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A DESIGN ALGORITHM TO CALCULATE THE STATIC STIFFNESS OF MULTI-BOLTED JOINT.

Part. I Joint Stiffness Analysis

By

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ABSTRACT

The multi-bolted joint is commenly used in various machine design problems. In order to predict the static behaviour of the machine tool structures, it is necessary to know the stiffness of the joints between their components.

This paper is concerned with the establishment of mathematical models which enable to analysis the static behaviour of the multi-bolted joints in machine tools. Several assumptions have been taken into consideration to simplify the proposed mathematical models.

INTRODUCTION

Bolted joints are quite commonly encountered in various machine tool design problems. Determination of the static stiffness of the multi-bolted joints is of extreme importance in the design of such joints. The problem has not been satisfactory treated analytically.

Many investigators reported solutions based upon the experemental results; Wasner [15] , IzyKowski [8], and Plock [11] . The basic work in these investigations are that; testing of simple joint models to derive some experimental relations which can help the designer to prodict the magnitude of the static stiffness. Recently, Back [1] and Schulz [12] used the finite element technique for analysis of the deformation and the stress distribution in machine tool joint. The iteration cycles of the schulz-Calculations is very long into the relative deference of deformation smaller than the desired limit. Also, the system of stiffness matrix must be, for every new iteration step, constructed and inversed. Therefore, this solution is very complicated and required a long time for calculations using high capacity computers. It is clear that , no simple totally reliable formulae exsits that will give the designer the static stiffness value for the joints to be designed, at the specified conditions. Thus, the objective of this work is construction of mathematical models to calculate the static stiffness of the multi-bolted joint.

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DESIGN PARAMETERS OF THE MULTI-BOLTED JOINT AND ASSUMPTIONS:

Fig.(1) shows some typical examples of a multi-bolted joint used in machine tools, where the loads are transmitted a cross the joint interfaces. These examples of joint may be discriped by the following design parameters [2-4-6-7-9-10]:

1. Construction and quality of the contact surfaces of joint.
2. Geometry, arrangement and number of fastened bolts used.
3. Preload of fastened bolt.
4. Construction of bolt packet.
5. Distribution and value of the external loads.
6. Material.

These parameters have been taken into consideration to derive the proposed mathematical model, which has been constructed by the following assumptions:

1. In the contact interface, the stress is concentrated around the fastenning bolts [3-9-11-14].
2. The static stiffness of each fastened bolt in the multi-bolted joint is calculated by using the mathematical model of the single bolted joint. This model can be specified by the following equation [13] .

$$C_{sv} = C_2 + \frac{F_v - F_R}{\frac{F_v - F_R}{C_3} - \frac{\alpha}{A_k^m}(F_v^m - F_R^m)} \qquad \text{..............(1)}$$

3. The external loads are linear-distributed on the contact joint interface.
4. The non-linear deformation in the contact interface has been considered [1-5-10] .

Fig.(2) represents the geometric model of the multi-bolted joint. The calumn flange(1) is fastened with the machine base (2) by several fastening bolts (3). These bolts are made of same material and have the same geometry and dimensions. The fastening bolts are inserted in the flange hole with the suitable clearance to avoid any contact stresses due to assembly. The preload (F_v) is produced due to the bolt tightening by a torque rench to a designed magnitude. This preload is usually distributed on the joint interface (4) to produce the required tightening.

The external loads (F_x, F_y, F_z) are applied on the culumn wall(5) in the three-directions. The magnitudes of these loads are to be defined according to the working requirements of the machine.

These forces are transmitted accross the machine culumn to the joint members (flange1; base 2 ; bolt 3), which can be defined finally as the joint external loads (see Fig.3). The magnitude of joint loads may depend on the culumn rigidity and over-all dimensions. By analysis of the joint loads, the magnitude of the axial tension force (F_B) on each bolt in the joint could be calculated. Therefore, according to the single bolt model, the static stiffness of each bolt could be estimated.

LOAD DISTRIBUTION ON THE CONTACT JOINT INTERFACE:

Several load types are applied on the machine during the working. These loads are the cutting forces, the dynamic forces of the moving parts of the machine and other loads due to the machine enviromental conditions. By analysing all load types in the 3-directions systems, these loads can be classified into Tension, Turnover, Shear and Torsion loads, which affect the joint interface.

Fig.(3) shows the direction of the main forces and moments which affect the joint interface. The F_x and F_y represent the shear forces, the F_z represents the tension or compresion force, the M_x and M_y are the turnover moments and M_z is the torsion moment.

To derive the mathematical expersion which defined the axial tension load on every fastenning bolt in the joint, the following linear relation for the static stiffness has been used (See Fig. 4):

$$F_{Bi} = C_{svi} \cdot \delta_{B_i} = C_{svi} \cdot X_{ni} \cdot \varphi_{ys} \qquad \dots\dots(2)$$

$$M_{ys} = - \sum_{i=1}^{n} F_{Bi} \cdot X_{ni} \qquad \dots\dots(3)$$

Assuming that all fastenning bolts in the joint having the same initial tightening force; thus:

$$M_{ys} = - C_{sv} \cdot \varphi_{ys} \sum_{i=1}^{n} X_{ni}^2 \qquad \dots\dots(4)$$

Thereof;

$$F_{Bi} = - \frac{M_{ys}}{\sum_{i=1}^{n} X_{ni}^2} \cdot X_{ni} \qquad \dots\dots(5)$$

in the Y-direction; and also,

$$F_{Bi} = \frac{M_{xs}}{\sum_{i=1}^{n} Y_{ni}^2} \cdot Y_{ni} \qquad \text{.........................(6)}$$

in the x-direction. The axial load in z-direction may be also divided between the number of fastenning bolts to be used; i .e.

$$F_{Bi} = F_z/n \qquad \text{.........................(7)}$$

Therefore, the total applied force of the fastenning bolt(i) in the joint may be calculated by the following relationship:

$$F_{Bi} = - \frac{M_{ys} \cdot X_{ni}}{\sum_{i=1}^{n} X_{ni}^2} + \frac{M_{xs} \cdot Y_{ni}}{\sum_{i=1}^{n} Y_{ni}^2} + \frac{F_z}{n} \qquad \text{...................(8)}$$

LOCATION OF THE CENTROID POINT OF THE BOLT GROUP.

Fig.(4) represents the mathematical model of multi-bolted joint and the centroid point (s) of the bolt group. It is clear that, at the centroid point , the turnover moment M_{ys} is equal to zero,. Therefore;

$$M_{ys} = \delta_s \sum_{i=1}^{n} C_{svi} (X_{oi} - X_s) + \varphi_{ys} \sum_{i=1}^{n} C_{svi} (X_{oi} - X_s)^2 +$$

$$+ C_{svi} \cdot \varphi_{xs} \sum_{i=1}^{n} (X_{oi} - X_s)(Y_{oi} - Y_s) = 0 \qquad \text{...........(9)}$$

i.e, the first term equal zero, thus

$$X_s = \frac{\sum_{i=1}^{n} C_{svi} \cdot X_{oi}}{\sum_{i=1}^{n} C_{svi}} \qquad \text{.........................(10)}$$

and,

$$Y_s = \frac{\sum_{i=1}^{n} C_{svi} \cdot Y_{oi}}{\sum_{i=1}^{n} C_{svi}} \qquad \text{.........................(11)}$$

Where X_s and Y_s are the coordinates of the centroid point of the bolt group, which depends on the cooridinates of each fastenning bolt in the joint.

LOCATION OF THE MAIN JOINT COORDINATES.

The main joint coordinates are the coordinates which have pure tension, turnover, shear and torsion loads on the joint interface.

The location of these coordinates may be determined from the equation of the turnover moment as follows: (See Fig.4)

$$M_x = \delta_s \sum_{i=1}^{n} C_{svi} \cdot Y_i + \varphi_{ys} \sum_{i=1}^{n} C_{svi} \cdot X_i Y_i + \varphi_{xs} \sum_{i=1}^{n} C_{svi} \cdot Y_i^2 \quad \ldots\ldots\ldots(12)$$

It is clear that, the scond term with respect to the x-direction is very small. Therefore, it may be neglected. From Fig.(4), the following coordinate relations can be obtained:

$$Y_i = Y_{ni} \cdot \cos \varphi_s - X_{ni} \cdot \sin \varphi_s \quad \ldots\ldots\ldots(13)$$

$$X_i = X_{ni} \cdot \cos \varphi_s + Y_{ni} \cdot \sin \varphi_s \quad \ldots\ldots\ldots(14)$$

Substituting equations (13 and 14) in the second term of equation (12), the location of the main joint coordinate can be determined firstly as follows:

$$\tan 2\varphi_s = \frac{2 \sum_{i=1}^{n} C_{svi} \cdot X_{ni} \cdot Y_{ni}}{\sum_{i=1}^{n} C_{svi} \cdot X_{ni}^2 - \sum_{i=1}^{n} C_{svi} \cdot Y_{ni}^2} \quad \ldots\ldots\ldots(15)$$

Where φ_s is rotation angle of the main joint coordinates with respect to the centroid point coordinates.

If φ_{xs} in equation (9) is very small, the term $C_{svi} \sum_{i=1}^{n} (X_{oi} - X_s)(Y_{oi} - Y_s)$ tends to zero and also in the horizontal plane there is no vertical deflection , i.e, $\delta_s = 0$. Therefore, the equation (9) becomes;

$$M_{ys} = \varphi_{ys} \sum_{i=1}^{n} C_{svi} \, (X_{o_i} - X_s)^2 \quad \ldots\ldots\ldots(16)$$

or,

$$C\varphi_{ys} = M_{ys}/\varphi_{ys}$$

$$= \sum_{i=1}^{n} C_{svi} \cdot X_{ni}^2 \quad \ldots\ldots\ldots(17)$$

Also,

Also,

$$C\varphi_{xs} = \sum_{i=1}^{n} C_{svi} \cdot Y_{ni}^{2} \qquad \text{..........................(18)}$$

and ,

$$C_{xs.ys} = \sum_{i=1}^{n} C_{svi} \cdot X_{ni} \cdot Y_{ni} \qquad \text{..........................(19)}$$

Using the relations (17 , 18 , 19), the equation (15) becomes;

$$\tan 2\varphi_s = \frac{2 \; C\varphi_{xs.ys}}{C\varphi_{ys} - C\varphi_{xs}} \qquad \text{.................(20)}$$

MATHEMATICAL MODELS OF THE STATIC STIFFNESS BASED ON THE MULTI-BOLTED JOINT MODEL.

1. Static joint stiffness subject to tension force.

Fig.(4) represents the mathematical model used to determine the joint stiffness. From this Figure, the following formula can be obtained,

$$F_z = \sum_{i=1}^{n} C_{svi} \left[\delta_s + \varphi_{ys}(X_{oi} - X_s) \right]$$

$$= \delta_s \sum_{i=1}^{n} C_{svi} + \varphi_{ys} \sum_{i=1}^{n} C_{svi}(X_{oi} - X_s)$$

Thus,

$$\delta_s = \frac{F_z - \varphi_{ys} \sum_{i=1}^{n} C_{svi}(X_{oi} - X_s)}{\sum_{i=1}^{n} C_{svi}} \qquad \text{..............(21)}$$

where δ_s is the deflection of the centroid point(s) of the joint. Thereof, the static joint stiffness C_z against the tension force F_z may be determined as follows:

$$C_z = F_z / \delta_s$$

$$= \frac{F_z \sum_{i=1}^{n} C_{svi}}{F_z - \varphi_{ys} \sum_{i=1}^{n} C_{svi} (X_{oi} - X_s)} \qquad \text{...........(22)}$$

2. Turnover Stiffness:

Turnover stiffness of the joint can be determined from the moments M_x or M_y about the main joint coordinates as follows, (See Fig. 4)

$$M_y = \delta_s \sum_{i=1}^{n} C_{svi} \cdot Y_i + \varphi_{xs} \sum_{i=1}^{n} C_{svi} \cdot X_i \cdot Y_i + \varphi_{ys} \sum_{i=1}^{n} Y_i^2 \cdot C_{svi} \quad \text{.........(23)}$$

Thereof, the turnover stiffness can be defined as:

$$C\varphi_y = \frac{\partial M_y}{\partial \varphi_{ys}} = \sum_{i=1}^{n} Y_i^2 \cdot C_{svi} \quad \text{.................... (24)}$$

From Fig.(4) and the relations (13) , $C\varphi_y$ is obtained ,

$$C\varphi_y = \cos^2\varphi_s \sum_{i=1}^{n} X_{ni}^2 \cdot C_{svi} + \sin^2\varphi_s \sum_{i=1}^{n} Y_{ni}^2 \cdot C_{svi} - 2 \sin\varphi_s \cdot \cos\varphi_s \sum_{i=1}^{n} X_{ni} \cdot Y_{ni} \cdot C_{svi} \quad \text{...........(25)}$$

Using equations (17,18-19), equation (25) becomes;

$$C\varphi_y = C\varphi_{ys} \cdot \cos^2\varphi_s + C\varphi_{xs} \cdot \sin^2\varphi_s - 2 \sin\varphi_s \cdot \cos\varphi_s \cdot C\varphi_{xs.ys} \quad \text{..................(26)}$$

Also,

$$C\varphi_x = C\varphi_{xs} \cdot \cos^2\varphi_s + C\varphi_{ys} \cdot \sin^2\varphi_s + 2 \sin\varphi_s \cdot \cos\varphi_s \cdot C\varphi_{xs.ys} \quad \text{..................(27)}$$

Where $C\varphi_x$ is the turnover stiffness in X-direction and $C\varphi_y$ is the turnover stiffness in Y-direction.

3. Static Joint Stiffness Against The Shear Loading:

Joint stiffness against the shear forces F_x and F_y may be determined as a summation of the exisiting shear stiffness on each bolt in the joint (13). Thus,

$$C_{xsi} = \sqrt{\frac{A_K \cdot F_{Ri}}{R^2}} \quad \text{..................(28)}$$

where C_{xsi} is the shear stiffness of the bolt i in the joint. Thereof, the total joint stiffness can be defined as:

$$C_{xs} = \sum_{i=1}^{n} \sqrt{A_k . F_{Ri} / R^2} \quad \ldots\ldots(29)$$

where R is constant after kirsanova [9].

Where the shear stiffness of the same magnitude in X-and Y-directions is

$$C_x = C_y = C_{xs} \quad \ldots\ldots(30)$$

4. Torsional Stiffness:

Fig.(5) represents the mathematical model to determine the torsional joint stiffness. To simplify the model, the following assumptions have been made:

a- No plastic deformation occurres in the applied torsional stress; i.e. all deformation are elastic.

b- The resultant normal stress in the jointing surface is equally distributed.

With these assumption, the torsion moment acting on any bolt in the joint can be defined as follows (See Fig. 5);

$$\delta T_i = M_{zi} / R \, . \, C_{xsi}$$

$$M_{zi} = \frac{\delta T_i}{R_i} . R_i^2 . C_{xsi}$$

$$= \varphi_{zi} . R_i^2 . C_{xsi} \quad \ldots\ldots(31)$$

Therefore, the torsional stiffness of bolt i is;

$$C\varphi_{zi} = M_{zi} / \varphi_{zi} = R_i^2 . C_{xsi} \quad \ldots\ldots(32)$$

Accordingly, the torsional stiffness of the joint may be determined from the following equation.

$$C\varphi_z = \sum_{i=1}^{n} (X_{ni}^2 + Y_{ni}^2) \sqrt{A_k . F_{Ri} / R^2} \quad \ldots\ldots(33)$$

RESULTS:

The developement in machine tools required many investigations, which defined in a simplified method the problem and its acceptable solving. Thus, a model of the multi-bolted joint has been proposed based on the main joint design parameters. A simplified mathematical model to calculate the static stiffness of the multi-bolted joint has been derivaited in this paper. The advantages of the proposed models against the experimental

a- No material and manufacturing costs required.
b- The required time and effort is very low.
c- It represents the better data to make a comparison between the different joint models.

A suitable mathematical method to evaluate the joint stiffness with respect to the machine rigidity requirements has been developed to use easily by the machine designers.

A comparison of the computed values with the experimental data and the proposed design algorithms to calculate the static stiffness will be discussed later in another paper *part*.

NOMENCLATURE:

A_k : Contact area (mm^2)

C_z : Stiffness of the bolt under external load ($N/\mu m$).

C_3 : Stiffness of the members to be jointed under external load ($N/\mu m$).

C_k : Stiffness of the joint contact surface ($N/\mu m$)

C_{sv} : Single-bolted joint stiffness ($N/\mu m$).

C_{xs} : Shear stiffness of single-bolted joint ($N/\mu m$).

$C\varphi_{zi}$: Torsional stiffness of single-bolted joint (N m/ rad)

C_x : Static shear stiffness of the multi-bolted joint in X-direction ($N/\mu m$).

C_y : Static shear stiffness of the multi-bolted joint in Y-direction ($N/\mu m$).

C_z : Static tension stiffnessof the multi-bolted joint in Z-direction ($N/\mu m$).

$C\varphi_x$: Static turnover stiffness about X-direction(N.m/ rad).

$C\varphi_y$: Static turnover stiffness about Y-direction (N.m/ rad).

$C\varphi_z$: Static torsion stiffness (N.m/ rad).

F_B :: External tension load (N).

F_V : Preload on bolt due to tightenning (N).

F_R : Resultant load on themembers to be jointed (N).

$F_{x,y,z}$: Forces acting on the joint in x-; y- and z-direction. (N.m).

n : Number of bolts in the joint.

R : Constant after Kirsanova ($mm^2/\sqrt{N}$)

R_i : Radial coordinate of bolt i in the joint (mm).

S : Centeriod point of the multi-bolted joint.

X,Y,Z : Main joint coordinates.

X_o, Y_o, Z_o : Basis joint coordinates.

X_n, Y_n, Z_n : Neutral joint coordinates.

X_i , Y_i : Coordinates of the bolt i in the joint-w.r.t. the main coordinates.

X_{oi} , Y_{oi} : Coordinates of the bolt i in the joint w.r.t. the Basis coordinates.

X_{ni} , Y_{ni} : Coordinates of the bolt i in the joint w.r.t. the neutral-coordinates.

δ_B : Normal deflection under the external load (μm).

δ_{Ti} : Torsion deformation of bolt i (μm).

φ_s : Rotation angle of the main joint coordinates (Grad).

φ_{xs} : Turnover angle of the joint in X-direction (rad).

φ_{ys} : Turnover angle of the joint in Y-direction (rad).

φ_{zi} : Torsion angle of bolt i (Grad).

α, m : Constants.

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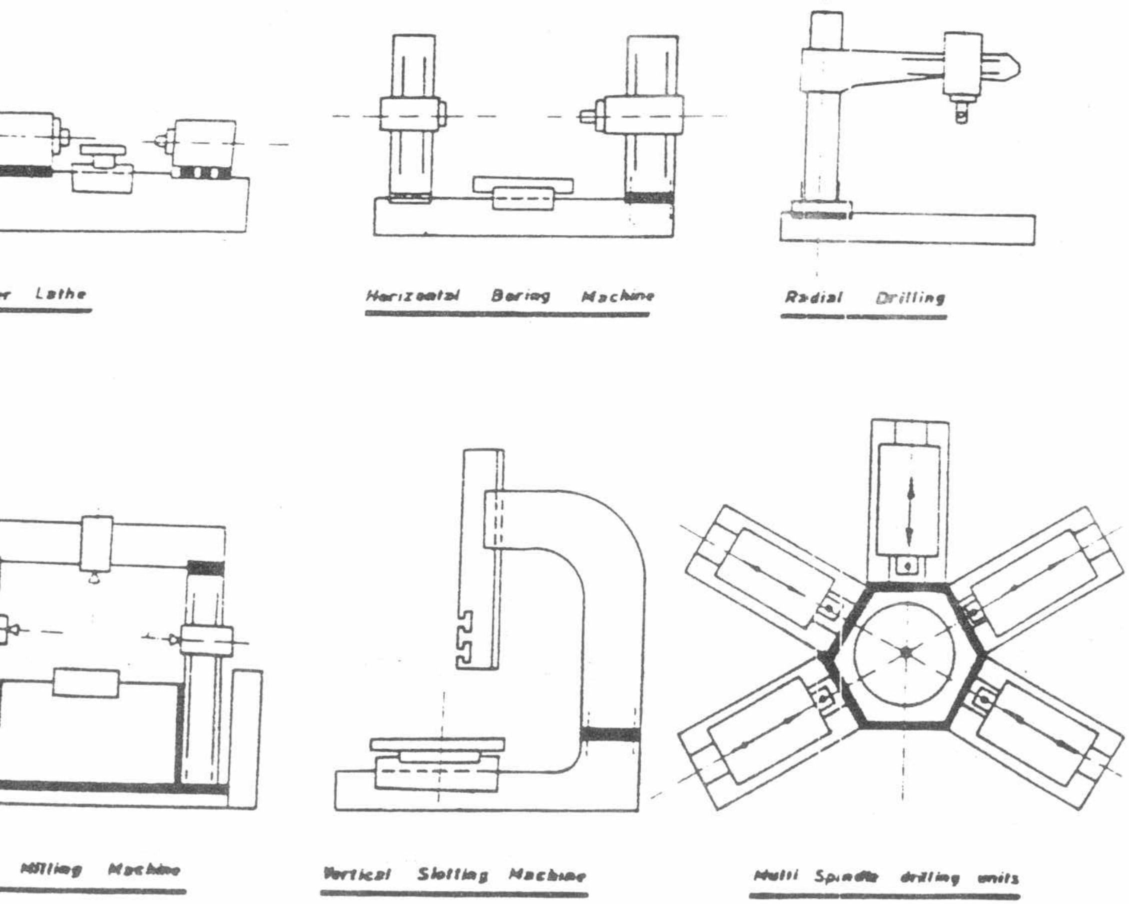


1) Examples of bolted joints in machine tool structures.

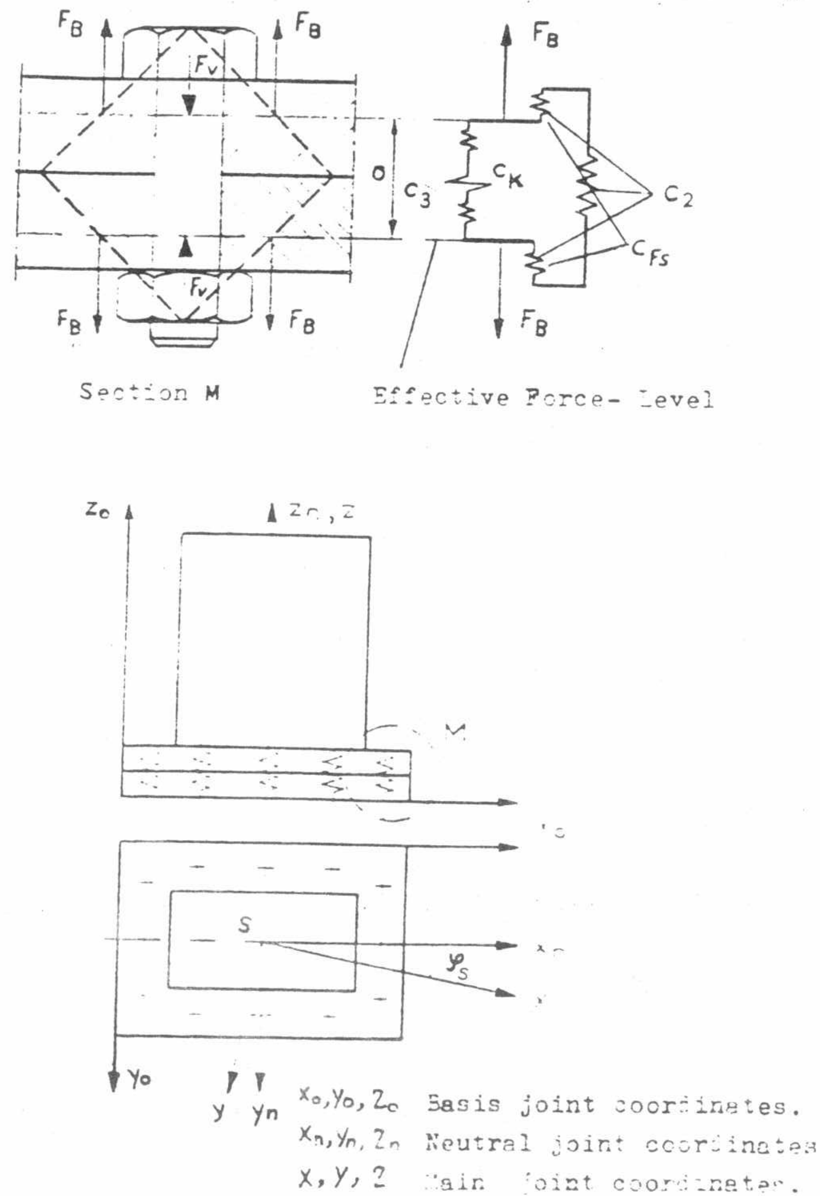


Fig.(2) Multi-bolted joint.

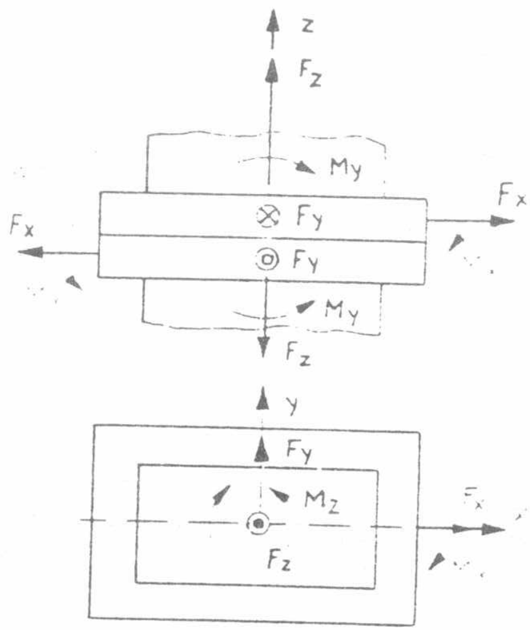
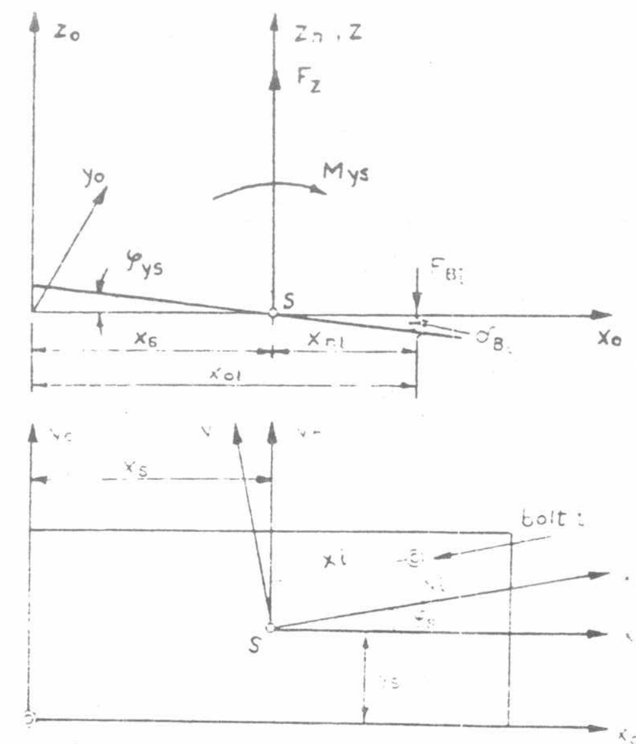


Fig.(3) Effecting load types on the joint.



x_o, y_o, z_o Basis joint coordinates.

x_n, y_n, z_n Neutral joint coordinates.

x, y, z Main joint coordinates

Fig.(4) Mathematical model of tension turnover stiffness.

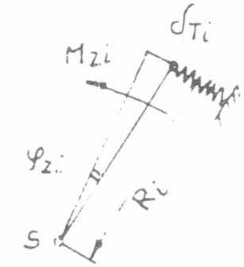


Fig. 5) Mathematical model of the torsional joint stiffness.