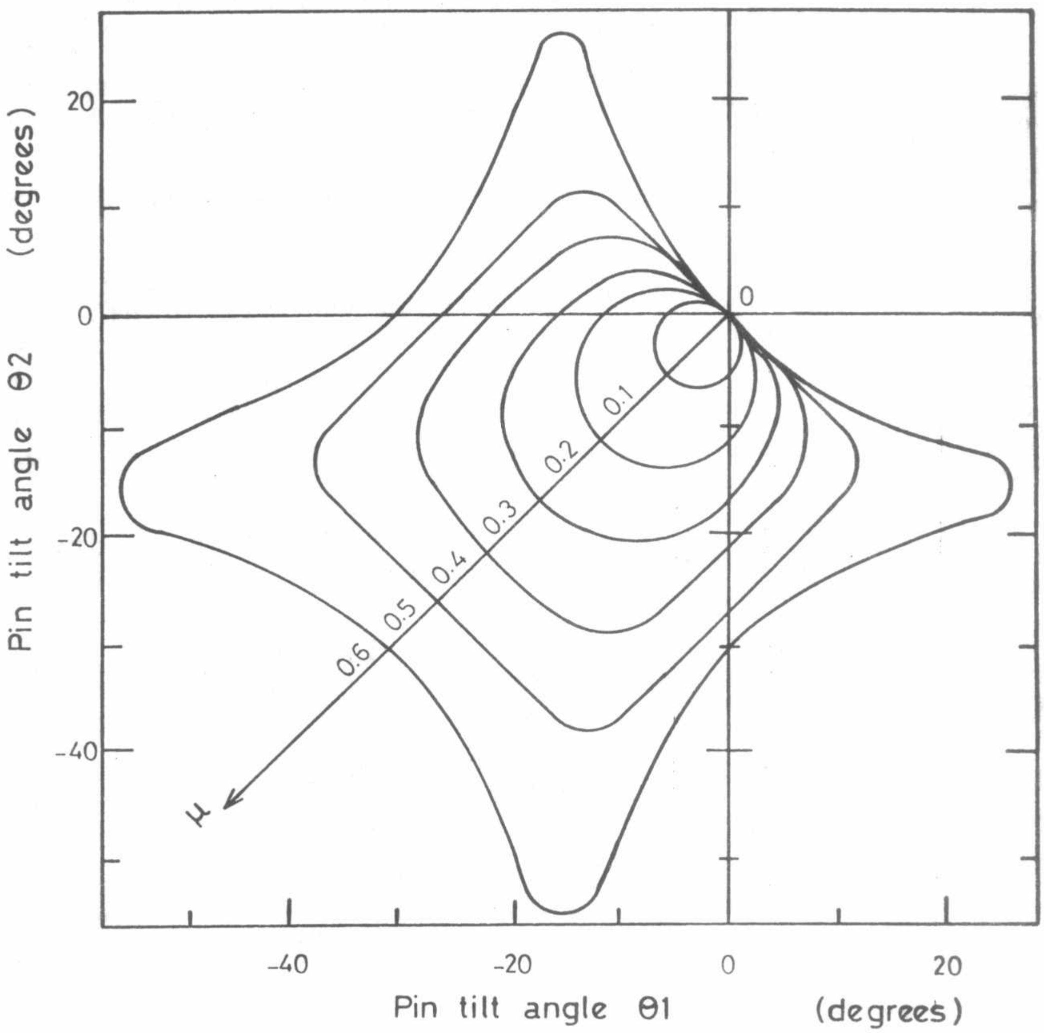

FIG. 10 Instability regions for the disc brake at different coefficients of friction

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